SIMULATIONS OF MAGNETIC-FLUX TRANSPORT IN SOLAR
ACTIVE REGIONS

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Abstract. We simulate the evolution of several observed solar active regions by solving a transport equation for magnetic flux at the photosphere. The rates of rotation, meridional flow, and diffusion of the flux are determined self-consistently in the calculations. Our findings are in good quantitative agreement with previous measures of the rotation rate and diffusion constant associated with photospheric magnetic fields. Although our meridional velocities are consistent in direction and magnitude with recently reported poleward flows, relatively large uncertainties in our velocity determinations make this result inconclusive.

1. Introduction

The transport model for the magnetic field at the solar photosphere originated in the effort to understand the observed shearing and spreading of the flux in active regions (Babcock and Babcock, 1955; Babcock, 1961). Leighton (1964) calculated the effects of differential rotation and diffusion by the supergranulation on idealized bipolar regions, and he was able to account for the basic facts about their evolution. Later, Schatten et al. (1972) used the surface flux distribution measured at the Mount Wilson Observatory to initialize the model, and they simulated the subsequent evolution of the field over a nine-month interval. They found good agreement between the large-scale features of the simulated and observed fields.

In their calculations, Leighton and Schatten et al. specified a priori the value of the diffusion constant and the variation of the rotation rate with latitude. Leighton had estimated the diffusion constant to lie in the range 770–1540 km$^2$ s$^{-1}$ from his simulations of the reversal of the polar magnetic fields during the sunspot cycle. The rate of rotation had been determined by Newton and Nunn (1951) from observations of long-lived sunspots.

Subsequently, Mosher (1977) estimated the rate of increase of area covered by the field in active regions to derive a value of 200–400 km$^2$ s$^{-1}$ for the diffusion constant. So that the polar fields would reverse at the appropriate epoch in the cycle, he postulated the existence of a poleward meridional surface flow of about 3 m s$^{-1}$. Since then,

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possible evidence for a poleward flow of magnitude 10–20 m s$^{-1}$ has been reported by several investigators (Duvall, 1979; Howard, 1979; Howard and LaBonte, 1981; LaBonte and Howard, 1982; Topka et al., 1982). Snodgrass (1983) recently measured the rate of rotation of general magnetic features by cross-correlating Mount Wilson magnetograms. His rate is in good agreement with the Newton and Nunn sunspot rate at low solar latitudes.

In this paper, we shall simulate the evolution of a sample of magnetic regions observed at the National Solar Observatory at Kitt Peak. As part of our modeling procedure, the parameters of the transport model will be self-consistently determined. We shall find good agreement with Mosher's estimate for the diffusion constant and with Snodgrass' expression for the rotation rate. The direction and magnitude of our best-fit meridional velocities are generally consistent with the findings of Duvall, Howard, and the others. However, uncertainties in our derived velocities at the 20 m s$^{-1}$ level render this result inconclusive.

2. Procedure

We began by selecting several large, relatively isolated magnetic regions which were observed near central-meridian passage for at least three consecutive solar rotations. We acquired digitized, full-disk magnetograms which were taken as part of the daily observing program at the National Solar Observatory's Vacuum Telescope (Livingston et al., 1976). The measured line-of-sight values of the magnetic field were corrected for the projection angle as seen from Earth, assuming that the field is radial at the photosphere (cf. Howard and LaBonte, 1981). We then interpolated the radial magnetic fluxes onto the full-sphere simulation grid of 128 × 256 pixels, uniformly spaced in latitude and in longitude. The data were provided on a grid of 180 × 180 pixels, uniformly spaced in sine of latitude and in longitude.

For each of the regions, we extracted pairs of magnetograms which were suitable for use with the model. Many possible pairs had to be excluded from the sample due to the emergence of substantial amounts of new magnetic flux between the observations. For each selected pair of magnetograms, we use the data from earlier observation to initialize the magnetic field in the model and use the data from the later observation to evaluate the simulated field at the later time. By adjusting the parameters of the model and repeating the calculation, we determine the values which give the best agreement between the observed and simulated fields.

The transport equation for the magnetic field is (Leighton, 1964; DeVore et al., 1984)

$$\frac{\partial B}{\partial t} + \frac{1}{R \sin \theta} \frac{\partial}{\partial \theta} \left( B v(\theta) \sin \theta \right) + \omega(\theta) \frac{\partial B}{\partial \phi} = \kappa \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial B}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 B}{\partial \phi^2} \right]$$

where $v(\theta)$ is the meridional velocity; $\omega(\theta)$, the angular rate of rotation; $\kappa$, the diffusion
constant; \( R \), the radius of the Sun. We solve this equation by specifying the initial distribution of flux over the surface, the profiles for the rotation and meridional flow, and a value for the diffusion constant. At each step of the calculation, we increment the time and evolve the field according to Equation (1). We solve it using a fast Fourier transform along azimuths, and a flux-conserving convection algorithm and an implicit diffusion algorithm along meridians.

For each pair of observations, we simultaneously optimize the rate of rotation and the meridional velocity. We assume that the profiles take the forms

\[
\omega(\theta) = \omega_0 - 2.77 \cos^2 \theta \text{ deg day}^{-1}
\]  

(2)

and

\[
v(\theta) = v_0 \sin 2\theta \text{ m s}^{-1},
\]  

(3)

where \( \omega_0 \) and \( v_0 \) are constants to be determined. Our assumed coefficient of differential rotation was measured by Newton and Nunn (1951), who found

\[
\omega(\theta) = 14.38 - 2.77 \cos^2 \theta \text{ deg day}^{-1},
\]  

(4)

for the sidereal angular rotation rate of sunspots. At the later observation of each pair, we calculate the centroidal coordinates of the total flux, in latitude and longitude, for both the observed and simulated field patterns. The best-fit values of \( \omega_0 \) and \( v_0 \) are determined by trial and error as the values which cause these centroids to coincide. This procedure may be carried out for any reasonable value for \( \kappa \), the diffusion constant having essentially no effect on the position of the centroid of flux of the region.

We optimize the diffusion constant for fixed rotation and meridional flow profiles, using the expressions in Equations (2) and (3) and the best-fit values of \( \omega_0 \) and \( v_0 \). At the later observation, we calculate a correlation coefficient \( C \) defined by (cf. Bevington, 1969)

\[
C \equiv \frac{\sigma_{os}^2}{\sigma_{oo} \sigma_{ss}},
\]  

(5)

where

\[
\sigma_{os}^2 \equiv \langle os \rangle - \langle o \rangle \langle s \rangle
\]  

(6)

and

\[
\langle x \rangle \equiv N^{-1} \sum_{k=1}^{N} x_k;
\]  

(7)

\( o \) and \( s \) represent the observed and simulated fluxes, respectively, on the individual pixels, \( k \) is a pixel index, and \( N \) is the number of pixels overlying the region. The best-fit value of the diffusion constant is determined by trial and error as the value which maximizes this correlation coefficient.

One measure of the fit of the simulated to the observed field patterns is the maximum value of the correlation coefficient already determined. Another which we shall use is
the relative error in the total surviving flux of the region. It is defined by

\[
\varepsilon = \left( \sum_{k=1}^{N} |s_k| - \sum_{k=1}^{N} |o_k| \right) \div \left( \sum_{k=1}^{N} |o_k| \right),
\]

where \(o\) and \(s\) are again the observed and simulated fluxes.

3. Results

We obtained magnetic data for the seven regions listed in Table I. Our sample spans a wide range in heliographic latitude and dates from the declining phase of sunspot cycle 20 through the rising phase of cycle 21, when the Sun was relatively quiet. From the data for these regions, we selected fifteen pairs of magnetograms which were suitable for use with the model. The other possible combinations of deposition and comparison dates were ruled out to either the injection of additional new flux into the regions or the emergence of new regions in close proximity to those in the sample.

We applied the modeling procedure to these fifteen pairs of magnetograms and obtained the results listed in Table II. The rotation rate and meridional velocity in the table were determined by evaluating Equations (2) and (3) at the average centroidal latitude of the region over the simulation interval. We display the rates of rotation, the meridional velocities, and the values for the diffusion constant as data points in Figure 1.

**TABLE I**

<table>
<thead>
<tr>
<th>Pair no.</th>
<th>Initial latitude</th>
<th>Initial longitude</th>
<th>Initial rotation</th>
<th>Initial date</th>
<th>Deposition date</th>
<th>Comparison date</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>19 Aug., 1977</td>
<td>15 Sep., 1977</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14 Jan., 1977</td>
<td>9 Mar., 1977</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>23 Feb., 1977</td>
<td>23 Mar., 1977</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14 Feb., 1977</td>
<td>13 Apr., 1977</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14 Mar., 1977</td>
<td>13 Apr., 1977</td>
</tr>
</tbody>
</table>
Listed for each of the fifteen pairs of observations used are the average heliographic latitude over the simulation interval; the best-fit values for the sidereal rotation rate (deg day$^{-1}$), meridional flow velocity (m s$^{-1}$), and diffusion constant (km$^2$ s$^{-1}$); the correlation coefficient between the simulated and observed fields; and the percentage error in the total surviving flux.

<table>
<thead>
<tr>
<th>Pair No.</th>
<th>Latitude</th>
<th>Rotation rate</th>
<th>Meridional velocity</th>
<th>Diffusion constant</th>
<th>Correlation coefficient</th>
<th>Flux error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5 N</td>
<td>14.45</td>
<td>0.0</td>
<td>425</td>
<td>0.62</td>
<td>+ 3</td>
</tr>
<tr>
<td>2</td>
<td>12 N</td>
<td>14.29</td>
<td>7.8</td>
<td>300</td>
<td>0.66</td>
<td>+ 5</td>
</tr>
<tr>
<td>3</td>
<td>20 S</td>
<td>14.06</td>
<td>9.4</td>
<td>275</td>
<td>0.71</td>
<td>+ 20</td>
</tr>
<tr>
<td>4</td>
<td>23 S</td>
<td>14.02</td>
<td>5.6</td>
<td>375</td>
<td>0.62</td>
<td>- 17</td>
</tr>
<tr>
<td>5</td>
<td>23 S</td>
<td>14.15</td>
<td>12.7</td>
<td>150</td>
<td>0.81</td>
<td>+ 55</td>
</tr>
<tr>
<td>6</td>
<td>28 S</td>
<td>13.86</td>
<td>19.3</td>
<td>325</td>
<td>0.65</td>
<td>+ 8</td>
</tr>
<tr>
<td>7</td>
<td>30 S</td>
<td>13.77</td>
<td>6.4</td>
<td>175</td>
<td>0.68</td>
<td>+ 25</td>
</tr>
<tr>
<td>8</td>
<td>30 S</td>
<td>13.75</td>
<td>6.4</td>
<td>375</td>
<td>0.66</td>
<td>- 18</td>
</tr>
<tr>
<td>9</td>
<td>31 S</td>
<td>13.77</td>
<td>1.6</td>
<td>425</td>
<td>0.60</td>
<td>- 34</td>
</tr>
<tr>
<td>10</td>
<td>33 N</td>
<td>13.62</td>
<td>15.3</td>
<td>1100</td>
<td>0.51</td>
<td>- 22</td>
</tr>
<tr>
<td>11</td>
<td>34 N</td>
<td>13.47</td>
<td>13.7</td>
<td>1050</td>
<td>0.42</td>
<td>- 37</td>
</tr>
<tr>
<td>12</td>
<td>35 N</td>
<td>13.36</td>
<td>20.6</td>
<td>425</td>
<td>0.53</td>
<td>- 30</td>
</tr>
<tr>
<td>13</td>
<td>43 S</td>
<td>12.80</td>
<td>12.1</td>
<td>275</td>
<td>0.58</td>
<td>+ 2</td>
</tr>
<tr>
<td>14</td>
<td>45 S</td>
<td>12.62</td>
<td>12.3</td>
<td>175</td>
<td>0.39</td>
<td>+ 71</td>
</tr>
<tr>
<td>15</td>
<td>46 S</td>
<td>12.56</td>
<td>14.2</td>
<td>275</td>
<td>0.62</td>
<td>- 10</td>
</tr>
</tbody>
</table>

The best-fit angular rotation rates clearly exhibit the characteristic decrease from equator to pole of other measures of the rotation of the Sun (cf. Howard, 1978). Moreover, the agreement with Snodgrass’ (1983) result for the magnetic rotation of the photosphere is quantitatively very good, to within 2%. He found the sidereal rate

\[
\omega(\theta) = 14.37 - 2.30 \cos^2 \theta - 1.62 \cos^4 \theta \text{ deg day}^{-1},
\]

shown as the solid curve in Figure 1.

The best-fit meridional velocities are all poleward-directed, with speeds ranging from 0 to 21 m s$^{-1}$. The direction and magnitude of the flow are generally consistent with the results of several recent studies (Duvall, 1979; Howard, 1979; Howard and LaBonte, 1981; LaBonte and Howard, 1982; Topka et al., 1982). However, as can be seen in Figure 1, there is no systematic variation of the flow speed with latitude. Indeed, the two values determined at 31° S and 35° N span essentially the entire range of speeds in the sample.

The best-fit diffusion constants, with two exceptions, lie in the range 150–425 km s$^{-1}$. These values are in good agreement with Mosher’s (1977) estimated range of 200–400 km$^2$ s$^{-1}$, which is shown in Figure 1. The remaining two values exceed 1000 km$^2$ s$^{-1}$. They lie within Leighton’s (1964) estimated range of 770–1540 km$^2$ s$^{-1}$, a portion of which is also shown.
The correlation coefficients range from a minimum of 0.39 to a maximum of 0.81. Their average value is 0.60. The errors in the total surviving flux range from $-37\%$ to $+71\%$. Their mean signed value is $+1\%$, and their mean absolute value is $24\%$.

4. Discussion

We have obtained values for the parameters of the flux-transport model which are in good quantitative agreement with previous measures of the rates of rotation (Snodgrass, 1983) and dispersal (Mosher, 1977) of the magnetic field at the solar photosphere. Our

Fig. 1. Best-fit values of the sidereal angular rotation rate in deg day$^{-1}$ (upper left) and the meridional flow velocity in m s$^{-1}$ (upper right), evaluated at the centroid of total flux, and of the diffusion constant in km$^2$ s$^{-1}$ (lower) are shown as data points. Snodgrass' (1983) rotation rate, Mosher's (1977) range for the diffusion constant, and part of Leighton's (1964) range for the diffusion constant are also shown for comparison.
results are also consistent in direction and order of magnitude with reported rates of meridional convection of the photospheric plasma (Duvall, 1979; Howard, 1979; LaBonte and Howard, 1982), the photospheric magnetic field (Howard and LaBonte, 1981), and Hx filaments (Topka et al., 1982). However, the variation in the meridional velocities over the sample is comparable to the peak velocity obtained and does not reflect any systematic dependence of the flow speed upon latitude.

Our best-fit angular rotation rates differ from the rate measured by Snodgrass (1983) by less than 2% at all latitudes. The maximum absolute error is 0.20 deg day\(^{-1}\). However, these relatively small differences in the angular rates correspond to residual rotational speeds as large as 20 m s\(^{-1}\). We find similar differences among the best-fit rates themselves, in cases in which multiple determinations are made over a narrow range in latitude. Thus the variation in the rotational velocities is of the same magnitude as that in the meridional velocities, suggesting a common origin.

We may readily estimate the uncertainty in the velocities due to the finite numerical resolution of the simulations. The position of the centroid of total flux is only known to within one-half of a pixel width, or 0.7°, at both the initial and final observations. Over a 27-day rotation the corresponding velocity uncertainty is ± 7.3 m \(^{-1}\), while over a two-rotation interval it is only ± 3.6 m s\(^{-1}\). Random numerical error arising from the limited resolution can, therefore, account for some, but not all, of the variation in the best-fit velocities.

It may be significant that the residual rotational velocities and the meridional velocities vary substantially from one region to another, but are rather consistent from rotation to rotation for a given region. Indeed, the variation among the best-fit values for the individual regions generally falls within the limits imposed by the numerical resolution. This suggests that the differences from region to region are systematic rather than random. Although it is conceivable that our procedure leads to best-fit velocities which are systematically in error, there are no obviously suspect steps in the analysis. Alternatively, these differences may be real, originating in some nonstationary or nonaxisymmetric pattern of convection on the Sun. There is insufficient evidence from our small sample, however, to provide compelling support for this hypothesis.

Our results for the rotational and meridional velocities should, therefore, be considered uncertain at the 20 m s\(^{-1}\) level. Consequently, the relatively minor differences between our best-fit rotation rates and Snodgrass’ rate may not be meaningful. At the same time, however, our measures of the meridional flow must be regarded as inconclusive.

Of our fifteen best-fit values for the diffusion constant, thirteen lie within or adjacent to Mosher’s (1977) range of 200–400 km\(^2\) s\(^{-1}\). This is surprisingly good agreement, considering that very different methods were used in the two studies. The total-flux errors associated with the best-fit diffusion constants are rather large in absolute value, yet average to nearly zero over the sample. This suggests that our procedure neither consistently under- nor overestimates the diffusion constant, but does allow substantial random scatter in the best-fit values. Determining the diffusion constant by requiring the total-flux error to vanish would not eliminate this scatter, however. For example,
the diffusion constant would have to be increased from 425 km$^2$ s$^{-1}$ for the region at 5$^\circ$ N and decreased from 275 km$^2$ s$^{-1}$ for the region at 46$^\circ$ S, in order to meet this alternate criterion.

The remaining two values for the diffusion constant lie within Leighton's (1964) range of 770–1540 km$^2$ s$^{-1}$. Both are associated with the first month of existence of a single, magnetically complex region. It initially appeared as a pair of neighbouring bipolar structures, but by its next central-meridian passage the region had evolved into a simple bipolar configuration. The best-fit diffusion constant exceeded 1000 km s$^{-1}$ over its first rotation, but decreased to only 425 km$^2$ s$^{-1}$ over its second. It is possible that additional activity in the region during the first rotation may have biased the determinations of the diffusion constant. However, an inspection of the 8-cm photographic prints of the series of magnetograms revealed no evidence for such activity while the region was in view from Earth.

The correlation coefficients associated with our best-fit diffusion constants are consistently rather small. A comparison of the field contours in the regions invariably reveals the presence of small-scale structure in the observations which is absent in the simulations, yet the large-scale organization of the field is very similar. Schatten et al. (1972) also noted this characteristic in their calculations with the model, and a turbulence analysis of the evolution of the photospheric magnetic field (DeVore et al., 1984) suggests that corrections to the transport equation are likely to become important on spatial scales comparable to the radius of supergranules. Evidently, this inability of the model to describe the fine structure in the field is responsible for the relatively poor correlation between the observed and simulated distributions of flux. It may thereby be the cause of much of the scatter in our best-fit diffusion constants.

Finally, we recall that Leighton obtained his estimate for the diffusion constant by timing the reversal of the polar magnetic fields in simulations of the sunspot cycle. When Mosher later concluded from his measurements of active regions that the diffusion constant was much smaller than Leighton's value, he estimated that a poleward meridional flow of about 3 m s$^{-1}$ was necessary to supplement the reduced rate of diffusive transport of flux. The peak flow speed of 10–20 m s$^{-1}$ deduced from recent observations is, however, substantially greater than he required, and such a flow could lead to an early reversal of the polar fields during the cycle, even at Mosher's rate of diffusion. Thus, simulations of the magnetic field over an entire sunspot cycle, which Leighton used to study the supergranular diffusion, might also provide some constraints on the solar meridional circulation.

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