The surface gravity of Arcturus from MgH lines, strong metal lines and the ionization equilibrium of iron

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Summary. The surface gravity of Arcturus is estimated from the strengths of MgH features (the magnesium abundance being derived from MgI lines), from strong metal lines, and from the FeI — FeII ionization equilibrium. The MgH lines give log $g = 1.8$ (cgs units), for the effective temperature of 4375 K found by Frisk et al. This value of log $g$ is consistent with the gravity derived from a sample of strong pressure-broadened lines of FeI, CaI and NaI which give log $g = 1.6$, and what we obtain from the ionization equilibrium of Fe, log $g = 1.5$. Reasons for the differences are discussed. The mass of Arcturus is found to be 0.95 $M_\odot$ when the gravity determination from MgH is used. The effective temperature is uncertain by about 50 K; a reduction of the temperature by this amount reduces the logarithmic gravity found from MgH to 1.6 and the mass to 0.7 $M_\odot$. It is concluded that the MgH features offer promising possibilities for determining gravities of late-type stars, when good estimates of effective temperatures are available.

1 Introduction

The two methods commonly used to determine the surface gravities of K giants are based on the ionization equilibria of elements such as Fe and the strengths of the wings of strong lines. Both of these methods require knowledge of the stellar temperature. At a given temperature, a model with lower gravity will have lower electron pressure and a lower gas pressure at the same optical depth. The change of electron pressure causes a change in ionization. The gravity of the star is then found by demanding that the abundances found from ionized lines equal those found from neutral lines. The collision broadening of spectral lines depends upon the gas pressure. Consequently, the strengths of the wings of strong lines can be used as a measure of the gas pressure and hence the surface gravity, although other forms of line broadening must also be taken into account.

A third method for finding the gravities of at least the cooler K giants is the use of"
molecular features, which can be used since many molecular equilibria are also pressure sensitive. In the present paper we have used MgH to find the gravity of Arcturus. MgH is well known as a luminosity discriminant in cool stars Öhman 1934; Spinrad & Wood 1965). We have used Mg i lines to determine the Mg abundance. Similar work could be carried out using other molecular features, e.g. OH, where the oxygen abundance could be found from the forbidden O i lines. We plan to examine these and other possibilities in later publications. It is of interest to compare the results from MgH/Mg i with those from other methods, based on atomic lines, of estimating spectroscopic gravities. Such a comparison should give valuable guidance as regards what methods should be preferred in the analysis of red giants in general. We have therefore undertaken such a comparison, and the results will be presented below.

The temperature of Arcturus has recently been discussed by Frisk et al. (1982), who deduce \( T_{\text{eff}} = 4375 \) K with an uncertainty of about 50 K. In order to judge the importance of the value adopted for \( T_{\text{eff}} \) on the final results, we have carried out our calculations for \( T_{\text{eff}} \) values ranging from 4375 to 4250 K, the latter value being a generally accepted one until recently.

2 Observations and calculations for magnesium

In order to determine the gravity of Arcturus from MgH features, we must first establish the Mg abundance. The solar abundance of Mg has recently been determined by Lambert & Luck (1978), who used both Mg i and Mg ii lines. We restrict our Arcturus analysis to lines which are contained both in their Mg i list and the lists of measurements of Arcturus equivalent widths of Mackle et al. (1975a). The five lines used are identified in Table 1. The \( gf \)-values for Lambert & Luck (1978) were adopted. The mean solar Mg abundance was found by the same authors to be 7.62 and the five lines in our analysis give 7.61. Curves of growth were computed for these lines for the following series of model atmospheres (denoted by \( T_{\text{eff}}, \log g \) and \([A/H]\)): 4350/0.75/−0.5, 4350/1.5/−0.5, 4350/2.0/−0.5, 4250/0.75/−0.5, 4250/1.5/−0.5 and 4250/2.25/−0.5. The models used were either from the grid of Bell et al. (1976) or were computed using the methods of Gustafsson et al. (1975) and are thus fully consistent with the Bell et al. grid. In addition to these models, some other models with special abundances or opacity sources were used (see below). The microturbulence velocity used was 1.7 km s\(^{-1}\). The Mg abundance deduced for each model is given in Table 2. These values are considerably lower than \([Mg/H] = −0.20\) found by Mackle et al. (1975b); to compare with \([Mg/H] = −0.37\) which is deduced from Table 2 after interpolation to their parameters (\( T_{\text{eff}} = 4260 \) K, \( \log g = 0.9 \)). The Mg/Fe abundance ratio of Arcturus is still significantly greater than the solar value (see below).

We computed the abundance of MgH in our Arcturus model atmospheres using the relevant values of the Mg abundance. We adopted a dissociation energy \( D_0 \) of 1.27 eV (Balfour & Lindgren 1978). The molecular partition function was obtained from Dreiling (1981, private communication), being based on the molecular constants of Balfour & Cartwright (1976) and of Rosen (1970). Dreiling gives the fit

\[
\log \Omega_{\text{MgH}} = 4.72191 - 2.76463 \log T + 0.669345 (\log T)^2
\]

Table 1. Mg I lines.

<table>
<thead>
<tr>
<th>( \lambda ) (Å)</th>
<th>( \log g f \lambda )</th>
<th>( \chi ) (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6318.7</td>
<td>1.83</td>
<td>5.11</td>
</tr>
<tr>
<td>6319.2</td>
<td>1.60</td>
<td>5.11</td>
</tr>
<tr>
<td>7657.6</td>
<td>2.60</td>
<td>5.11</td>
</tr>
<tr>
<td>8712.7</td>
<td>2.85</td>
<td>5.93</td>
</tr>
<tr>
<td>8717.8</td>
<td>3.08</td>
<td>5.93</td>
</tr>
</tbody>
</table>
Table 2. Chemical abundances of Arcturus found from different models.

**Constant microturbulence velocity 1.7 km/s:**

<table>
<thead>
<tr>
<th>Model</th>
<th>[Mg/H] from Mg I</th>
<th>[Fe/H] from Fe I</th>
<th>[Fe/H] from Fe II</th>
</tr>
</thead>
<tbody>
<tr>
<td>4250/0.75/-0.5</td>
<td>-0.38</td>
<td>-0.93</td>
<td>-1.08</td>
</tr>
<tr>
<td>4250/1.50/-0.5</td>
<td>-0.32</td>
<td>-0.80</td>
<td>-0.65</td>
</tr>
<tr>
<td>4250/2.25/-0.5</td>
<td>-0.23</td>
<td>-0.64</td>
<td>-0.19</td>
</tr>
<tr>
<td>4350/0.75/-0.5</td>
<td>-0.35</td>
<td>-0.90</td>
<td>-1.18</td>
</tr>
<tr>
<td>4350/1.50/-0.5</td>
<td>-0.31</td>
<td>-0.80</td>
<td>-0.78</td>
</tr>
<tr>
<td>4350/2.00/-0.5</td>
<td>-0.27</td>
<td>-0.70</td>
<td>-0.49</td>
</tr>
<tr>
<td>s.d. for last model</td>
<td>0.07</td>
<td>0.13</td>
<td>0.11</td>
</tr>
</tbody>
</table>

**Using microturbulent velocity = (1-log r) km/s:**

<table>
<thead>
<tr>
<th>Model</th>
<th>[Mg/H]</th>
<th>[Fe/H]</th>
<th>[Fe/H]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4350/0.75/-0.5</td>
<td>-0.36</td>
<td>-0.96</td>
<td>-1.11</td>
</tr>
<tr>
<td>4350/1.50/-0.5</td>
<td>-0.33</td>
<td>-0.87</td>
<td>-0.70</td>
</tr>
</tbody>
</table>

To his calculations. Dreiling's work is identical to that of Tripathi & Gaur (1979), except that he sums over all vibrational energy levels with \( v < (\omega_e/2\omega_e x_e) - 0.5 \) (cf. Tatum 1966). Dreiling's values are essentially equal to those of Tripathi & Gaur at 2000 K and become steadily greater with increasing temperature, being 25 per cent greater at 5000 K. The values of \( Q_{\text{MgH}} \) are not essential for the present analysis since the partition function cancels in the calculation of line strengths. It is important, however, that the partition function used in the dissociation equilibrium calculations (i.e. in the 'dissociation constants') and in the excitation equilibria must be consistently calculated for the partition functions to cancel. We adopted the calculated value of Kirby, Saxon & Liu (1979), \( f_{00} = 0.161 \), for the oscillator strength of the (0,0) band of the \( A-X \) system. The Hön-London factors were computed from the formulae given by Kovacs (1969), and normalized according to the conventions of Whiting et al. (1980) and Schadee (1978, see also Larsson 1983). The magnesium isotopic abundance ratios in Arcturus were taken to have the values of \( ^{24}\text{Mg}:/^{25}\text{Mg}:/^{26}\text{Mg} = 83:5:12 \), since Tomkin & Lambert (1976), using low-noise Reticon data obtained at the McDonald Observatory, found the \( ^{24}\text{Mg}/^{26}\text{Mg} \) value to be terrestrial but concluded that \( ^{24}\text{Mg}/^{25}\text{Mg} \) was twice terrestrial. In our analysis of \( \text{MgI} \) lines the isotopic shifts are negligible.

The basic observational data which we have used for \( \text{MgH} \) lines are published by Tomkin & Lambert (1976). The features which we have employed are the blends of \( Q_1 (30), R_1 (17) \) at 5101.4 Å, the \( Q_2 (23) \) feature at 5134.2 Å and the \( Q_1 (23), R_2 (11) \) feature at 5134.6 Å. The actual wavelengths of the \( ^{24}\text{MgH} \) lines were taken from Balfour (1970) and Branch (1970). The full width at half maximum of the instrumental profile was taken to be 0.03 Å, the profile being Gaussian (Tomkin, private communication). A macroturbulence parameter of 2 km s\(^{-1}\) was adopted, and its profile was assumed to be exponential, following Tomkin & Lambert (1976).

Tomkin & Lambert have pointed out that when their data are used for analysis of \( \text{Mg isotopic ratios} \), it is necessary to include \( \text{C}_2 \) lines in the calculations. We have not done so in the present work, since inspection of their calculations suggests that \( \text{C}_2 \) features will not seriously interfere in the gravity determinations when the \( ^{24}\text{MgH} \) contributions to the features are mainly used. In more metal-rich stars, however, it may well be necessary to include \( \text{C}_2 \) lines. The \( \text{MgH} R_1 (11) \) line at 5135.06 Å seen in Figs 1 and 2, is presumably blended, possibly with a TiI line. It is this basic problem of blending which has caused us to use only a small number of carefully chosen features instead of employing the large sample of \( \text{MgH} \) lines in Mäckle et al. (1975a).

Synthetic spectra were then calculated for the \( \text{MgH} \) features for the various Arcturus...
Figure 1. Synthetic spectra for MgH features for models with $T_{\text{eff}} = 4350$ K and $\log g = 0.75, 1.50$ and 2.00. Details of the MgH lines are given in the text. The $^{25}$MgH and $^{26}$MgH lines lie approximately 0.1 and 0.2 Å, respectively, redward of the main $^{24}$MgH features. The observations of Arcturus by Tomkin & Lambert (1976) are indicated by the dots.

Figure 2. Synthetic spectra for MgH features for models with $T_{\text{eff}} = 4250$ K and $\log g = 0.75, 1.50$ and 2.25. The observations of Arcturus by Tomkin & Lambert (1976) are indicated by the dots.

models, and these spectra were convolved with the combined macroturbulence, rotational broadening, and instrumental profile. Sample calculations for the models with $T_{\text{eff}} = 4250$ and 4350 K are shown in Figs 1 and 2. The strengths of the MgH features increase sharply with increasing gravity and decreasing temperature, in agreement with the known variation of MgH with spectral type and luminosity class.

The calculations are compared with the observations in Figs 1 and 2. The best fit at $T_{\text{eff}} = 4350$ K is found for $\log g = 1.67$ (cgs units), whereas the best fit at $T_{\text{eff}} = 4250$ K is at $\log g = 1.27$. In estimating these gravities we have put the greatest weight on the fit of the $^{24}$MgH features, owing to the effect of C$_2$ lines on the other isotopic components.

The determination of $\log g$ from MgH lines is affected by systematic errors, which are somewhat different from those of other methods. It has been demonstrated above that the temperature sensitivity of the abundance of MgH is considerable. The uncertainty in $T_{\text{eff}}$ thus leads to one of the most important uncertainties of the gravity when this is estimated.
Surface gravity of Arcturus

from MgH lines. This also means that the estimated gravity could be expected to be affected by errors in the temperature structures of the model atmospheres.

The model atmospheres used above were originally calculated under the hypothesis that all elements other than H and He were deficient relative to the Sun by the same amount of \([A/H] = -0.5\). Our abundance of Mg, one of the most important electron donors, departs considerably from this. Moreover, the results of Mäckle et al. (1975b) suggest significantly non-solar abundance ratios for Arcturus. Therefore, as an alternative to the standard models, two models with abundances following Mäckle et al. (1975b), although scaled to the appropriate effective temperature and gravity, were constructed and used in the analysis. Both models have \(\log g = 1.5\) and one has \(T_{\text{eff}} = 4250\) K while the other has \(T_{\text{eff}} = 4375\) K. The latter model was the one described by Frisk et al. (1982), in which the effects of an extra opacity source in the blue—violet—ultraviolet spectral range were included in order to match the observed fluxes. The results using these models agreed with those based on the standard models with the same \(T_{\text{eff}}\) and \(\log g\) within 0.05 in \([\text{Mg/H}]\) and 0.06 in \(\log g\). We also found that the use of the semiempirical model of Mäckle et al. (1975b) would lead to \([\text{Mg/H}]\) and \(\log g\) values very close to those of corresponding standard models.

A more general discussion of the effects of model uncertainties on the gravity estimates from different methods is given in the Appendix.

In all calculations described up to this point for Arcturus we have used a microturbulence velocity of 1.7 \(\text{km s}^{-1}\). This value is similar to that found by Mäckle et al. (1975b) of 1.8 \(\text{km s}^{-1}\), and the mean value determined for this parameter for G8 III to K2 III stars by Gustafsson, Kjærgaard & Andersen (1974) (1.7 \(\text{km s}^{-1}\)), and of Gray & Martin (1979) for Arcturus (1.7 \(\text{km s}^{-1}\)). To check the effect of a different value of the microturbulence on the abundances, some of the calculations were repeated using 2 and 1.5 \(\text{km s}^{-1}\). This reduced (and increased, respectively) the Mg abundance derived from the Mg I lines by 0.05 (0.02) dex, which leads to weaker (stronger) MgH lines and consequently to an increase (decrease) of \(\log g\) derived from the models by about 0.15 dex (0.05) dex. The error discussions in the sources quoted above suggest that these errors should be upper limits or overestimates. The effects of a possible depth variation of the microturbulence parameter were investigated by adopting \(\xi_\tau = (1 - \log \tau) \text{ km s}^{-1}\), for \(-4 < \log \tau < 1\), a relation expected to be a reasonable upper bound for the depth variation in Arcturus by fig 1 of Stencel (1980). We find that such a depth variation would lead to an increase of log \(g\) by 0.1 dex. In the comparison in Figs 1 and 2 the equivalent widths of the MgH lines could not be used directly due to blending C\(_2\) lines, but we used instead the line depth of the 24 MgH components. Therefore the choice of macro-turbulence parameter (as well as instrumental profile function) is of some importance. We have found that an increase of this parameter by 50 per cent leads to an increase of log \(g\) by 0.05 dex.

The systematic errors due to departures from LTE may be of some importance, affecting the solar Mg I lines and the lines in Arcturus differently. For example over-ionization of Mg I in the solar atmosphere would lead to a smaller Mg abundance and a greater log \(g\) in Arcturus (cf. the situation for iron, as described by Lites 1973). However, there is no indication of such an over-ionization in the analysis of solar Mg I and Mg II lines of Lambert & Luck (1978). An over-ionization in Arcturus would not be of importance since the effect on the abundance determination would lead to a corresponding compensation in the calculated MgH abundances. The dissociation equilibrium of MgH is affected by photodissociation from the \(X\) state to the nuclear continuum of the \(B'\) state. Kirby, Saxon & Liu (1979) have given the photodissociation cross-section from the \(v'' = 0\) level of the \(X\) state as a function of photon energy. Using these cross-sections and detailed synthetic spectrum calculations of the radiative field in our Arcturus models (which tend to somewhat overestimate the UV-fluxes)
we have found that the departures from LTE through photodissociation from the X state could at most reduce the MgH abundance in the line-forming region by 0.20 dex, since the radiative field in the atmosphere of Arcturus is rather well thermalized in the relevant wavelength region (2600—3100 Å), even relatively high up in the atmosphere. Basically, the thermalization is the result of the heavy metal-line blocking in the UV — the corresponding effects in more metal-poor stars, as well as in hotter stars such as the Sun, may well be more significant. The maximum effects of departures from LTE in the dissociation equilibrium is an increase of log g by about 0.3 dex. A considerably smaller effect is probable if collision processes and all relevant opacities could be taken into account. The effects of departures from LTE for the source functions of the MgH lines are found to be rather small when assuming, following Hinkle & Lambert (1975), that the lines are formed in scattering processes. This assumption would lead to an increase of log g by less than 0.1. It should be noted that the effects discussed here of departures from LTE in the solar Mg I or in MgH for Arcturus would increase the value of log g.

What effect does the uncertainty in $f_{00}$ of the MgH A-X system have on the value deduced for the gravity of Arcturus? Kirby, Saxon & Liu (1979) expect an accuracy of ±20 per cent in their calculated oscillator strengths, implying $0.13 < f_{00} < 0.19$. Their value is in fairly reasonable agreement with the older values of 0.25 (Henneker & Popkie 1971) and 0.192 (Popkie 1971) and is also consistent with the measurements by Nedelec & Dufayard (1978) of the radiative lifetime of the A state. Some check on $f_{00}$ can be obtained from solar observations. Lambert, Mallia & Petford (1971) studied MgH in the solar spectrum. For log $N$(Mg) = 7.59 and $D_0$ = 1.27 eV their results give $f_{00} = 0.23$. Our calculations, adopting the Holweger-Müller (1974, HM subsequently) model and $f_{00} = 0.16$, give $W = 4.4$ mÅ for $Q(15)$ of $^{24}$MgH, in agreement with the value observed by Lambert, Mallia & Petford and with the values given by Grevesse & Sauval (1973). There may be some uncertainty, however, in the equivalent widths of the MgH lines in the photospheric spectrum. Sinha (1981) has used the KPNO solar atlas (Brault & Testerman 1972) and the Junfraujoch atlas (Delbouille, Neven & Roland 1973) and finds that the latter gives equivalent widths which are 40 per cent larger. The mean ratio of Lambert, Mallia & Petford equivalent widths to these KPNO values is 0.6. Basically, the differences reflect the difficulties of defining the continuum level. If we assume conservatively, that the errors in the value used for $f_{00}$ might amount to ±0.06 we find that this corresponds to errors in log g of ±0.3.

In concluding the discussion of possible errors in the surface gravity of Arcturus as deduced from MgH lines, we find that, apart from errors caused by the possible errors in $T_{\text{eff}}$ which may amount to 0.3 dex and errors in the LTE dissociation equilibrium which might lead to an underestimate of log g by up to 0.3 dex (while an overestimate is improbable), the contributions from the rest of the sources of error probably do not sum up to more than 0.3 dex. An estimated total maximum error of about 0.5 dex seems realistic.

3 Observations and calculations for pressure-broadened lines

The gravity derived above from the MgH features is rather close to the gravities derived using other methods (Spite & Martin 1981; Blackwell & Willis 1977). However, significantly lower values have also been proposed (Mäckle et al. 1975b; Ruland et al. 1980). Therefore, and since the MgH lines have not been used for this purpose before, we have also analysed a number of strong lines in the spectrum of Arcturus in order to get independent estimates of the gravity. Further details of this study are given by Edvardsson (1983). Here, we have used a method similar to that developed by Blackwell & Willis (1977, hereafter BW) and applied it to several strong lines. In this method the damping wings of strong van der Waals-broadened
lines are used as gravity criteria. The wing profile of such a line is a function of the chemical abundance of the element, the oscillator strength and the damping correction factor $F_6$. $F_6$ is an empirically-determined factor to be multiplied with the value of $C_6$ in Unsold’s classical expression for the damping constant (cf. Mihalas 1978). The abundance is derived differentially relative to the Sun, by using weak spectral lines with excitation energies as close as possible to the energy of the lower level of the strong line. In principle, this lower level should be identical for the weak and strong lines in order to fully compensate for the temperature errors and departures from the LTE populations. For most of the useful strong lines this requirement is difficult or impossible to fulfill in practice. The oscillator strengths of the strong lines are adopted from laboratory experiments and the damping correction factors are determined from fitting solar line profiles, allowing for radiative damping and macro turbulence. The observational data for the Sun were taken from the atlas of Delbouille, Neven & Roland (1973) for $\lambda < 8000\text{ Å}$ and from the solar atlas of Delbouille & Roland (1963) for the remaining two lines. The Arcturus data were taken from the Arcturus atlas (Griffin 1968).

The equivalent widths of the weak lines were measured from the solar atlas from 1973 while those of Arcturus were adopted from Mäckle et al. (1975a), except for the Ca I $\lambda 6156$ line which is not in their list. Corrections for scattered light were applied in the abundance analysis according to Mäckle et al. (1975b).

The strong lines were chosen to fulfil the following requirements: The wavelength should be within the interval 5000—8825 Å in order to make it possible to define a reliable con-

Table 3. Spectral lines used for determining the surface gravity of Arcturus from the pressure broadening of strong lines. The errors given are estimated $1\sigma$ errors. The errors quoted in $[X/H]$ include various possible systematic effects; the formal statistical errors in this quantity are considerably smaller.

<table>
<thead>
<tr>
<th>Element</th>
<th>$\lambda$ (Å)</th>
<th>$\chi$ (eV)</th>
<th>$\log gf_{\odot}$</th>
<th>$w_{Q,\odot}$ (mA)</th>
<th>$w_{\odot}$ (mA)</th>
<th>$[X/H]$</th>
<th>$F_6$</th>
<th>$\log g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe I</td>
<td>5269.55</td>
<td>0.86</td>
<td>-1.32</td>
<td>39.1</td>
<td>1.6</td>
<td>-0.58±0.12</td>
<td>1.3±0.3</td>
<td>1.7±0.34</td>
</tr>
<tr>
<td></td>
<td>6221.68</td>
<td>0.86</td>
<td>1.12</td>
<td>39</td>
<td>1.5</td>
<td>-0.58±0.12</td>
<td>1.2±0.3</td>
<td>1.7±0.20</td>
</tr>
<tr>
<td></td>
<td>6358.84</td>
<td>0.91</td>
<td>1.14</td>
<td>36</td>
<td>1.5</td>
<td>-0.58±0.12</td>
<td>1.2±0.3</td>
<td>1.7±0.20</td>
</tr>
<tr>
<td>Fe I</td>
<td>8688.63</td>
<td>2.18</td>
<td>-1.21</td>
<td>34.1</td>
<td>4.3</td>
<td>-0.58±0.12</td>
<td>2.5±0.3</td>
<td>1.7±0.38</td>
</tr>
<tr>
<td></td>
<td>6015.25</td>
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<td>2.90</td>
<td>32</td>
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<td>2.5±0.3</td>
<td>1.7±0.38</td>
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<tr>
<td></td>
<td>6667.42</td>
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<td>6746.96</td>
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<td>29</td>
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<td>2.5±0.3</td>
<td>1.7±0.38</td>
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<td></td>
<td>6860.29</td>
<td>2.60</td>
<td>3.40</td>
<td>44</td>
<td>6.3</td>
<td>-0.58±0.12</td>
<td>2.5±0.3</td>
<td>1.7±0.38</td>
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<tr>
<td>Ca I</td>
<td>6162.17</td>
<td>1.90</td>
<td>-0.09</td>
<td>41</td>
<td>9.5</td>
<td>-0.37±0.12</td>
<td>8.4±0.3</td>
<td>1.6±0.39</td>
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<tr>
<td></td>
<td>6156.02</td>
<td>2.52</td>
<td>3.89</td>
<td>44</td>
<td>9.0</td>
<td>-0.37±0.12</td>
<td>8.4±0.3</td>
<td>1.6±0.39</td>
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<tr>
<td></td>
<td>6508.85</td>
<td>2.52</td>
<td>3.85</td>
<td>44</td>
<td>9.0</td>
<td>-0.37±0.12</td>
<td>8.4±0.3</td>
<td>1.6±0.39</td>
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<td></td>
<td>6709.87</td>
<td>2.93</td>
<td>3.61</td>
<td>9.0</td>
<td>2.3</td>
<td>-0.37±0.12</td>
<td>8.4±0.3</td>
<td>1.6±0.39</td>
</tr>
<tr>
<td>Na I</td>
<td>5889.95</td>
<td>0.00</td>
<td>0.12</td>
<td>39</td>
<td>13.8</td>
<td>-0.40±0.15</td>
<td>1.3±0.1</td>
<td>1.1±0.56</td>
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<tr>
<td></td>
<td>5895.92</td>
<td>0.00</td>
<td>-0.18</td>
<td>39</td>
<td>13.8</td>
<td>-0.40±0.15</td>
<td>1.3±0.1</td>
<td>1.1±0.56</td>
</tr>
<tr>
<td></td>
<td>4751.82</td>
<td>2.10</td>
<td>4.27</td>
<td>39</td>
<td>13.8</td>
<td>-0.40±0.15</td>
<td>1.3±0.1</td>
<td>1.1±0.56</td>
</tr>
<tr>
<td></td>
<td>5148.84</td>
<td>2.10</td>
<td>4.20</td>
<td>33</td>
<td>12.3</td>
<td>-0.40±0.15</td>
<td>1.3±0.1</td>
<td>1.1±0.56</td>
</tr>
<tr>
<td>Mg I</td>
<td>8806.76</td>
<td>4.35</td>
<td>0.12</td>
<td>22</td>
<td>9.1</td>
<td>-0.30±0.12</td>
<td>5.7±0.3</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>5509.60</td>
<td>5.11</td>
<td>4.82</td>
<td>22</td>
<td>9.1</td>
<td>-0.30±0.12</td>
<td>5.7±0.3</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>6319.24</td>
<td>5.11</td>
<td>5.36</td>
<td>45</td>
<td>45</td>
<td>-0.30±0.12</td>
<td>5.7±0.3</td>
<td>2.8</td>
</tr>
</tbody>
</table>

a) Blackwell et al. (1979)  
b) Blackwell et al. (1982)  
c) Smith and O’Neill (1975)  
d) Gehren (1975)  
e) Kurucz and Peytremann (1975)  
f) Allen (1976)
tinuum and to find the line in the Arcturus atlas; the line should be broad enough to be on
the damping part of the curve of growth both in Arcturus and in the Sun; the line should not
be heavily blended or asymmetric; the pressure sensitivity should be large enough, relative
to the observational accuracy, to allow a good gravity determination. These criteria were found
to be fulfilled by the following lines: Fe I λ5269.55 (the line used by BW), Fe I λ8688.63
(discussed by Simmons & Blackwell 1982), Ca I λ6162.17, the Na D-lines and Mg I λ8806.76.
These lines and the weak lines used for abundance analysis are tabulated in Table 3. It is seen
in the table that only for the Fe I lines has it been possible to find suitable weak lines for
abundance determinations with excitation energies close to those of the strong lines.

The solar HM model was used in the analysis, including its depth-dependent microtur-
bulence. We have found that the use of other contemporary models of the solar atmosphere
may lead to significant changes in the abundance and Fe abundance determinations — typically, the
effects may be 0.2 to 0.3 dex in both quantities. However, these effects cancel each other
for the strengths of the strong-line wings and thus only contribute less than 0.05 dex to the
uncertainty in the gravity determination for any one of the lines.

In analysing the strong lines we basically used the detailed model atmosphere for Arcturus
of Frisk et al. (1982). In addition to these models a number of other models with other
fundamental parameters were used, including those used in the MgH analysis. Somewhat
inconsistently with the latter analysis, a microturbulence parameter of 2 km s$^{-1}$ was adopted.
This parameter was found, however, to be of little importance. The macroturbulence in the
Sun and in Arcturus was taken into consideration following Gray (1977, 1978 and 1981).
The theoretical line profiles were also folded with the instrumental profile given in the
Arcturus atlas, and corrected for 5 per cent scattered light (Griffin 1969).

### Table 4

Effects on log $g$, as determined from pressure-broadened lines, when estimated maximum errors in the
parameters and models are introduced into the analysis. The estimated maximum errors in log $g$
caused by uncertainties in the fit to the observations are also presented.

<table>
<thead>
<tr>
<th>Line</th>
<th>$\lambda$</th>
<th>$T_\text{eff}$</th>
<th>[A/H] $\pm 150$ K</th>
<th>[X/H] $\pm 0.25$</th>
<th>$F_\text{c}$ $\pm 2 \times$ error</th>
<th>$dT/d\log r$ Observations $\pm 100$ K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe I</td>
<td>5269</td>
<td>±0.09</td>
<td>±0.10</td>
<td>±0.50</td>
<td>±0.30</td>
<td>±0.04</td>
</tr>
<tr>
<td>Fe I</td>
<td>8688</td>
<td>±0.08</td>
<td>±0.17</td>
<td>±0.54</td>
<td>±0.30</td>
<td>±0.07</td>
</tr>
<tr>
<td>Ca I</td>
<td>6162</td>
<td>±0.10</td>
<td>±0.23</td>
<td>±0.58</td>
<td>±0.20</td>
<td>±0.06</td>
</tr>
<tr>
<td>Na I</td>
<td>5889</td>
<td>±0.36</td>
<td>±0.14</td>
<td>±0.93</td>
<td>±0.35</td>
<td>±0.30</td>
</tr>
<tr>
<td>Mg I</td>
<td>8806</td>
<td>±0.09</td>
<td>±0.17</td>
<td>±0.59</td>
<td>±0.36</td>
<td>±0.10</td>
</tr>
</tbody>
</table>

The main results of the analysis of the strong lines are given in Table 3. The errors in the
surface gravities are estimated mean errors (rms), derived in the following way:

For each strong line $2\sigma$ errors in effective temperature, overall metal abundance,
abundance of the relevant element, damping correction factor and micro- and macroturbulence
are estimated and transformed to corresponding errors of $2\sigma$ in log $g$. In addition, the
maximum errors found in the observations and resulting from fitting problems were estimated.
Finally, the effects of possible errors in the temperature structures were studied by steepening
the temperature gradient over the interval $0.01 < \tilde{r} < 2.0$ by about 100 K per unit in log $\tilde{r}$
(cf. Gustafsson 1983). The error estimates are given in Table 4 except for the effects of
errors in the micro- and macroturbulence parameters ($2\sigma$'s errors 0.5 and 2.0 km s$^{-1}$ respectively)
which were found to lead to $2\sigma$ errors less than 0.03 and 0.04, respectively, in log $g$
for all the lines. Finally, for each line, the resulting $1\sigma$ error in log $g$, $\sigma_{\text{Line}}$, was calculated.
from the equation

$$\sigma_{\text{Line}} = \left[ \Sigma \frac{1}{2} (2\sigma)^2 \right]^{1/2}. \quad (2)$$

These are given in the last column in Table 3.

In Figs 3–7 the determination of the damping correction factors and of log g are illustrated for the strong lines. Some comments will subsequently be given for each of them.

Our value of $F_6$ for $\lambda 5269.55$ agrees well with those of Blackwell & Shallis (1979), who obtained $F_6$ values between 1.2 and 1.9 depending on which solar model was used. The iron abundance and gravity of BW, when interpolated to our effective temperature, are $[\text{Fe/H}] = -0.56$ and $\log g = 1.46$. For $\lambda 8688.63$ Simmons & Blackwell (1982) found $F_6 = 2.3$. This line is less sensitive to $\log g$ than $\lambda 5269$. The Ca abundance derived agrees very well with that of Mäckle et al. (1975b), scaled to the proper temperature and gravity ([Ca/H] = −0.37) and that found by Smith & Lambert (1983, [Ca/H] = −0.42). Our $F_6$ value for $\lambda 6162.17$
agrees very well with the solar determination of O'Neill & Smith (1980, $F_6 = 8.4$) but not with their laboratory value ($F_6 = 5.0$). The sodium abundance also agrees very well with the scaled Mäckle et al. value ([Na/H] = −0.38), but in our case this value is only based on two lines, one shortwards of 5000 Å. The $F_6$ values found for the $D$-lines are smaller than that used by Gehren (1975, $F_6 = 3.2$). The resulting gravity for the $D$-lines is lower than for the other lines used. However, the log $g$ determination from the $D$-lines is quite uncertain — e.g., a lower sodium abundance by 0.2 dex would increase log $g$ to 1.7, and errors in the temperature structure have serious effects due to very different excitation energies for the strong and the weak lines. The weak Mg I lines used here and given in Table 3 are severely blended,
Figure 7. Calculated flux profiles for Na I D lines in Arcturus. See caption of Fig. 3.
especially in the spectrum of Arcturus. The derived abundance is 0.1 dex smaller than that of Mäckle et al., when scaled to our effective temperature but similar to that obtained in Section 2 from a greater sample of Mg i lines. The logarithmic gravity one would derive from the $\lambda$ 8806.76 line is astonishingly high (in excess of 2.2). Obviously, the result from this line is not consistent with those from the other strong lines. The reason for this is not well understood at present, and it should be further investigated. Calculations of neighbouring Fe i lines rule out the possibility that the bad fit is due to errors in the instrumental profile at this last page of the Arcturus atlas.

Excluding the inconsistent Mg i $\lambda$8806.76 line and combining the remaining log $g$ determinations in a mean with weights proportional to $(\sigma_{\text{Line}})^2$, we find log $g = 1.6 \pm 0.19$. A 100 K change in the assumed effective temperature for Arcturus makes this determination of log $g$ change by only 0.06 dex.

As a consistency check the profile of the $\lambda$5172 Mg b triplet line was calculated. This line was chosen since it is less blended than $\lambda$5167 and more symmetric than $\lambda$5183. It was found that log $g = 1.6$ appears consistent with the observations, but the severe blending prevents any further conclusions.

4 Observations and calculations for weak iron lines

The extensive debate concerning the accuracy of the determinations of log $g$ for Arcturus from ionization equilibria during the last decade (cf., for example Ruland et al. 1980; Spite & Martin 1981; Trimble & Bell 1981, and references cited therein) has been inspired by the fact that different analyses, even when based on similar observational material, give different logarithmic gravities ranging over an interval from 0.9 to 1.8. The studies mentioned have shown that much of this scatter may be explained by the model sensitivity of the procedure used. The choices of model atmospheres for Arcturus and the Sun, of microturbulence parameters and of the sample of spectral lines are of importance for the result. Here, we shall attempt to circumvent some of these problems by using high-excitation Fe i lines in the infrared. This sample of lines was chosen in order to minimize the temperature sensitivity (and thus the sensitivity to the details of the model temperature structure) and the difficulties in defining the continuum.

A major obstacle in an analysis of this type is that accurate oscillator strengths are not available for the weak lines which we use. We must consequently deduce the oscillator strengths from the observed solar equivalent widths.

Branch, Bonnell & Tomkin (1978) have shown that accurate abundances can be derived for K giants from Fe i lines in the near-infrared. Problems of line blending are minimized in this wavelength region. We used the Delbouille, Neven & Roland (1973) solar atlas to measure equivalent widths of these lines with $\lambda < 7850$ Å and measured the remaining lines from the

<table>
<thead>
<tr>
<th>$\lambda$ (Å)</th>
<th>log $W/\lambda$</th>
<th>log gf$\lambda$</th>
<th>$\chi$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7719.04</td>
<td>-5.41</td>
<td>2.82</td>
<td>5.03</td>
</tr>
<tr>
<td>7723.21</td>
<td>-5.34</td>
<td>0.29</td>
<td>2.28</td>
</tr>
<tr>
<td>7733.74</td>
<td>-5.88</td>
<td>2.24</td>
<td>5.06</td>
</tr>
<tr>
<td>7802.61</td>
<td>-5.68</td>
<td>2.50</td>
<td>5.08</td>
</tr>
<tr>
<td>7820.81</td>
<td>-6.19</td>
<td>1.19</td>
<td>4.29</td>
</tr>
<tr>
<td>7832.21</td>
<td>-4.73</td>
<td>4.01</td>
<td>4.43</td>
</tr>
<tr>
<td>7844.56</td>
<td>-5.78</td>
<td>2.15</td>
<td>4.83</td>
</tr>
<tr>
<td>8699.46</td>
<td>-5.08</td>
<td>3.47</td>
<td>4.95</td>
</tr>
<tr>
<td>8763.98</td>
<td>-4.86</td>
<td>3.88</td>
<td>4.65</td>
</tr>
</tbody>
</table>
Surface gravity of Arcturus (Braatz & Testerman 1972). The values found are given in Table 5. We then derived the $gf$ values with the HM solar model, with a microturbulence velocity of $1.0 \, \text{km s}^{-1}$. This value is based on recent work on the solar Fe I spectrum (Blackwell & Shallis 1979). Individual Fe abundances were next determined for each Fe I line with the series of Arcturus models mentioned above by comparison with the equivalent widths of Mäckle et al. (1975a) and the mean Fe abundance was computed for each model. The values found are given in Table 2. The standard deviation of the result for the model $4350/2.00/-0.5$ is also given. As a check of the sensitivity of the abundances to the microturbulence parameter, the abundances were also calculated for two models $4350/0.75/-0.5$ and $4350/1.50/-0.5$ assuming a depth-dependent microturbulence calculated as $(1-\log r) \, \text{km s}^{-1}$ within the range 0–5 km s$^{-1}$. The result of these calculations are also shown in Table 2.

Table 6. Fe II lines. Equivalent widths and $gf$-values. A solar logarithmic iron abundance of 7.60 was assumed.

<table>
<thead>
<tr>
<th>$\lambda$ (Å)</th>
<th>$\log W/\lambda$</th>
<th>$\log gf\lambda$</th>
<th>$\chi$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5256.9</td>
<td>-5.42</td>
<td>-0.47</td>
<td>2.88</td>
</tr>
<tr>
<td>5264.8</td>
<td>-5.08</td>
<td>0.45</td>
<td>3.22</td>
</tr>
<tr>
<td>5325.6</td>
<td>-5.07</td>
<td>0.47</td>
<td>3.21</td>
</tr>
<tr>
<td>5414.1</td>
<td>-5.32</td>
<td>0.00</td>
<td>3.21</td>
</tr>
<tr>
<td>5991.4</td>
<td>-5.30</td>
<td>0.07</td>
<td>3.14</td>
</tr>
<tr>
<td>6084.1</td>
<td>-5.46</td>
<td>-0.13</td>
<td>3.19</td>
</tr>
<tr>
<td>6113.3</td>
<td>-5.79</td>
<td>-0.51</td>
<td>3.21</td>
</tr>
<tr>
<td>6149.2</td>
<td>-5.23</td>
<td>0.89</td>
<td>3.87</td>
</tr>
<tr>
<td>6247.6</td>
<td>-5.06</td>
<td>1.32</td>
<td>3.87</td>
</tr>
<tr>
<td>6369.4</td>
<td>-5.53</td>
<td>-0.47</td>
<td>2.88</td>
</tr>
<tr>
<td>6432.6</td>
<td>-5.19</td>
<td>0.11</td>
<td>2.88</td>
</tr>
</tbody>
</table>

Very few Fe II lines are available in the near-infrared. We consequently analysed lines in the 5000–6000 Å region with solar equivalent widths (as measured by Mäckle et al. 1975a) within the range 10 to 66 mÅ. The procedure described above for the Fe I lines was repeated for the Fe II lines. The lines used and their $gf$-values are given in Table 6. These values are consistent with those of Blackwell, Shallis & Simmons (1980). The abundances found from the Fe II lines are given in Table 2, together with the standard deviation of the result for the $4350/2.00/-0.5$ model.

The iron abundances found are plotted versus log $g$ in Fig. 8, for the models with $T_{\text{eff}} = 4350 \, \text{K}$. The abundance found from the Fe I lines agrees well with that found from the Fe II lines at log $g = 1.45$. A similar plot at 4250 K gives log $g = 1.1$. This temperature sensitivity is somewhat smaller than that of the corresponding gravity determination of Mäckle et al. (1975b) which is a consequence of our sample of Fe lines with relatively high excitation energies.

Study of Fig. 8 shows that errors in the abundance results — the shaded areas correspond to formal mean errors in the abundance mean from different lines — cause a large uncertainty in the values deduced for the gravities. An error of 0.5 in log $g$ is by no means impossible even if systematic errors (departures from LTE, errors in the model atmospheres, etc.) are neglected.

Fig. 8 shows that at $T_{\text{eff}} = 4350 \, \text{K}$ and log $g = 1.45$ the Fe I and Fe II lines yield [Fe/H] = $-0.81$, while at $T_{\text{eff}} = 4250 \, \text{K}$, log $g = 1.1$ and [Fe/H] = $-0.87$. These abundances are lower than that found by Mäckle et al. (1975b), who obtained [Fe/H] = $-0.70$. The gravity derived from the calculations with a depth-dependent microturbulence ($\xi = 1 - \log r$) gives log $g = 1.1$ and [Fe/H] = $-0.92$, at 4350 K which is a much greater sensitivity than that found for the other methods for determining the gravity. Mäckle et al. (1975b) give [Fe/H] for 6 of the 9...
Figure 8. The iron abundance of Arcturus, [Fe/H], has been derived from Fe I and Fe II lines, for models of different temperatures and surface gravities. The microturbulence assumed was 1.7 km s\(^{-1}\). The abundances found are plotted versus log g, using solid lines. The shaded areas show the formal errors of the abundance mean of the individual determinations from the different spectral lines. The dashed lines show the results found when using a depth-dependent microturbulence = (1 -\(\log r\)) km s\(^{-1}\) (log \(r < 1\)). All results in this figure refer to models with \(T_{\text{eff}} = 4350\) K.

infrared lines that we use and the mean [Fe/H] from these 6 lines is \(-0.89\). There thus appears to be no systematic difference between our values and theirs when the same lines are used and allowance is made for the error in each determination. One reason why these [Fe/H] values depart from those given in Table 3 is the different gravity. However, most of the difference seems to be related to the different wavelengths and excitation energies of the 2 samples of lines. For reasons explained above we ascribe the greatest significance to the lines in the infrared. We have found that the use of model atmospheres with chemical abundances according to Mäckle et al. (1975b), though scaled to the relevant \(T_{\text{eff}}\) and log \(g\) and with consistent iron abundances with those obtained here and with an increased ultraviolet opacity, would decrease the gravities from the Fe I/Fe II equilibrium by about 0.1 dex. Thus, our study of the ionization equilibria suggests a logarithmic gravity close to 1.4 for \(T_{\text{eff}} = 4350\) K. The model changes resulting from the increased Mg abundance and increased continuous opacity cause the deduced Fe abundance to be increased by about 0.1 dex.

Spite & Martin (1981) have discussed the gravity of Arcturus, using the ionization equilibrium of both iron and titanium. They found a gravity of 1.6, using our models. The main difference between their analysis and the present one is our use of the restricted set of infrared Fe I lines which seems to give a somewhat smaller Fe abundance, and thus a lower gravity. Spite & Martin also show that Arcturus models from different sources give log \(g\)
values ranging from 1.2 to 1.6 with the ionization equilibrium method applied to their large sample of lines.

The difference between the gravity deduced from our sample of iron lines and the result of other determinations including the results discussed in Section 2 and 3 above may suggest that more fundamental systematic errors than those discussed above could be of some importance. In particular, departures for LTE for Fe I and in the ionization equilibrium could be important in this respect — there are several other indications that lines of neutral metal atoms are significantly affected by non-LTE effects in differential studies of red giants relative to the Sun (cf. Ruland et al. 1980; Rutten & Kostik 1982; Brown, Tomkin & Lambert 1983).

5 The mass of Arcturus

Basic data for Arcturus is given by Mäckle et al. (1975b). With \( V = -0.04 \) and a parallax of 0.092 arcsec, \( M_V = -0.22 \). The bolometric corrections from Bell & Gustafsson (1978), give \( M_{\text{bol}} = -0.72 \) if \( T_{\text{eff}} = 4375 \) K and \(-0.83\) if \( T_{\text{eff}} = 4250 \) K. Adopting \( M_{\text{bol}}^{\odot} = 4.72 \), the stellar bolometric magnitude can be written as

\[
M_{\text{bol}} = 4.72 + 2.5 \log g - 10 \log T_{\text{eff}} - 2.5 \log \left( \frac{M}{\odot} \right),
\]

where quantities within brackets denote the logarithm of the ratio between the corresponding stellar parameter and the solar one. Combining the results of the three methods, based on MgH, strong lines and ionization equilibrium respectively, we find log \( g = 1.6 \pm 0.2 \), corresponding to an Arcturus mass of \( 0.42 \leq M \leq 1.05 \odot \) for the effective temperature 4375 K.

Figure 9. The gravity determination as a function of \( T_{\text{eff}} \) resulting from the three different methods. The error bars indicate the estimated errors.
6 Conclusions

Clearly, a dominant source of error in using the MgH lines for determining stellar gravities is the error in temperature. This is illustrated in Fig. 9 where the temperature sensitivity of the 3 methods is shown. Another major source of error, which could tend to lead to an underestimated gravity by up to 0.3 dex, may be departures from LTE in the dissociation equilibrium. The overall uncertainty from other sources of error — in the consideration of microturbulence and in the LTE assumption when calculating ionization equilibria — may add up to about 0.3 dex in the gravity. The agreement between the present results from MgH — suggesting a value of log g in the interval 1.8 — 1.5 for $T_{\text{eff}}$ values between 4375 and 4300 K — and those from pressure-broadened strong lines ($\log g = 1.6 \pm 0.2$) is very satisfactory. The discrepancy is however somewhat larger between the gravity determination from MgH lines and determinations from ionization equilibria, respectively. It seems natural to ascribe this discrepancy to the uncertainties in the ionization equilibrium method referred to above. Thus, a reasonable value for the gravity of Arcturus is $\log g = 1.6$ with an error not greater than 0.3 dex.

The corresponding spectroscopic mass of Arcturus is $0.6 \, M_\odot$, but this value is still uncertain by a factor of 2 or somewhat less. Obviously, we are far from accurate mass determinations for single stars from stellar spectroscopy, although the MgH method seems competitive with other methods, especially in differential studies when the colour difference, and thus the temperature difference, between the 2 stars under comparison is well known.

Acknowledgments

Special thanks are due to Professor Yngve Öhman who urged us to study the possibilities of the MgH lines many years ago. We are grateful to Dr Jocelyn Tomkin for sending us information on the instrumental profile as well as details of the model atmosphere used by Tomkin & Lambert. We also wish to thank Professor David Lambert and Dr Kjell Eriksson for valuable comments on the manuscript. This project has been supported by the National Science Foundation under grant AST 80-19570 (prior to 1981 September 8) and by the Swedish Natural Science Research Council. The University of Maryland Computer Center and the Faculty for Natural Sciences at Uppsala University supplied the computer time.

References


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Surface gravity of Arcturus


Appendix: The effects of uncertainties in the temperature structure on gravity determinations for Arcturus

In order to study the effects of model-atmosphere uncertainties on the gravity deduced for Arcturus using different methods, we here assume that the effective temperature is obtained from observed near-infrared colours of the star, as done by e.g. Frisk et al. (1982), and that only criteria formed at rather great photospheric depths (weak spectral lines, damping wings of strong lines) are being used for the gravity determination. In the general case, we may write for the strength \( S \) of any spectral feature

\[
S = S(T_{\text{eff}}, \log g, [\alpha], D),
\]

where \([\alpha]\) is a relevant abundance parameter (such as \([\text{Fe/H}]\) or \([\text{Mg/H}]\)) and \( D \) represents any quantitative measure of a model perturbation. Similarly, we may write for any quantitative measure of the infrared flux gradient \( I \):

\[
I = I(T_{\text{eff}}, \log g, [\alpha], D).
\]

Representing \( T_{\text{eff}}, \log g \) and \([\alpha]\) by \( T, G \) and \( A \), respectively, we write:

\[
\Delta S = \frac{\partial S}{\partial T} \cdot \Delta T + \frac{\partial S}{\partial G} \cdot \Delta G + \frac{\partial S}{\partial A} \cdot \Delta A + \frac{\partial S}{\partial D} \cdot \Delta D
\]

and a corresponding expression for \( I \). Now, requiring that the observed criteria \( S \) and \( I \) be given and are thus constant we easily derive the following expression:

\[
\Delta G = \left[ \left( \frac{\partial S}{\partial A} - \frac{\partial I}{\partial A} \cdot \frac{\partial S}{\partial T} \right) \left( \frac{\partial S}{\partial G} - \frac{\partial I}{\partial G} \cdot \frac{\partial S}{\partial T} \right) \right] \cdot \Delta A
\]

\[
- \left[ \left( \frac{\partial S}{\partial D} - \frac{\partial I}{\partial D} \cdot \frac{\partial S}{\partial T} \right) \left( \frac{\partial S}{\partial G} - \frac{\partial I}{\partial G} \cdot \frac{\partial S}{\partial T} \right) \right] \cdot \Delta D.
\]

Here, \( \Delta G \) represents the change in the logarithmic gravity that would be deduced from a perturbed model grid (perturbation \( \Delta D \)) and \( \Delta A \) the corresponding change in logarithmic abundance, as compared with the standard grid.

Equation (A4) has been used to estimate the effects of perturbations of the model atmosphere on gravity determinations by three different methods: First, the method with MgH and Mg I lines, chosen to be typical for the present approach, secondly the method of Blackwell & Willis (1977, BW) based on the strength of the pressure-sensitive wing of a strong neutral iron line, as compared with very weak lines from the same multiplet, and finally the ionization equilibrium method, with Fe I and Fe II lines, again chosen to be typical for our sample of lines.

The evaluation of the derivatives of Equation (A4) was made numerically using a small set of ‘standard’ model atmospheres with fundamental parameters defining a region around the point \( T_{\text{eff}}/\log g/\text{[A/H]} = 4350/1.50/-0.75 \), and a corresponding set of perturbed models with an extra opacity for \( \lambda < 5000 \text{ Å} \) added according to Equation (1) of Frisk et al. (1982). This perturbation introduced an extra backwarming and thus steepening of the temperature gradient of the models, amounting to 15–20 K in the optical depth region \( 0.01 < \tau_{5000} < 1.0 \). The infrared gradient \( I \) was chosen to be the R–I colour of the UBVRI system, but this specific choice does not affect the results appreciably.

For the three methods of gravity determination the following results were obtained, assuming that ‘standard models’ were adopted in the analysis and that perturbed models with extra backwarming represent the true stars: our MgH/Mg I method and our Fe II/Fe I
method would both lead to an underestimate of $\log g$ by 0.05 dex while the BW method would lead to an overestimate by 0.01 dex. It was also found that, even if a considerably steeper temperature gradient were adopted (i.e., still more ad hoc opacity introduced) the effects on the derived gravities would hardly be greater than 0.15 dex. The very small model sensitivity of the BW method (when used for the outer wings of strong lines) is worth noting, although other uncertainties, pointed out by Ruland et al. (1980), may be of importance for this method.