NON-LTE LINE TRANSFER WITH PARTIAL REDISTRIBUTION

II. An Equivalent-Two-Level-Atom Approach

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ПЕРЕНОС ИЗЛУЧЕНИЯ В СПЕКТРАЛЬНЫХ ЛИНИЯХ ВНЕ ЛТР ПРИ НЕПОЛНОМ
ПЕРЕРАСПРЕДЕЛЕНИИ ПО ЧАСТОТАМ

2. Метод эквивалентных двухуровневых атомов

A formulation of equations for radiative transfer in a gas of multilevel atoms, taking into account recent developments of theoretical description, is presented. It is shown that for a simple case where one chosen transition is allowed to depart from complete redistribution, the global multilevel problem may be solved by suitably modified complete redistribution numerical techniques. In particular, we formulated a modification of the equivalent two-level-atom approach that enables a multilevel transfer to be solved by a simple iteration scheme. Various approximate forms of the line source function are also discussed.

1. Introduction

In the previous paper of this series (Hubený, 1981a – hereinafter referred to as Paper I), we attempted to formulate, in a somewhat heuristic way, a general form of the emission coefficient for multilevel atoms. The formalism developed in Paper I is based on a semi-classical picture which assumes that an atom is practically always in one of its energy eigenstates, and that a well-defined velocity distribution can be ascribed to atoms in a particular energy state. The process of radiation scattering, which is in fact linked with a time development of the atomic wave functions, is regarded as instantaneous, and is simply described by means of suitably introduced redistribution functions.

Recently, considerable progress has been made in several directions. First, Cooper et al. (1982 – CBBH) presented a quantum-mechanical derivation of the absorption and emission coefficients and of the statistical equilibrium equations, starting with the density matrix equation. On the other hand, the semi-classical formulation has been considerably extended to include general chains of radiation processes (i.e. general n-photon correlations), velocity-changing collisions, and non-local phenomena (Hubený, O xenius and Simonneau, 1983a, b – HOS I and II, respectively). The latter approach represents a generalization of the formalism of Paper I. The relation between the quantum-mechanical and the semi-classical approach is discussed by Cooper, Hubený and Oxenius (1983 – CHO), who analysed a controversial point of the stimulated emission profile. They showed that, for a low-intensity radiation field (the case usually met in astrophysical applications), both approaches practically agree. Conclusions of CHO are important for actual applications because they justify a considerable simplification of the stimulated emission rate without impairing the overall physical consistency of the formalism. Finally, from the computational point of view, a convenient form of the redistribution functions has been derived for both resonance fluorescence (Heinzel, 1981; Heinzel and Hubený, 1982), and for general two-photon processes (Hubený, 1982), and a simple and accurate method for their numerical evaluation has been developed (Heinzel and Hubený, 1983).

From the astrophysical point of view, the most important problem is to assess the interpretational
significance of such a refined physical description. However, the formalisms developed in the above cited papers appear to be very complicated, and, consequently, they usually do not offer a simple way of solving actual diagnostic problems. Therefore, it seems reasonable to concentrate on developing approximate schemes for handling these problems. The present and the next paper of this series are intended to provide a practical guide to possible approximations of the general partial redistribution problem. Considerable attention is devoted to formulating suitable approximate schemes in a form similar to traditional complete-redistribution approaches.

2. Specification of the Problem

Solving any actual transfer problem requires the specification of various terms of the basic set of equations, which comprises the radiative transfer equation, a set of statistical equilibrium equations, and the definition equations for line profile coefficients for absorption and emission. We shall discuss them in turn.

a) Radiative Transfer Equation

The time-independent transfer equation for unpolarized light of a specific intensity \( I(v, n) \) in a spectral line that originates due to an atomic transition between a lower level \( i \) and an upper level \( j \) of a multilevel atom can be written (HOS I) as

\[
\nabla I(v, n) = \frac{h \nu_0}{4\pi} \left[ n_i A_{ji} \psi_{ji}(v, n) \right] 
- \left[ 1 + \frac{B_{ji}}{A_{ji}} I(v, n) \right] - n_i B_{ji} \varphi_{ij}(v, n) I(v, n) \right].
\]

Here \( v \) is the frequency of the photon, \( n \) is the unit vector in the direction of propagation; \( A_{ji}, B_{ji}, B_{ij} \) are the Einstein coefficients for spontaneous emission, stimulated emission, and absorption, respectively. \( n_i \) and \( n_j \) are the atomic level populations for levels \( i \) and \( j \), respectively, and \( \varphi_{ij} \) and \( \psi_{ji} \) are the line profile coefficients for absorption and emission, respectively (often called simply absorption and emission profiles). The profile coefficient for stimulated emission is identical to that for spontaneous emission, as demonstrated by Oxenius (1965).

b) Statistical Equilibrium Equations

The atomic level populations can be determined from the set of statistical equilibrium equations (also called rate equations) which represent the zero-order steady-state moment equations of corresponding kinetic equations for an atom in its individual energy states. Neglecting the streaming term (see Hubený, 1981b), one can write for the level \( a \):

\[
\frac{d}{dt} \left( n_a \sum_{b \neq a} (C_{ab} + R_{ab}) - \sum_{b \neq a} n_b (R_{ba} + C_{ba}) \right) = 0,
\]

where \( R_{ab} \) and \( C_{ab} \) are the radiative and collisional rates for the transition \( a \rightarrow b \), respectively (including continuum states).

According to the semi-classical formulation developed in Paper I and HOS I, II, the radiative rates for transitions between two discrete levels are given by

\[
R_{iu} = B_{iu} \int I(v, n) \varphi_{ia}(v, n) \, dv \, d\Omega / 4\pi ,
\]

and

\[
R_{ai} = A_{ai} + B_{ai} \int I(v, n) \psi_{ai}(v, n) \, dv \, d\Omega / 4\pi ,
\]

for \( i < u \).

However, as discussed by CHO, the stimulated emission rate [the second term on the r.h.s. of Eq. (2.4)] can simply be given by the absorption profile \( \varphi_{ia}(v, n) \) instead of the emission profile \( \psi_{ai}(v, n) \) in the integrand. One thus obtains

\[
R_{ai} = A_{ai} + B_{ai} \int I(v, n) \varphi_{ia}(v, n) \, dv \, d\Omega / 4\pi .
\]

CHO showed that, strictly speaking, the stimulated emission rate is given by

\[
B_{ai} \int I(v, n) \int d^3v f_a(v) \alpha_{ai} [v - (v_0/c) n \cdot v] \, dv \, d\Omega / 4\pi ,
\]

where \( \alpha_{ai} \) is the atomic absorption profile, while the absorption rate is given by

\[
B_{ia} \int I(v, n) \int d^3v f_a(v) \alpha_{ia} [v - (v_0/c) n \cdot v] \, dv \, d\Omega / 4\pi .
\]

Equation (2.5) is thus valid only if \( f_a(v) = f_a(v) \).

Further, it should be stressed that Eq. (2.5) represents a consistent approximation, and not only a convenient computational simplification, due to the following reasons: i) in the low-intensity limit (i.e. if \( B_{ai} I(v, n) / A_{ai} \ll 1 \), the rate of stimulated emission is negligible compared to the spontaneous rate; differences between the emission and the absorption profiles are of no consequence in the rate equations; ii) for higher radiation intensities, the simple rate equations can only be derived assuming broadband irradiation (i.e. the radiation intensity is assumed almost constant within the natural width of the line); in this case \( \varphi_{ia}(v, n) = \psi_{ai}(v, n) \) anyway; iii) in situations, where
the stimulated emission is important, i.e. beyond the approximation of the low-intensity, broadband radiation field, more fundamental changes of the theory will be required. Particularly, in addition to the correlated chain of radiation transitions \( l \rightarrow u \rightarrow l \) (described by the usual redistribution functions), more complicated chains like e.g. \( l \rightarrow u \rightarrow l \rightarrow u \rightarrow \rightarrow l \), etc., should be considered (because the stimulated emission leads, in general, to deviations from the natural population of the lower level — see HOS I).

Concluding, it is meaningless to deal with the very complicated Eq. (2.4) and, at the same time, to adopt the simple semi-classical formalism that is based on the two-photon redistribution functions only. Equation (2.5) can thus be viewed as an internally consistent approximation for most cases of astrophysical interest.

c) Expressions for Profile Coefficients

The main difficulty in determining atomic line profile coefficients is connected with the existence of correlations between photons that are involved in consecutive radiative transitions of an atom. This phenomenon was thoroughly discussed by HOS I, II, who showed that convenient formulas for profile coefficients can be written in terms of suitably introduced generalized redistribution functions. Moreover, the absorption and emission profiles depend on the velocity distribution of atoms in various excited states and on the radiation field. The resulting formulas are very complicated even in the simple case of a three-level atom with neglected stimulated emission.

In this paper, we shall be concerned with a simplified problem that consists in assuming absorption and emission profiles in all transitions except the one of interest, say \( i \leftrightarrow j \), to be given by corresponding natural excitation profiles (see HOS I). In other words, the generalized redistribution functions for radiation transition chains are given by (schematically)

\[
R_{abcd\ldots} = \phi_{ab}^* \phi_{bc}^* \phi_{cd}^* \cdots.
\]

The only transition chain taken into account explicitly is that of \( i \rightarrow j \rightarrow i \), for which

\[
R_{ijij} = \phi_{ij}^* \phi_{ji}^*.
\]

Here \( \phi_{ab}^* \) denotes the natural excitation profile coefficient. As stressed in HOS II, \( \phi_{ab}^* \) is not, in general, equal to \( \phi_{ab}^* \) unless the velocity distribution of atoms in levels \( a \) and \( b \) are identical.

To simplify the problem further, we shall assume the velocity distributions for all levels to be Maxwellian, and neglect velocity-changing collisions. Further, we shall assume that the rate of stimulated emissions \( j \rightarrow i \) is sufficiently lower than the rate of spontaneous emissions \( j \rightarrow i \), so that level \( i \) can be assumed to be populated naturally. In other words, deviations of the stimulated emission profile from the natural excitation profile are only taken into account in the transfer equation, otherwise the stimulated emission is treated as negative absorption.

Under these assumptions, the absorption profile \( \varphi_{ij}(v, n) = \varphi_{ij}^*(v, n) = \varphi_v \) is independent of direction, and is simply given by a prespecified function that is independent of the radiation field and atomic level populations (e.g., the Voigt profile in the case of isolated atomic levels). The emission profile is given by [see, e.g. HOS II, Eq. (4.15)]

\[
\psi(v, n) = \varphi_v + \frac{n_{ij} R_{ij}}{n_{ij} P_{ij}} \varphi_v \left[ \mathcal{B}(v, n) - J \right],
\]

where

\[
\mathcal{B}(v, n) = \frac{1}{n_{ij}} \int \int R(v', n', v, n) I(v', n') dv' d\Omega'/4\pi,
\]

\[
J = \int \int I(v', n') \varphi_v \, dv' d\Omega'/4\pi,
\]

and

\[
P_{ij} = \sum_{k+j} (R_{jk} + C_{jk})
\]

Here, \( R \) is the velocity-averaged redistribution function for the transition chain \( i \rightarrow j \rightarrow i \).

Another simplification consists in adopting the isotropic approximation (Hummer, 1969; Milkey et al., 1975b), which is introduced here for purposes of comparison with other approaches and for notational convenience rather than for necessity. In this case

\[
\mathcal{B}(v, n) \equiv \mathcal{B} = \int R(v', n) J_{v'} \, dv'/\varphi_v,
\]

\[
J = \int J_{v} \varphi_v \, dv,
\]

\[
J_v = \int I(v, n) \, d\Omega/4\pi.
\]

It should be stressed that the physical basis of the present approach is contained in Eq. (2.7). For more realistic models the transfer equation (2.1) and the rate equations (2.2) remain unchanged, while the profile-definition equation (2.7) should be replaced by a more general equation that would involve further two-photon redistribution functions and, in general, multi-photon generalized redistribution functions corresponding to various transition chains ending with the transitions \( j \rightarrow i \) and \( i \rightarrow j \) (see HOS II).
On the other hand, the classical multilevel transfer problem assuming complete redistribution (CRD) is described by Eqs. (2.1) and (2.2), and by the equality \( \psi(v, n) = \psi_s = \varphi_s \) instead of Eq. (2.7).

3. Formulation of the Equivalent-Two-Level-Atom Approach

a) Preliminary Considerations

There are, in principle, two types of effects that arise due to the difference of the emission profile (2.7) from the absorption profile, namely i) direct effect that involves changes of the radiation intensity, and to a lesser extent the radiative rates, in the line \( i \leftrightarrow j \) itself; and ii) indirect effect — an overall switch of the population balance induced by changes of the radiative rates for the transition \( i \leftrightarrow j \) (sometimes called interlocking effect).

Nevertheless, under most astrophysically important circumstances, the direct effect is far more important than the indirect one. Indeed, numerical experience with simple partial redistribution transfer solutions (see, e.g., Hummer, 1969; Mihalas, 1978) indicates that the radiation intensity in the line is affected primarily in the line wings while there are almost negligible differences between partial redistribution (PRD) and CRD solutions in the line core. On the other hand, a dominant contribution to the radiative rates comes from the line core. Consequently, these rates are usually much less affected by deviations from CRD than the radiation field itself.

The above considerations indicate that once a CRD solution for a given multilevel atom is known, a simultaneous solution of Eqs. (2.1), (2.2), and (2.7) can be obtained by an iterative process: influence of the emission profile (2.7) on the transfer equation (direct effect) should be treated explicitly, while the interlocking effect (indirect effect) can be treated iteratively. This is indeed the idea of the well-known equivalent two-level-atom approach (ETA).

Our approach should not be confused with the use of ETA to solve the CRD transfer problems. In our case, the CRD solution is already given, thus the dominant part of the interlocking between various transitions is already taken into account. Indeed, in the extreme case, a PRD solution can be obtained by simply re-evaluating the transfer equation with the emission profile (2.7) and assuming CRD populations held fixed. Numerical experience shows that even this simplest approach yields quite satisfactory first-order estimates for departures from CRD in many cases of interest.

b) Classical ETA

We shall briefly outline the basic formulas of the standard ETA procedure for CRD in all transitions. For a detailed description and thorough discussion see, e.g., Mihalas (1978).

The transfer equation (2.1) is written as

\[
\frac{dI_s}{dz} = -\chi_s(I_s - S_s),
\]

where the absorption and emission coefficients are given by

\[
\chi_s = \frac{h \nu_0}{4\pi} B_{ij} \varphi_s \left( n_i - n_j \frac{g_i}{g_j} \right),
\]

\[
\eta_s = \frac{h \nu_0}{4\pi} A_{ji} n_i \varphi_s,
\]

and the (frequency-independent) source function is given by

\[
S_s \equiv S_s^{CRD} = \frac{n_j A_{ji}}{B_{ij} (n_i - n_j g_i / g_j)}.
\]

The rate equations for the levels \( i \) and \( j \) are written as

\[
a_2 + n_i (R_{ji} + C_{ji}) - n_i (a_1 + R_{ij}) = 0,
\]

\[
a_4 + n_i (R_{ij} + C_{ij}) - n_i (a_3 + R_{ji}) = 0,
\]

where

\[
a_1 = \sum_{i \neq i, j} (R_{ii} + C_{ii}) + C_{ij};
\]

\[
a_2 = \sum_{i \neq i, j} n_i (R_{ii} + C_{ii});
\]

\[
a_3 = \sum_{i \neq i, j} (R_{ji} + C_{ji}) + C_{ji},
\]

\[
a_4 = \sum_{i \neq i, j} n_i (R_{ij} + C_{ij}).
\]

By eliminating populations from Eq. (3.4) using Eq. (3.5) and substituting Eqs. (2.3) and (2.5), and using the well-known relations between Einstein coefficients, one obtains

\[
S_s^{CRD} = \frac{\bar{J} + \eta}{1 + \varepsilon},
\]

where

\[
\alpha = \frac{g_i}{g_j} \frac{\beta}{A_{ji}}; \quad \eta = \frac{\beta}{B_{ij}}
\]

and

\[
\alpha = \frac{a_2 a_3 + a_4 C_{ji}}{a_2 + a_4}; \quad \beta = \frac{a_1 a_4 + a_2 C_{ij}}{a_2 + a_4},
\]

and \( \bar{J} \) is given by Eq. (2.12).
The most important property of the ETA scheme is that the line source function is linear in the mean intensity of radiation in the given line. On the other hand, this property also represents its basic drawback, because an overall coupling between different transitions is included implicitly in the parameters \(\varepsilon\) and \(\eta\).

c) ETA for partial redistribution

As discussed above, the rate equations remain in the form of Eqs. (3.5) – (3.6), while the absorption and emission coefficients now become [using Eqs. (2.1), (2.7), (2.10), (2.11)]

\[
\chi_v = \frac{h\nu_0}{4\pi} B_{ij} \varphi_v \left( n_i - n_j \frac{g_i}{g_j} \varphi_v \right),
\]

and

\[
\eta_v = \frac{h\nu_0}{4\pi} \left[ n_j A_{ji} \varphi_v + n_i B_{ij} \frac{A_{ji}}{P_j} \varphi_v \right] \left( \frac{\varphi_v}{\varphi_v} - J \right),
\]

where

\[
\varphi_v \equiv \varphi_v = 1 + \frac{n_i}{n_j} \frac{B_{ij}}{P_j} \left( \frac{\varphi_v}{\varphi_v} - J \right).
\]

Applying an algebraic procedure analogous to that in the case of CRD, one obtains

\[
S_v = \frac{J + \mu(\varphi_v - J) + \eta}{1 + \varepsilon},
\]

where \(\eta\) is given by Eq. (3.8), and

\[
\varepsilon_v = \frac{\alpha - \beta \varphi_v}{A_{ji} g_i g_j} + J \frac{c^2}{2h\nu_0^2} (1 - \varphi_v),
\]

and

\[
\mu = \frac{A_{ji} + B_{ij} J + \alpha}{P_j}.
\]

The most important difference of the partial-redistribution form of ETA with respect to the classical ETA is that the line source function is no longer linear in \(J_v\), owing to the dependence of \(\varepsilon_v\) and \(\mu\) on \(J_v\). Physically, this is caused by the deviations of the stimulated emission profile from the absorption profile. Consequently, in contrast to the case of CRD, where the radiative transfer equation with the source function (3.4) can be solved in a single step, the transfer equation for PRD is, in general, not easily solvable. However, in cases where the stimulated emission is of secondary importance, one can proceed in a simple iterative way: the parameters \(\varepsilon_v\) and \(\mu\) are given by a previous step and held fixed during the solution of the transfer equation. As the initial values, one may adopt \(\mu = (A_{ji} + \alpha)/P_j\), and \(\varphi_v = 1\).

We end this section with expressing the line source function (3.13) by means of the CRD source function, viz.\(\)

\[
(3.16) \quad S_v = \delta_v S^{\text{CRD}} + \frac{\mu \delta_v}{1 + \varepsilon} \left( \frac{\varphi_v}{\varphi_v} - J \right),
\]

where

\[
(3.17) \quad \delta_v = \frac{1 + \varepsilon_v}{1 + \varepsilon}.
\]

The parameter \(\delta_v\) does not usually differ very much from unity; it becomes exactly equal to unity either for \(\varphi_v = 1\) (which corresponds to assuming the stimulated emission profile equal to the absorption profile), or if the stimulated emission is neglected altogether. Equation (3.16) clearly shows that for a strict two-level atom, for which \(\mu = 1\), departures from complete redistribution are potentially greatest. On the other hand, for subordinate lines in a multi-level atom, \(\mu\) is typically less than unity, due to the possibility of transitions from various levels to the upper level. Consequently, the importance of partial redistribution effects is smaller.

Finally, we note that in putting \(\delta_v = 1, \mu = 1\), one recovers the line source function employed by Vernazza et al. (1981) — their Eq. (A 25) that is based on the formulation of Heasley and Kneer (1976).

4. Approximate Schemes

a) Approximate OSC Form

The appropriate atomic-frame redistribution function for an isolated line in the impact approximation, and with neglected velocity-changing collisions, has been derived by Omont, Smith and Cooper (1972 – OSC). Heinzl and Hubený (1982) showed that the OSC redistribution function, averaged over the Maxwellian velocity distribution (so-called laboratory-frame redistribution function), can generally be written as

\[
(4.1) \quad R(v', v) = \gamma R^*(v', v) + (1 - \gamma) R_{\text{III}}(v', v),
\]

where

\[
(4.2) \quad R^* = \begin{cases} R_{\text{II}} & \text{for negligible lower state broadening, i.e. resonance lines} \\ R_{\text{V}} & \text{for non-negligible lower-state broadening, i.e. subordinate lines} \end{cases}
\]

Here \(R_{\text{II}}\) and \(R_{\text{III}}\) are the redistribution functions introduced by Hummer (1962); \(R_{\text{V}}\) has been introduced by Heinzl (1981). The branching ratio \(\gamma\) is given by
Heinzel and Hubený (1982)

\[ \gamma = \frac{P_j}{P_j + Q_E}, \]

where \( Q_E \) is the elastic collision rate.

Substituting Eqs (4.1) and (4.3) into (3.13), and introducing

\[ \lambda = \mu \gamma = \frac{A_{ij}}{P_j + Q_E} \left( 1 + \frac{c^2}{2hv_0^2} \frac{J}{J + \frac{c}{A_{ij}}} \right), \]

one obtains

\[ S_v = \frac{J + \lambda (\mathcal{R}_v^e - J) + \eta}{1 + \epsilon_v}, \]

where \( \mathcal{R}_v^e \equiv \int R^e(v', v) J_{v'} \, dv'/\varphi_{v'} \).

In deriving Eq. (4.5), we made a common approximation

\[ R_{\text{li}}(v', v) \equiv \varphi_v \varphi_{v'}, \]

which has been shown to be quite satisfactory for most applications (Finn, 1967; Vardavas, 1976).

The final expression (4.5) for the line source function bears close resemblance to Eq. (3.13), from which it may be obtained formally by replacing \( \mu \to \lambda, \mathcal{R}_v \to \mathcal{R}_v^e \). Physically, this reflects the fact that collisions, both elastic and inelastic, redistribute the radiation completely. However, this is strictly valid in the atomic rest frame only. The simplicity of Eq. (4.5) follows from Eq. (4.6) that expresses the neglect of the additional correlation of photon frequencies measured in the laboratory frame. We note in passing that the collisional redistribution due to inelastic collisions was a point of controversy for some time (OSC; Cooper and Ballagh, 1978; Yelnik and Voslaumer, 1979). For a detailed discussion of this question, see (Heinzel and Hubený, 1982 – Appendix); their analysis shows that the present formulation is physically correct.

b) Partial Coherent Scattering Approximation

Another approximation that enables one to avoid the calculation of the redistribution functions entirely, is the so-called partial coherent scattering approximation (PCS). It was originally introduced by Jeffries and White (1960), and subsequently modified by Kneer (1975). Recently, its generalization for \( R_V \) has been presented by Hubený and Heinzel (1984).

The basic idea of this approach is to approximate the exact redistribution function \( R^e \) by a simple form

\[ R^e(v', v) \approx \xi a(v', v) \varphi_{v'} \delta(v' - v) [1 - \xi a(v', v)] \varphi_v \varphi_{v'}, \]

where

\[ a(v', v) = \bar{a}[\max \{ |x_v|, |x_{v'}| \} ]; \]

\[ a(v, v) = \int_{-\infty}^{\infty} a(v', v) \varphi_{v'} \, dv'. \]

Here, \( \delta \) is the Dirac \( \delta \)-function, \( x = (v - v_0) / \Delta v_p \) is the frequency displacement from the line center measured in units of Doppler widths, \( \Delta v_p \). The function \( \bar{a}(x) \) is an empirical function of \( x \) that has to satisfy the conditions \( \bar{a}(x) \to 0 \) for \( x \to 0 \), and \( \bar{a}(x) \to 1 \) for large \( x \). Kneer (1975) suggested a simple analytical form

\[ \bar{a}(x) = 1 - \exp \left( -\frac{(x - x_0)^2}{A} \right), \]

for \( x \geq x_0 \); \( \bar{a}(x) = 0 \) for \( x < x_0 \),

where \( x_0 \) and \( A \) are empirical parameters. He showed that the choice \( x_0 = 2, A = 2 \) yields the best agreement with solutions using the exact \( R_{\text{li}} \); Hubený and Heinzel (1984) found that the same values are also convenient for the case of \( R_V \). Yet several other representations of \( \bar{a}(x) \) are possible (see, e.g., Vernazza et al., 1981).

The parameter \( \zeta \), equal to unity for \( R_{\text{li}} \), reflects the fact that the resonance scattering between two broadened levels is not coherent (in the atomic frame) even in the absence of collisions. A numerical evaluation of \( R_V \) (Heinzel and Hubený, 1983) revealed that \( \zeta \) can be given simply by

\[ \zeta = \frac{P_j + Q_E}{P_i + P_j + Q_E}, \]

In the case of resonance lines, \( P_i \ll P_j \), and consequently \( \zeta \approx 1 \).

On the other hand, in the limit of very large elastic rates, \( Q_E \gg P_i, P_j \), one obtains \( \zeta \approx 1 \), i.e. one again obtains the maximum contribution of partial redistribution [see Eq. (4.7)]. At first sight, it might seem that this contradicts the fact that elastic collisions yield the complete redistribution. However, this is not the case because the parameter \( \zeta \) enters the line source function [see Eq. (4.11)] only through the product \( \gamma \zeta = P_j / (P_j + P_i + Q_E) \), which goes to zero for \( Q_E \gg P_i, P_j \). One thus obtains \( S_v \approx S_{\text{CRD}} \) regardless of the detailed form of the function \( R^e \). This consideration also indicates that the approximation (4.10) is only useful for sufficiently low elastic collision rates, \( Q_E \ll P_j \) (the case usually met in astrophysical applications).

By substituting Eq. (4.7) into (4.5), one obtains

\[ S_v = \frac{J + \lambda \xi a_{v'} J_{v'} - J^e + \eta}{1 + \epsilon_v}, \]
where

\[ J^* v = \int_0^\infty a(v', v) J_{v'} \varphi_{v'} dv'. \]

Again, in putting \( \xi = 1, \delta = 1, \) and \( \mu = 1, \) one recovers Eq. (A 36) of Vernazza et al. (1981).

The same partial coherent scattering approximation comes from the fact that an actual correlation of photon frequencies is approximated by laboratory-frame coherent scattering for sufficiently large frequency displacements from the line center (in practice, for \( x \gtrsim 6 \)). However, in reality, the frequency of a wing photon is shifted, on the average, by one Doppler width during each scattering. Therefore, this approximation has to be used with caution, for it does not allow for such a photon diffusion in line wings. Consequently, it may lead to incorrect results for strong lines whose wings are optically very thick (Milkey, 1976; Basri, 1980; Frisch, 1980). Nevertheless, it can be used as a first-order estimate of the PRD effects in a wide range of astrophysically relevant problems. A detailed discussion of this question and a further modification of this approach is presented elsewhere (Hubený, 1984).

5. Solution of the Multilevel Transfer Problem

In this section, we outline an application of the formalism developed in the previous sections to the solution of line formation in multilevel atoms, where one chosen transition is allowed to depart from CRD. In solving any actual transfer problem, one has to take into account effects of the background continuum. This may be done quite easily by putting

\[ \chi^\text{tot}_v = \chi_v + K_v, \]

and

\[ \eta^\text{tot}_v = \eta_v + E_v, \]

where \( \chi^\text{tot}_v \) and \( \eta^\text{tot}_v \) represent total absorption and emission coefficients, respectively; \( \chi_v \) and \( \eta_v \) are the line contributions given by Eqs (3.10) and (3.11), and \( K_v \) and \( E_v \) are the background opacity and emissivity, respectively. The total source function is given by

\[ S^\text{tot}_v = \frac{\varphi_v}{\varphi_v + r_v} S^L_v + \frac{r_v}{\varphi_v + r_v} S^\text{cont}_v, \]

where \( S^L_v \) is the line source function given by Eq. (3.13) [or its approximate forms (4.5) or (4.11)]; \( S^\text{cont}_v = E_v/K_v \) is the continuum source function, and

\[ r_v = K_v \left[ \frac{\hbar \nu_0}{4\pi} n_i B_{ij} \left( 1 - \frac{n_j}{n_i} \frac{\varphi_i}{\varphi_0} \right) \right]. \]

In contrast to the case of CRD, the parameter \( r \) depends on frequency even for frequency-independent background opacity. Fortunately, this frequency dependence, which is induced by deviations of the stimulated emission profile from the absorption profile, is usually quite weak. Therefore, parameter \( r \) can be treated iteratively like \( \varepsilon \) and \( \mu \).

Solution of the multilevel transfer problem may thus be obtained by applying the following scheme:

I) Solve the standard CRD multilevel problem, for example by the complete linearization method (Auer, 1973; Auer and Heasley, 1976).

II) For a chosen PRD transition, iteratively solve the radiative transfer equation (2.1) with the source function (3.13) (or its approximate forms). One can use current estimates of the level populations and mean intensities of radiation to determine \( \varphi_v, \mu, \eta, \varepsilon, \) and \( r \). In this inner iteration loop, only the radiation field in a chosen transition and corresponding populations of the lower and upper levels are updated. This yields corresponding improvements of \( \varphi_v, \mu, \eta, \varepsilon, \) and \( r \) (not \( \eta \)). The process is then iterated to convergence.

III) Then perform the next iteration of the complete linearization method, in the standard CRD form, with \( \psi_v = \varphi_v \varphi_e \), where \( \varphi_e \) is held fixed (i.e. it is not linearized). This only requires a trivial modification of the CRD complete linearization code. One thus obtains new values for all the populations. If necessary, one may return to step II and repeat the procedure until overall convergence is achieved.

It should be emphasized that in some applications step III may be reduced to just one further iteration, or be omitted entirely. For example, in calculating the PRD formation of the Mg II resonance lines (see, e.g., Milkey and Mihalas, 1974), one does not take the influence of the PRD in resonance lines on populations of high levels and vice versa into account at all. Notice that the latter authors employed the complete linearization technique to solve just step II.

6. Discussion

We have presented the formulation of a modified equivalent-two-level-atom approach for a frequency-dependent source function that is appropriate to treating departures from complete redistribution. Further, we have shown that this approach can also be employed in solving radiative transfer in a multilevel atom, provided that just one transition is allowed to depart from CRD. The proposed iterative scheme may easily be incorporated in existing computer codes.
for solving the standard CRD multilevel transfer problems. The only substantial modification concerns the formal solution step (i.e. a solution of the radiative transfer equation with a given source function) where one should employ numerical methods that are capable of handling the frequency-dependent source function of the form of Eq. (3.13).

It is worth stressing that the chosen “PRD transition” need not necessarily be a resonance line; the formalism is capable of treating a subordinate line as well.

At first sight, it might seem that the proposed scheme possesses several drawbacks as compared to the substate formulation of Milkey and Mihalas (1973). However, this is not the case. The latter method in fact solves a problem analogous to step II of the preceding section, [i.e. gives a solution of the transfer equation with a non-linear source function analogous to our form (3.13)]. Moreover, it uses an incorrect description of stimulated emission [see Bachek et al., (1981); Paper I], which yields much more complicated and apparently a “more non-linear” source function. In other words, the use of the complete linearization method was required mainly by the non-linearity of the emission coefficient (or the source function) in the line of interest itself, rather than by the interlocking between various transitions.

Our approach is closest in spirit to that of Heasley and Kneer (1976) [see also Mihalas (1978, p. 437)], yet ours differs from the latter in three important aspects: i) more consistent treatment of stimulated emission; ii) inclusion of multilevel effects; and iii) a possibility to treat subordinate lines.

Further, our lagging technique, consisting in considering a fixed value of a current estimate of the parameter \( q_* = \psi_* / \sigma_0 \) in a subsequent iteration of complete linearization, should not be confused with the seemingly similar approach of Ayres and Linsky (1976). The latter authors applied the idea of fixed \( q_* (\beta, \text{in their notation}) \) to solve a problem that is, again, equivalent to our step II, while the present treatment employs fixed \( q_* \) to evaluate the overall coupling between various levels. Moreover, our solution of step II only employs the parameter \( q_* \) for treating the stimulated emission term; the basic effect – departures of the spontaneous emission profile from CRD – is treated in a fully explicit way.

We stress again that the apparent simplicity of our approach is due to two facts, namely: i) simpler but more consistent treatment of stimulated emission, and ii) treatment of just one transition with PRD. As far as the stimulated emission is concerned, we feel that either the approach adopted in this paper may be employed, or a more fundamental formalism has to be developed. In semi-classical terminology, one has to take into account not only correct profile coefficients, but also deviations from the natural population induced by stimulated emission. Such a procedure was outlined in HOS I, but the corresponding velocity averaging would be, even in the simplest case of Maxwellian velocity distributions, extremely complicated.

Allowing for several transitions to depart from complete redistribution would yield problems quite analogous to those met in applying the classical CRD equivalent-two-level-atom approach to multilevel atoms. In such a case, several transitions would be coupled by redistribution (more precisely, by a photon correlation). Thus a new phenomenon, which may be called redistribution interlocking, would arise. To this category belongs, for example, the coupling of several lines that originate as transitions from a common upper level to different lower levels [e.g., the case of Ca II H and K lines, treated by Milkey et al. (1975a); and Ayres and Linsky (1976)]. In general, the relevant equations have been given by HOS II; they form a highly coupled nonlinear set which does not generally allow the idea of the equivalent-two-level-atom to be applied. Yet there are several cases where ETA can be applied. They will be discussed in the next paper of this series.

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REFERENCES

DOUBLE-STATION OBSERVATIONS OF 454 TV METEORS

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ТЕЛЕВИЗИОННЫЕ НАБЛЮДЕНИЯ 454 МЕТЕОРОВ С ДВУХ СТАНИЦ

I. Траектории

Описываются наблюдения метеоров с двух станций, сделанные с помощью двух телевизионных систем для низкого уровня яркости (диапазон абсолютных магнитуд 0,5 ÷ 8,5). Кроме того, что настоящее телевизионное наблюдение метеоров точнее предыдущего, рассмотренная совокупность данных состоит из большего числа метеоров, чем раньше. Траектории в атмосфере, скорости, видимый блеск метеоров и их массы представляют вместе с начальной и окончательной высотами и с высотами соответствующими максимальным светимостям.

Double-station meteor observations made with two different low-light-level television systems are described which together span the magnitude range 0.5 to 8.5 absolute magnitudes. Besides being more accurate than previous television meteor surveys the present data set constitutes a much larger sample than was previously available. The atmospheric trajectories, velocities, brightnesses and mass are presented together with the beginning and ending heights and the heights of maximum luminosity.

1. Introduction

The observation of meteors by low-light-level television provides a way of investigating meteors which is relatively free of observational bias. This is important since for meteor magnitudes fainter than +5 the other possibilities — radar and telescopic observations — are subject to serious sources of error. The radar method suffers greatly from the masking effect of the "underdense echo ceiling" (Greenhow, 1961) which not only causes many meteors to go undetected but it also makes it very difficult to salvage reliable information about the ionization profiles of those meteors which are detected. Telescopic meteor observations suffer from many of the same sorts of drawbacks experienced by unaided visual observations — for example slow response time of the eye and uncertainties in the direction and brightness estimates.

Hawkes and Jones (1980) and Hawkes, Jones and Ceplecha (1983) have described some limited double-station TV meteor observations which have pointed towards exciting possibilities concerning the constitution of meteoroids and also the evolution of interplanetary dust complex. This present paper describes the results obtained so far in this on-going program. The results in Hawkes et al. referred to relatively bright