FREQUENCY SPLITTING IN Ap STARS

W. DZIEBOWSKI
N. Copernicus Astronomical Center, Warsaw

AND

PHILIP R. GOODE
Department of Physics, New Jersey Institute of Technology, Newark

Received 1985 March 6; accepted 1985 June 24

ABSTRACT

We generalize the oblique pulsator model by Kurtz to account for the observed properties of rapidly oscillating Ap stars. In our model, which includes advection and an oblique magnetic field, the modes are represented, in general, by a superposition of all spherical harmonics having the appropriate degree, \( l \). We predict that an observer will report a mode splitting into \( (2l + 1) \) frequency components with the spacing equal to the rotation frequency of the star. The relative amplitudes at these frequencies are predicted following from a postulated selective excitation mechanism due to the field, and the diagnostic potential of the data on these Ap stars is discussed.

Subject headings: stars: peculiar A — stars: pulsation

I. INTRODUCTION

Rapid light variations having periods between 6 and 12 minutes have been accurately observed by Kurtz (1982) in several slowly rotating Ap stars. Each of these stars is known to have a strong magnetic field which is inclined to its axis of rotation. A typical period of rotation is about 20 days. The largest amplitude light fluctuations observed are synchronous with the largest magnetic field effects. The spacing, in frequency, between adjacent peaks in the power spectrum is precisely once or twice the rotational frequency of the star.

Kurtz (1982) has attributed the rapid light variations to low-degree \((l = 1 \text{ or } 2)\), high radial order \((n)\) modes which are phase locked to the magnetic field. Kurtz (1982) argues that the detailed frequency pattern in his data is due to the magnetic field exciting a single acoustic mode in each of many \((nl)\)-multiplets and each mode is split into uniformly spaced components as viewed from the observer's frame of reference.

Objections to Kurtz's model have been raised arguing that rotation implies advection, which he ignores, and advection disperses the oscillations which, therefore, cannot be phase locked to the magnetic field. Dolez and Gough (1982) suggested a nonadiabatic mechanism to account for Kurtz's result. They argued that the presence of the field results in the excitation of modes that could have a large amplitude in the region of the field. As the field rotates, these modes de-excite and other modes, now favored, are excited. Mathys (1984) has suggested a different explanation for Kurtz's data. Like Kurtz, he assumed that the excitation of a single mode, which, however, is symmetric about the axis of rotation. The apparent mode splitting is due to modulation of the oscillation signal due to large spots on the surface of the star.

Another work concerning these Ap stars is due to Aizenman et al. (1984). They wanted to determine the rotational splitting of a mode in a star with an inclined rotation axis. Their formula is the Ledoux (1951) formula multiplied by \( \cos \beta \), where \( \beta \) is the angle of inclination of the rotation axis with respect to the pulsation axis. Their formula is not applicable to any real system because it ignores the implicit effect which fixes the pulsation axis.

In our view, the models of Dolez and Gough (1982) and Mathys (1984) introduce unnecessary complications to the problem and do not appear to account for the observed long-term phase coherence of the data. In this Letter, we show that simultaneously accounting for the effect of rotation and the magnetic field on adiabatic pulsation yields a simple and natural explanation of the observed splittings. This model is similar to that of Kurtz while accounting for its shortcomings. We also propose a simple model of mode selection and comment on the observed amplitudes for each frequency in a multiplet.

II. FORMALISM

We show here a self-contained treatment of the problem of a nonaxisymmetric steady perturbation due to a corotating oblique magnetic field. Because the perturbation is nonaxisymmetric, the normal modes are coupled which implies that degenerate perturbation theory be used to solve the problem. Our treatment here is similar to that of Gough and Taylor (1984) and Dziembowski and Goode (1984) and is consistent with, but different from, the earlier treatment by Dicke (1982).

We start by writing \( \xi_{nlm}(r, \theta, \phi, t) \), abbreviated as \( \xi_{lm} \), the unperturbed \((nlm)\)-oscillation eigenfunction in the intrinsic frame with respect to the rotation axis of the field. Defining \( O_{jm} \) to be an element of the perturbing operator, we write the
real, symmetric matrix equation

\[
(\delta_{j,m} \omega_l^{(s)} - O_{jm}) a_m^{(s)} = 0, \quad (1)
\]

where the normalized, perturbed eigenfunction in the same frame as \( \xi_m \) is given by

\[
\xi_i^{(s)} = \sum_{m=-l}^{+l} a_{m}^{(s)} \xi_m. \quad (2)
\]

For each \((nl)\)-multiplet there are \((2l + 1)\)-eigenvalues, \( \omega_l^{(s)} \), and eigenvectors, \( \xi_i^{(s)} \). The \((2l + 1)\)-eigenvalues and eigenvectors are labeled by \( s \). The matrix element \( O_{jm} \) has two components, a diagonal one due to rotation and the other due to magnetism, that is

\[
O_{jm} = \omega_{j,m}^{\text{rot}} \delta_{j,m} + O_{jm}^{\text{mag}}. \quad (3)
\]

Assuming a constant rotation rate, the Ledoux (1951) formula applies and the splitting due to advection, \( \omega_l^{\text{rot}} \), is given by \( m \Omega \), where \( \Omega \) is a constant. Centrifugal effects of rotation are negligible. The matrix \( O_{jm}^{\text{mag}} \) is not diagonal in the chosen frame. Assuming the magnetic field is axially symmetric, \( O_{jm}^{\text{mag}} \) is, generally, most easily evaluated by transforming to the frame in which it is diagonal, that is, the frame in which the \( z \)-axis is the field’s axis of symmetry.

The transformation to the axisymmetric system of the field is obtained by applying Wiener functions to the spherical harmonics of the eigenfunction, that is,

\[
\xi_m(\theta) = \sum_{m'} d_{m't, m}^{(s)}(\beta) \xi_{m'}(\theta'), \quad (4)
\]

and where \( \beta \) is the angle between the rotation and magnetic field axes. For a treatment of the Wiener functions see Edmonds (1960). Using equations (1) and (4), we write

\[
O_{jm}^{\text{mag}} = \sum_{m'} d_{m't, m}^{(s)} a_{m'}^{(s)} \omega_{1,m'}^{\text{mag}}. \quad (5)
\]

For a dipole-like field, we have

\[
\omega_{1,m}^{\text{mag}} = \frac{l(l+1) - 3(m')^2}{4l(l+1) - 3} K_{m',l}, \quad (6)
\]

where the coefficient of \( K_{m',l}^{\text{mag}} \) follows from a \( |Y_{m'}^{(s)}|^2 \)-weighted integration over the \( P_l \)-distortion of the matter caused by the field. The splitting is due to the nonadiabatic effects. For high-order \( p \)-modes, e.g., Dziembowski and Goode (1984) have shown that \( K_{m',l}^{\text{mag}} / \omega_0 \) is proportional to the ratio of the magnetic pressure to the gas pressure, where \( \omega_0 \) is the unperturbed oscillation frequency. Even if the ratio of the magnetic pressure to the gas pressure is quite small, magnetic field effects may well dominate those of rotation because \( \omega_{j,m}^{\text{rot}} / \omega_0 \approx 10^{-7} \) to \( 10^{-6} \) for the case of Ap stars.

The relative change in luminosity due to oscillations as given by Dziembowski (1977) is

\[
\Delta L^{(s)} = \sum_m \text{Re} \left[ Y_{m'}^{(s)}(\theta, \phi) e^{i\omega t} \right] a_m^{(s)}, \quad (7)
\]

where \( \theta, \phi \) are the inclination angles of the rotation axis with respect to the observer. In accounting for the transformation to the inertial frame of the observer, we note that

\[
\Delta L^{(s)} = \sum_m \text{Re} \left[ Y_{m'}^{(s)}(\theta, \phi) e^{i\omega t} \right] a_m^{(s)}, \quad (8)
\]

Thus, the splitting of the excited modes is described by

\[
\Delta L^{(s)} = \sum_m a_{m'}^{(s)} P_m(\cos \theta) [((l + m)!/(l - m)!)]^{1/2}
\]

\[
\times \cos \left[ (\omega_0 - \omega_{l}^{(s)} - m \Omega) t + \alpha^{(s)} \right]. \quad (9)
\]

where the \( \alpha \)'s are arbitrary constants and the \( a \)'s are determined from equation (1). Thus, we see that if a single mode is excited for an \((nl)\)-oscillation, the observer will report \((2l + 1)\)-frequencies from it each of which is separated by \( \Omega \) from its nearest neighbors.

In the model proposed here, like that of Kurtz (1982), in order to explain the Ap star data, we assume a single mode is excited due to the magnetic field in the intrinsic frame. Unlike Kurtz, we account for advection. A schematic model for such a selective excitation has been proposed by one of us (Dziembowski 1984) for high-order, low-degree oscillations. For such oscillations, it can be assumed that the driven and/or damped oscillations may be treated in a locally plane-parallel approximation. Moreover, we assume that only the magnetic field plays an important role in these nonadiabatic effects. The local contribution to driving can be approximated by \( P_{r} \) and \( P_{r}\)-terms which represent the distortion caused by the field. Our generalization over the earlier suggestion of a selective mechanism arises from the fact that an individual mode in the magnetic system is not given by a single \( Y_{m'}^{(s)} \), but rather by a linear combination of them. The driving rate is determined by integration over angle which is weighted by the \( |Y_{m'}^{(s)}|^2 \) from the oscillation. The result for the driving rate is

\[
\gamma^{(s)} = \gamma_0^{(s)} + \gamma_2^{(s)} D^{(s)}, \quad (10)
\]

where

\[
D^{(s)} = \sum_m \left( \sum_m a_{m}^{(s)} \right)^2 \frac{l(l+1) - 3(m')^2}{4l(l+1) - 3}. \quad (11)
\]

From equation (11), we see that even if the radial modes are stable, there is still the possibility of nonradial mode driving if the magnetic field is destabilizing (\( \gamma_2^{(s)} D^{(s)} > 0 \)). For the simple case, \( a_{m}^{(s)} = \delta_{m',s} \), as considered by Dziembowski (1984), this may happen for \( m' = 0 \). In the more general case we considered here, the mode having the largest value of \( D \) may be driven.
Fig. 1. — $D$ from eq. (11) vs. $\sigma_r$. The curve labeled by "1" represents the first mode to be driven.

Fig. 2.—$|A_{-1}|/|A_0|$ and $|A_{+1}|/|A_0|$ vs. $\sigma_r$ assuming the solution corresponding to curve 1 from Fig. 1. The upper branch of the curve represents the lower frequency term for $\sigma_r$ and would represent the higher frequency term for $\sigma_r < 0$. The apparent divergence of the upper branch is a consequence of the modes switching identification.
III. A NUMERICAL EXAMPLE

To illustrate the general properties of our model, consider the special case of HD 24712 for which Kurtz determined the inclination angle $\beta(= 47^\circ)$ and the angle of the rotation axis of the star to the observer, $\theta_r(= 33^\circ)$. In this star, two $l = 1$ multiplets were identified. Solutions to the real symmetric matrix problem of equation (1) for $l = 1$ are quoted in terms of $\sigma_r$, the ratio of the splitting due to advection to that due to magnetism as if each were separately observed. For each of the three solutions to equation (1), we calculated the ratio $|A_{\pm 1}|/|A_0|$ and evaluated $D$ using equation (11), where

$$|A_{\pm 1}| = \frac{|a_{\pm 1}| \tan \theta_r}{|a_0|^{2^{l/2}}} \tag{15}$$

is the predicted relative peak height presented in same form as Kurtz’s observational results. In Figure 1, we show the behavior of $D$ as a function of $\sigma_r$ for all three modes assuming the perturbations due to advection and magnetism have the same sign. If the perturbations had opposite signs, Figure 1 would have symmetric branches for $\sigma_r < 0$. Since the branches are mirror images of those for $\sigma_r > 0$, the redundant branches are not shown in the figure. The curve labeled 1 in this figure represents the mode which is axisymmetric about the magnetic axis at $\sigma_r = 0$. Increasing $\sigma_r$ results in an increase in the contamination from the other spherical harmonics, thereby reducing the degree of axial symmetry in this mode. Consequently, $D$ begins to decrease; nonetheless this mode remains the first one to be excited for a wide range of values of $\sigma_r$.

In Figure 2, we plot the calculated amplitude ratios $|A_{\pm 1}|/|A_0|$ for the mode labeled 1 in Figure 1. The lower branch, labeled by $|A_{-1}|/|A_0|$, represents the highest frequency of the three. For $\sigma_r < 0$, the plots would again be the mirror image of those for $\sigma_r > 0$ except that the upper branch there would represent the highest frequency. If $\sigma_r = 0$, the prograde and retrograde amplitudes are equal (in general, $A_m = A_{-m}$).

However, for slight changes in $\sigma_r$, the difference between the ratios grows quite rapidly.

The two $l = 1$ modes identified by Kurtz, when averaged yield an amplitude ratio of $0.37 \pm 0.04$ for the highest frequency component of the multiplet and $0.33 \pm 0.04$ for the lowest frequency component. The difference between these two quantities is not statistically significant. Ignoring this and using Figure 2 anyway, we determine that these numbers correspond to $\sigma = -0.05$ implying that the effects of magnetism and advection have opposite signs and the relative amplitude of the magnetic effect is 20 times larger.

IV. DISCUSSION

We have shown that the combined effect of advection and magnetism imply that each normal mode is given by a linear combination of $Y_m^m$’s with the respective amplitudes of the modes determined from equation (1). Thus, a single $(nl)$-mode in the intrinsic frame is observed as $(2l + 1)$ uniformly spaced frequencies, separated by $\Omega$, in the inertial frame. Therefore, if such a spectrum is observed we must conclude that a single mode of the appropriate $(nl)$-multiplet is excited in the star. We have shown that it is reasonable to expect that a single mode of a multiplet may be excited under the destabilizing influences of the magnetic field. Our model reduces to Kurtz’s in the limit in which advection can be ignored. The model retains the essential features of his model unless the effect of rotation on splitting dominates that of magnetism. The measure of the importance of rotation is the relative amplitude of the prograde and retrograde frequencies. The measurement of the relative amplitudes of split frequencies provides useful information about the star. In particular, for the case in which advection is important, we can determine the relative size of the perturbations due to magnetism and advection. If the rotation is approximately rigid, we can assess the mean strength of the magnetic field in the stellar interior.

We wish to thank Boris Kuharetz and William Savin for critical readings of the manuscript.

REFERENCES


W. DZIEMBOWSKI: N. Copernicus Astronomical Center, Warsaw, Poland

PHILIP R. GOODE: Department of Physics, New Jersey Institute of Technology, Newark, NJ 07102

© American Astronomical Society • Provided by the NASA Astrophysics Data System