SPECTRA OF GAMMA-RAY BURSTS

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ABSTRACT

We show here that γ-ray burst spectra are basically thermal synchrotron spectra emitted in a cold "photosphere" by electrons excited to high Landau levels by high-energy (E \( \geq \) mc²) photons that are beamed along the magnetic field lines. This high-energy radiation is produced in a corona by the interaction of soft, thermal photons and synchrotron photons with one-dimensional, relativistic electrons in a strong magnetic field. These coronal electrons are accelerated by short-scale magnetic reconnection. We have been able to determine self-consistently the physical parameters of the coronal layer (i.e., temperature and column density); the only free parameters left are the temperature of thermal photons and the value of the magnetic field. Monte-Carlo simulations that include Compton and resonant scattering produce spectra in good agreement with the observations between 20 keV and 1 MeV. We speculate that the recently discovered high-energy tail of the spectrum is formed in the outer corona and the wind zone. We have also shown that the emission is strongly anisotropic, and consequently, the most intense bursts need not be the closest; one may also explain in this way the low value of \( L_x/L_\gamma \) observed for GB 781119.

Subject headings: gamma rays: bursts — radiation mechanisms

I. INTRODUCTION

Spectra of γ-ray bursts are obviously a very important clue to the nature of their sources. Attempts have been made to fit those spectra with optically thin thermal bremsstrahlung (Mazets et al. 1981a; Gilman et al. 1980), inverse Compton emission (Fenimore et al. 1982a), thermal synchrotron emission (Liang 1982; Liang, Jernigan, and Rodrigues 1983), and a superposition of a blackbody and thermal synchrotron (Lasota and Belli 1983); and they are also interpreted as nonthermal synchrotron emission (Bussard 1984). In all cases, it is assumed that γ-ray bursts are emitted at the surface of a neutron star, and the synchrotron models involve also the presence of a very strong (~10¹² G) magnetic field. None of these models is satisfactory. Models involving only bremsstrahlung or only inverse Compton radiation imply that the magnetic field at the surface of the neutron star is smaller than 10⁹ G (Fenimore et al. 1982a; Lamb 1984), although a much larger magnetic field (10¹² G) seems to be an essential ingredient of γ-ray bursts, making them different from X-ray bursts (Woosley and Wallace 1982; Bonazzola et al. 1984; Woosley 1984). Moreover, such large fields have to be present to explain the observed cyclotron lines (Mazets et al. 1981a, but see also Lasota and Belli 1983; Fenimore et al. 1982b, and Fenimore, Klebesadel, and Laros 1983 for a discussion of observational problems). In addition, bremsstrahlung models imply unacceptable distances (much less than 100 pc, see Fenimore et al. 1982b; Lamb 1984). Synchrotron models, as developed by Liang, imply large luminosities, which can be greater than 10⁴² ergs s⁻¹ for a 1 km² emitting region (Liang, Jernigan, and Rodrigues 1983), while the luminosities predicted by the thermonuclear model range form 10³⁷–10³⁹ ergs s⁻¹ km⁻² (Woosley and Wallace 1982; Hameury et al. 1982). Therefore these synchrotron models require a model of the energy source, which is yet to be found.

An additional difficulty arises from the observations themselves. Most of the spectra were observed by Mazets et al. (1981b). Their measurements were integrated over times larger than 4 s. It appears, however, that at least some γ-ray bursts show a spectral evolution on time scales of 0.25 s, making difficult the definition of an average spectrum (Barat et al. 1984; Vedrenne 1984, and references therein). This means that the best fits of Mazets et al. spectra do not necessary reflect the underlying physical mechanism of γ-ray production (Vedrenne 1984). Finally, it appears that the spectra do not fall exponentially at energies greater than a few hundred keV but are more adequately represented by a power law (Vestrand 1984).

In any case, none of the published models for fitting the γ-ray energy spectra give a consistent description of the physical mechanisms responsible for γ-ray bursts (the source of energy, time scales, transport mechanisms, etc.).

In this paper we adopt a different approach to the problem of γ-ray energy spectra: instead of trying to fit the observed points by a given simple physical law, we investigate the physical mechanisms that produce γ-rays and we calculate the emitted spectra.

These calculations are based on a model (presented in a series of papers: Hameury et al. 1982, 1983; Hameury, Bonazzola, and Heyvaerts 1983; Bonazzola et al. 1984; hereafter Papers I, III, II, and IV respectively), in which the burst energy is released by a thermonuclear explosion under the surface of a strongly magnetized neutron star, then transported to the surface by convectively driven Alfvén and sound waves. Small-scale magnetic-field deformations reconnect in a "corona,"...
giving rise to strong electric fields that accelerate electrons to energies of the order of few MeV.

In § II we discuss the physical conditions in the corona of a \( \gamma \)-ray burster. Following Paper IV, we derive equations that allow us to determine in a self-consistent way the corona’s \( \gamma \)-ray burster. Following Paper IV, we derive equations that allow us to determine in a self-consistent way the corona’s parameters: \( T_{\text{rad}} \), the temperature of the thermal radiation; \( T \), characterizing the energy of accelerated electrons; \( \tau \), the Thomson optical thickness (or column density) of the corona.

Relevant interactions between matter and radiation in a very strong magnetic field are discussed in § III. We take into account resonant and Compton scattering, and synchrotron radiation induced by collisions of backscattered high-energy photons with cold electrons in denser regions where the accelerating mechanism is no longer at work. In all cases, \( \tau \) is found to be low (a few times \( 10^{-2} \)), so that the nonresonant inverse Compton interactions are rather insignificant, contrary to models of Fenimore et al. (1982a).

The transfer of the radiation through the hot magnetized corona is treated by a Monte-Carlo method that is described in § IV. The results are presented in § V. They show that our model is capable of fitting a large variety of burst spectra. In particular, the strong focusing of the scattered photons along the magnetic field lines means that the strongest bursts need not be the closest, and explains the ratio of persistent X-ray to \( \gamma \)-ray luminosity.

II. PHYSICAL CONDITIONS IN THE ATMOSPHERE OF A \( \gamma \)-RAY BURSTER

Here we assume that \( \gamma \)-ray bursts (or at least most of them) are the result of a thermonuclear runaway releasing about \( 10^{48} \) ergs \( \text{cm}^{-2} \) (Paper I) under the surface of a slowly accreting \( (M \approx 10^{-15} M_\odot \text{ yr}^{-1}) \) (Paper II), strongly magnetized \( (B \approx 10^{12} \text{ G}) \) neutron star (Paper III). As a result of the runaway (detonation or deflagration of helium, Papers I, II, IV), a thin \((\sim 9 \text{ m})\) and very hot \((T \approx 5 \times 10^9 \text{ K})\) burnt layer (of mostly \( ^{56}\text{Ni} \)) forms. This layer (in the form of a “pilbox”) is separated from the neutron star surface by \( 58 \text{ m} \) of cool \((T \approx 10^7 \text{ K})\), almost pure, hydrogen. Due to the presence of a very strong magnetic field, the hot “pilbox” is stable against ordinary convection, but is unstable against oscillatory (overstable) convective motions (Paper IV).

Since the field lines are anchored in the neutron-star crust, and since the gas pressure is much larger than the magnetic pressure, overstable convection produces both oscillatory deformations of the magnetic field lines \( (\delta B/B \approx 10^{-2}) \), which propagate upward in the form of Alfvén waves, and “piston-like” motions of matter that are the source of sound waves.

Because of the high sound and Alfvén velocities \((10^6 \text{ cm s}^{-1}\) deep in the layer, about \( c \) near the surface for the Alfvén speed), the energy flux from the hot layer reaches the surface very rapidly \((\sim 10^{-4} \text{ s})\). Using the values of the magnetic field perturbation as given in Paper IV, we obtain for the Alfvén wave energy flux leaving the convective layer:

\[
F = \frac{\delta B}{8\pi} e_A = 1.12 \times 10^{37} B_{12}^2 \rho_6^{-1/2} \text{ ergs cm}^{-2} \text{ s}^{-1},
\]

where \( B = 10^{12} B_{12} \) is the unperturbed value of the magnetic field, and \( \rho = 10^6 \rho_6 \) is the value of density in the convective layer. We will neglect here the weak dependence on the Chandrasekhar number (Paper IV).

The energy flux carried by sound waves should be of the same order of magnitude, and therefore the burst duration is a few seconds. Propagating upward, the waves steepen into shocks and release their energy in optically thick regions, giving rise to a thermal, and relatively soft (a few keV), component of radiation.

The dissipation of energy by Alfvén waves requires a more detailed description.

As shown in Paper IV, the magnetic-field perturbations due to Alfvén waves can reconnect and create an electric field (parallel to the unperturbed large-scale magnetic field) strong enough to accelerate electrons to relativistic energies. We assume that the reconnection takes place when the characteristic time of the linearized ("tearing") mode of reconnection is shorter than the period of the Alfvén waves (equal to the period of the oscillatory convection). This seems to be a generous estimate, since the characteristic time of the nonlinear phase of reconnection is much shorter than the linear one (Norman and Smith 1978).

In Paper IV the tearing-mode time was evaluated using the Spitzer conductivity. This was an overestimate of this characteristic time, since, as we shall see, the effective electron collision frequency, and therefore the resistivity, is much higher.

In the following, we shall calculate the physical parameters of the reconnecting corona (temperature and column density) as well as the temperature of thermal photons that are produced in deeper layers. As a first approach, we shall consider a plane-parallel, isothermal corona. But first, we shall show that the distribution of electrons is one-dimensional.

a) Electron Distribution Function

Reconnection accelerates electrons along the magnetic field. They should therefore have a one-dimensional distribution, unless collisions with ions, photons, or phonons could isotropize it. This will be the case only if the collision time is shorter than the synchrotron time.

The electron-ion collision time is given by (Canuto and Ventura 1977; see also paper IV):

\[
t_e = 7.86 \times 10^{-10} \frac{E}{m_e c^2} B_{12}^{-2/3} \text{ s};
\]

in a strongly magnetized plasma, \( E \) is the average energy of electrons and \( \tau \) the Thomson optical depth. This is much longer than the lifetime of the first Landau level, \( t_{\text{sync}} = 5 \times 10^{-16} B_{12} \text{ s} \), and therefore the first collisions with ions are not frequent enough to isotropize the electronic distribution.

Collision with thermal photons of a few keV are very inefficient in producing a three-dimensional distribution. The reason is that the component of the electron momentum perpendicular to the magnetic field is, after a collision with a photon of energy \( h\nu_0 \), at most equal to \( \gamma h\nu_0/c \) (\( \gamma \) is the Lorentz factor of the electron). This is much smaller than the initial electron momentum (about \( \gamma m_e c \)), unless \( h\nu_0 \) is of the order of \( m_e c^2 \).

On the other hand, \( \gamma \)-ray photons are energetic enough to do the job, but they are not available in sufficient quantity. In fact, even if all the flux were carried by photons with energy \( m_e c^2 \), the collision time \( t_{e-\gamma} \) with photons, computed using the Thomson cross section \( \sigma_{\text{Th}} \), would be:

\[
t_{e-\gamma} = \frac{m_e c^2}{F \sigma_{\text{Th}}} = 1.23 \times 10^{-9} F_{27}^{-1} \text{ s},
\]

where \( F_{27} \) is the flux in units of \( 10^{27} \text{ ergs cm}^{-2} \text{ s}^{-1} \). Therefore, collisions with photons also cannot isotropize the electronic distributions.
Because the ions are not much accelerated by the electric field created by reconnection, one would expect to have a situation where a monokinetic beam of electrons has a much larger velocity than the ionic thermal velocity. This is known to be unstable, but the associated instabilities have a characteristic frequency of the order of the plasma frequency \( v_p = 9 \times 10^8 \left( n_e / 10^{22} \text{ cm}^{-3} \right)^{1/2} \text{ s}^{-1} \), or of the ionic cyclotron frequency \( v_i \). Therefore, the quantum of energy that can be given to electrons by these instabilities is either \( hv_p \) or \( hv_i \), which is smaller than the energy of the first Landau level by several orders of magnitude. Therefore, the instabilities can only thermalize the distribution function parallel to the magnetic field, but in no way could they isotropize it.

We conclude that (unless one considers some exotic coherent processes; Liang 1984), the electronic distribution function is one-dimensional. Moreover, since the growth time of the ion-cyclotron instability (which arises because the drift velocity of electrons is larger than the ionic thermal velocity) is of the order of \( v_i^{-1} \), the ions can be heated almost immediately and thus quench this instability. Due to such plasma instabilities, the electronic distribution function should be broad, and for the sake of simplicity, we shall assume in the calculations that it is a one-dimensional relativistic Maxwellian.

b) Coronal Parameters

The parameters of the reconnecting corona (temperature \( T \) and Thomson optical depth \( \tau \)) as well as the temperature \( T_{BB} \) of the thermal component of radiation, will be determined here by three conditions: (i) the reconnection time has to be shorter than the period of convective oscillations; (ii) the net radiative energy flux emitted by the reconnecting layer has to be smaller than the input energy flux (i.e., the Alfvén flux); (iii) the temperature of electrons is fixed by the value of the accelerating effective potential.

We shall see that these conditions are not independent, and that we are left with \( T_{BB} \) as a free parameter (in addition to the magnetic field \( B \)).

The tearing mode time, \( t_{TM} \), is given by the classical formula (see Paper IV):

\[
t_{TM} = \left( \frac{4\pi \sigma}{c^2 \delta B} \right) \left( \frac{a B}{c \delta B} \right)^{1/2},
\]

where \( a \) is the length of the magnetic shear, \( \delta B \) the amplitude of the magnetic field perturbation induced by the Alfvén wave, and \( \sigma \) is the electrical conductivity, \( \sigma = n_e e^2 / m_e t_{coll} \). \( n_e \) being the electron number density and \( t_{coll} \) the collision time. In the classical (Spitzer) conductivity, \( t_{coll} \) is the electron-electron collision time, \( t_{e-e} \), which we will take to be equal to \( t_{e-e} \) as given by equation (2).

Since the electron mean energy is, as we shall see, much greater than the mean energy of thermal photons, the electron-photon collision time can be estimated as the ratio of the internal energy per unit surface of the hot, reconnecting corona to the energy flux \( F \) given to photons by collisions with the electrons, i.e.:

\[
t_{e-e} = \frac{(kT) n_e H}{F} = 1.23 \times 10^{-9} F_{27}^{-1} kT m_e^2 c^2 \tau  \text{ s},
\]

where \( H \) is the height of the corona and \( T \) its temperature. This formula resembles equation (3) very closely, but, contrary to equation (5), equation (3) applies to the case of hot photons and cold electrons.

Electron-photon collisions dominate if \( t_{e-e} < t_{e-e} \), i.e., if the Thomson optical thickness \( \tau \) of the corona is such as:

\[
\tau < 0.80 F_{27}^{-1/2} B_{12}^{3/4},
\]

which is going to be satisfied for the cases of interest (see, for instance, eq. [14]). Using equation (5) in the expression for electrical conductivity and the values for \( \alpha \) and \( \delta B \) as given in Paper IV, one gets:

\[
t_{TM} = 3.31 \times 10^{-5} F_{27}^{-1/2} B_{12}^{-1/4} \rho_6^{1/8} d_2^{3/2} \text{ s},
\]

where \( d_2 \) is the thickness of the convective layer in units of \( 10^2 \) cm.

Since the overstable convection frequency is \( 1.41 \times 10^6 B_{12} \rho_6^{-1/2} d_2^{-1} \text{ s}^{-1} \), in the region where collision with photons dominate, the ratio of \( t_{TM} \) to the convective time \( t_{conv} \), is given by:

\[
f_{TM} \frac{t_{TM}}{t_{conv}} = 46.7 F_{27}^{-1/2} B_{12}^{3/4} \rho_6^{-3/8} d_2^{3/2}.
\]

On the other hand, in the region where \( t_{e-e} \) is shorter than \( t_{e-e} \), the tearing-mode time scale is larger than the convective time (see Paper IV) by a factor slightly smaller than 10, and reconnection is thus almost certainly inhibited in this region. Therefore, electrons are accelerated and heated only in the Thomson optically thin part of the corona; the condition for reconnection to take place, \( t_{TM} / t_{conv} < 1 \), allows an estimation of the value of \( \tau \) as a function of \( F \).

Since the energy released in the corona comes from the dissipation of Alfvén waves, the net energy flux \( F \) is not greater than the Alfvén flux,

\[
F < F_A = 1.12 \times 10^{27} B_{12}^{3} \rho_6^{-1/2} \text{ erg cm}^{-2} \text{ s}^{-1},
\]

and therefore the second condition is \( F < F_A \).

Now, in order to produce relativistic electrons, one needs an electric field strong enough to accelerate electrons on a sufficiently long path. In an analogous way as in Paper IV, we find that the mean energy given to electrons, \( \epsilon \), is

\[
\epsilon < eEi = 5.51 F_{27}^{-1/2} B_{12}^{7/4} d_2^{-1/2} \rho_6^{-3/8} kT,
\]

where \( i \) is the electron mean free path. If the inequality (9) is fulfilled, then equation (10) is satisfied if

\[
B_{12} > 1.36 \times 10^{-3} \rho_6^{1/8} d_2^{-1/2}.
\]

Therefore, because the magnetic field required by our model is greater than \( 10^{11} \) G (see Paper III), the parameters of the reconnecting region need to satisfy only two conditions: \( t_{TM} / t_{conv} < 1 \) and \( F < F_A \). In order to estimate those parameters, and find self-consistent solutions, we will take:

\[
\frac{t_{TM}}{t_{conv}} = c_1,
\]

and:

\[
\frac{F}{F_A} = c_2,
\]

where \( c_1 \) and \( c_2 \) are two constants smaller than or equal to unity. As a reasonable estimate for these quantities, we shall take \( c_1 = c_2 = 1 \). Later we will comment on the dependence of the results on those parameters. Let us note for the moment that replacing \( F \) by \( c_2 F_A \) in equations (8) and (12) allows us to determine \( \tau \) as:

\[
\tau = 2.27 \times 10^{-2} c_1 c_2^{-1/2} B_{12}^{3/4} \rho_6^{1/8} d_2^{-1/2}.
\]
As the radiative energy flux $F$ is the result of complicated interactions between electrons and radiation in a strong magnetic field, as it is described in the next paragraph, the dependence of $F$ on $T$, and $T_{\text{BB}}$ (the temperature of the thermal component) cannot be expressed in an analytical form. Equation (13) will be solved by numerical simulations in § IV.

Since the photon flux as given by equations (1) and (13) is close to the Eddington luminosity for the whole surface of the star, the luminosity of the hot corona above the small polar caps will be very super-Eddington. This situation could lead to a relativistic wind formation (see, e.g., Woosley 1984), but things are very complicated since the radiative acceleration of electrons in the wind is negligible in comparison with the acceleration in the electric fields due to reconnection, and moreover, most of the luminosity is released in the corona in both up and down directions. In the following, therefore, we shall calculate the spectrum radiated by a hydrostatic, isothermal corona. As we shall see, the high energy part of the spectrum ($E > 1$ MeV) must be produced in the "wind zone." We leave for a future work a more detailed analysis of the effects of a super-Eddington luminosity in column-like structures (see Hameury, Bonazzola, and Heyvaerts 1980 for a discussion of this problem in a slightly different context of accretion columns).

III. RADIATION

We will be concerned mainly with the physical processes that form the observed spectrum in the hot "corona" of a γ-ray burster. The transfer of the radiation entering the corona from the underlying cool "photosphere" will be treated by a Monte-Carlo simulation. The radiation leaving the photosphere results from complex processes due to energy release by convectively generated waves as well as to the illumination of the photosphere by the hot, reconnecting corona. It will be assumed that the spectrum injected in the corona consists of two components: a thermal synchrotron (see § IIIId for justification) and a blackbody that represents the thermalized part of radiation. The form of the injected spectrum is therefore rather crude (especially at low energies), but it is reasonable, and it serves well our purpose of studying the formation of a γ-ray burst spectrum. Moreover, as we will see in § V, the low-energy photons do not have much influence on the final spectrum shape, except in the very special situation where the line of sight happens to coincide with the magnetic field direction.

In this section we will discuss the production of γ-rays, and we will study their interaction with matter.

a) Electron-Ion Collisions

Because the electrons are accelerated parallel to the magnetic field, they do not emit synchrotron radiation, and γ-rays have to be created by interactions of these high-energy electrons with ions, phonons, or thermal photons of a few keV.

After a collision with an ion, the electron velocity is no longer parallel to the magnetic field, and a significant fraction of the electron energy is converted into γ-rays via synchrotron emission. However, as mentioned in the previous section, the collision time between electrons and ions $t_{\text{e-i}}$ is very long. The energy flux emitted by this process is roughly

$$F_{\text{e-i}} = \frac{nHE}{t_{\text{e-i}}},$$

where $n$ is the electron number density, $H$ the thickness of the emitting layer, and $E$ the average energy of electrons. Since the Thomson optical depth of this layer is much smaller than unity, one finds:

$$F_{\text{e-i}} \ll 3 \times 10^{35} \left( \frac{E}{m c^2} \right)^{-3/2} \text{ ergs km}^{-2} \text{ s}^{-1},$$

which is much too low, and would put the distance of γ-ray bursters at less than 20 pc.

On the other hand, the collision time between electrons and photons is much shorter, and, as we will see, leads to the correct order of magnitude for the energy flux. Because of the presence of a strong magnetic field, the interaction between electrons and photons is more complex than in the non-magnetic case. In addition to the usual Compton scattering, electrons can be sent to excited Landau levels by resonant absorption, which is immediately followed by synchrotron emission. We shall show that, even if one neglects transitions from the $n = 0$ to $n \geq 2$ Landau levels, resonant scattering (transition from $n = 0$ to $n = 1$, and emission of a cyclotron photon) dominates over the normal Compton scattering.

b) Compton Scattering

When a photon with energy $\epsilon$ is scattered by an electron with a Lorentz factor $\gamma$, the final energy of the photon is about $\gamma^2 \epsilon$, provided that $\gamma \epsilon < m c^2$. For Lorentz factors of a few units and thermal photons of a few keV, the energies obtained can reach a few hundred keV. More precisely, the total Compton cross-section for a photon with energy $\epsilon$, propagating in a direction making the angle θ with the magnetic field, is given by:

$$\sigma_\epsilon(\epsilon, \theta) = \int_{-\infty}^{+\infty} N_\epsilon(p)(1 - \beta \cos \theta) \sigma_{\text{KN}}[(1 - \beta \cos \theta)\epsilon]dp,$$

where $\rho$ is the momentum of the electron, $\gamma$ is the Lorentz factor ($\gamma^2 = 1 + p^2/(m c^2)$), and $\beta = p/\gamma$. $N_\epsilon(p)$ is the electronic distribution function [for a relativistic one-dimensional Maxwellian, with temperature $T$, $N_\epsilon(p) = e^{-p^2/[2K_1(1/T)]}$, $K_1$ being the Hankel function of the first order]. $\sigma_{\text{KN}}$ is the Klein-Nishina cross section (Klein and Nishina 1929). It is found that $\sigma_\epsilon(\epsilon, \theta)$ depends only weakly on $\theta$ (no more than 10% variation for $\epsilon < 5 m c^2$, and $kT = mc^2$), but more strongly on $\epsilon$. Figure 1 gives $\sigma_\epsilon(\epsilon, \theta)$ for $\theta = 60^\circ$ and $kT = mc^2$.

Because the electrons have velocities parallel to the magnetic field, the angular distribution of scattered photons is very anisotropic and is strongly beamed along the field lines. After a Compton interaction, the photon is, in most cases, scattered in a cone whose axis is the electron velocity and whose solid angle is about $2\pi/\gamma^2$. When the electron distribution function is isotropic, there is no privileged direction, and, on the average, scattering is isotropic in the laboratory frame. On the other hand when the electrons are forced to move parallel to the magnetic field, the averaging preserves the $1/\gamma^2$ factor. More details on the angular distribution of photons having undergone an inverse Compton scattering will be given in § IV.

c) Resonant Absorption

A photon can be absorbed by a transition from the fundamental (0th) to the nth Landau level if its energy, in the electron rest frame, is a multiple of the cyclotron energy $h\nu_B$. This will happen if, in the laboratory frame:

$$\gamma(1 - \beta \cos \theta) \epsilon = nh\nu_B,$$

where $n$ is the electron number density, $H$ the thickness of the emitting layer, and $E$ the average energy of electrons. Since the Thomson optical depth of this layer is much smaller than unity, one finds:

$$F_{\text{e-i}} \ll 3 \times 10^{35} \left( \frac{E}{m c^2} \right)^{-3/2} \text{ ergs km}^{-2} \text{ s}^{-1},$$

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When a photon with energy $\epsilon$ is scattered by an electron with a Lorentz factor $\gamma$, the final energy of the photon is about $\gamma^2 \epsilon$, provided that $\gamma \epsilon < m c^2$. For Lorentz factors of a few units and thermal photons of a few keV, the energies obtained can reach a few hundred keV. More precisely, the total Compton cross-section for a photon with energy $\epsilon$, propagating in a direction making the angle θ with the magnetic field, is given by:

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A photon can be absorbed by a transition from the fundamental (0th) to the nth Landau level if its energy, in the electron rest frame, is a multiple of the cyclotron energy $h\nu_B$. This will happen if, in the laboratory frame:

$$\gamma(1 - \beta \cos \theta) \epsilon = nh\nu_B,$$
where $\epsilon$ is the photon energy and $\theta$ its angle with respect to the magnetic field. Since $h\nu_B \ll mc^2$, we have neglected the recoil of the electron. As the collision time is much longer than the synchrotron time (see § II), synchrotron emission immediately follows the absorption of a photon. Since transitions $n \rightarrow n - 1$ are more likely than transitions with $\Delta n > 1$, there exists a fundamental difference between the excitation of the first level and the higher ones. In the first case, de-excitation results in the emission of a single cyclotron photon, and the total interaction is equivalent to resonant diffusion. In the latter, de-excitation is likely to result in the emission of several photons (Daugherty and Ventura 1977), and the photon number is no longer conserved. We shall compute the cross sections for these processes and show that the corona is optically thin for transitions with $\Delta n > 1$.

The absorption cross section of a photon with energy $\epsilon$, propagating with an angle $\theta$ with respect to the magnetic field, is, for a transition to the nth Landau level, given by:

$$\sigma_a(\epsilon, \theta) = \int_{-\infty}^{+\infty} N(\epsilon, \theta) \sigma_a' \left[ (1 - \beta \cos \theta) \epsilon \sigma_a' \right] d\theta,$$

(19)

where $\sigma_a'(\epsilon', \theta')$ is the absorption cross section of a photon with energy $\epsilon'$ and angle $\theta'$ in the electron rest frame. The angle $\theta'$ is deduced from $\epsilon$, $\theta$, and $\rho$ by the Lorentz transformation formulae; $\sigma_a'(\epsilon', \theta')$ is given by:

$$\sigma_a'(\epsilon', \theta') = \frac{1}{8\pi} \frac{h^2c^2}{\epsilon^2} A_a(\epsilon', \theta') \phi(\epsilon'),$$

(20)

where $A_a$ is the Einstein coefficient for spontaneous emission, and $\phi(\epsilon')$ is the line profile, normalized by $\int \phi(\epsilon')d\epsilon' = h$. $A_a(\epsilon', \theta')$ is given by (Daugherty and Ventura 1977):

$$A_a(\epsilon', \theta') = \frac{\alpha(n, B) \epsilon^2 E_B \epsilon'}{4\pi h^2mc^2} \left( 1 + \cos^2 \theta' \right) \sin^{2n-2} \theta',$$

(21)

where $E_B$ is the cyclotron energy and $\alpha(n, B)$ is a numerical coefficient which depends on the magnetic field strength $B$; $\alpha$ is smaller than unity and decreases rapidly with increasing values of $n$ (as an example, for $B = 5 \times 10^{13}$ G, $\alpha(1) = 1$, $\alpha(2) = 0.04$, and $\alpha(3) = 0.003$). Equation (21) has been derived in the "cold plasma" approximation, in which only the extraordinary mode is resonant. In reality, due to relativistic effects such as vacuum polarization, both normal modes are resonant (Ventura, Nagel, and Mészáros 1979), and it is sufficient for our purpose to use only equation (21) in the transfer calculations.

Because the natural width of the line is very small, $\phi(\epsilon' - \epsilon)$, which enters in equation (11), varies much more rapidly than all the other quantities under the integral. Then, from equations (19), (20), and (21), one gets:

$$\frac{\sigma_a}{\sigma_{th}}(\epsilon, \theta) = \frac{3750}{K_1(1/T)} \frac{n B}{n B_{12}} \sum_{i=1}^{\infty} \gamma_i e^{-\nu_i T} (1 + \cos^2 \gamma_i) \times (1 - \beta_i \cos \theta' \sin \gamma_i) \left[ (1 - \beta_i \cos \theta') \sin^{2n-2} \theta' \right]^3,$$

(22)

where $(\gamma_1, \beta_1)$ and $(\gamma_2, \beta_2)$ are the two solutions of equation (18), and $\sigma_{th}$ the Thomson cross section. For $n = 1$,

$$\frac{\sigma_a}{\sigma_{th}}(\epsilon, \theta) = \frac{3750}{B_{12} K_1(1/T)} \sum_{i=1}^{\infty} \gamma_i e^{-\nu_i T} (1 - \beta_i \cos \theta') \times (1 + \cos^2 \gamma_i \epsilon)^3,$$

(23)

which is represented in Figure 1.

Note that for $\beta_i = \cos \theta$, the cross section given by equation (23) is infinite. This is due to the fact that the approximation used to obtain equation (22) is not valid in this case. In reality, this cross section is very large, but not infinite; this, however, does not create any difficulty, since the situation when $\beta_i$ is equal to $\cos \theta$ can last only until the next scattering.

For the column densities relevant to our problem (see §§ II and IV) the first harmonic is optically thick. As can be seen from equations (22) and (14), the higher harmonics are optically thin. In the numerical computations, we shall therefore neglect the transitions with $\Delta n > 1$ (but see below), although we shall still take into account optically thin inverse Compton interactions, which can give the high energy tail of the spectra.

d) Synchrotron Radiation

Most of the scattered, high-energy photons (with $\epsilon > mc^2$) propagate along the magnetic field lines; those directed outward escape almost freely; those directed toward the neutron star will reach regions where the Thomson optical depth is close to unity. In these denser regions, reconnection is no longer efficient; the electrons are much cooler than in the outer zones, and they are in equilibrium with the blackbody photons at a temperature of a few keV. Compton scattering of the high-energy photons ($\epsilon > mc^2$) on cool electrons produces relativistic electrons. These electrons lose very rapidly their energy perpendicular to the magnetic field by synchrotron emission. The energy parallel to the magnetic field is lost more slowly, by either collisions on ions (and therefore synchrotron emission), or as a result of instabilities. All these processes form the secondary component of the spectrum; part of it is a syn-
Fig. 2—Calculated spectra. For parameters, see Table 1.
chrotron emission, and the remaining goes into thermal emission, which we included in the blackbody flux. Therefore the blackbody temperature should be high enough to make the thermal flux a reasonable fraction of the high-energy flux.

It is very difficult to treat exactly this secondary component; $S_{\text{thermal}}$ is a reasonable fraction of the high-energy flux.

We have treated the transfer of radiation in the hot, reconnecting region by a Monte-Carlo method, which is particularly well suited to a case where the energy of a photon can change by a large amount in a single scattering (see, for instance, Fenimore et al. 1982a). Since the higher cyclotron harmonics are optically thin, the photon number is conserved during the transfer, and thus one can track each photon from the moment it enters the hot corona. The corona was assumed to have a constant temperature $T$ and a total Thomson optical depth $\tau$, given by equation (14).

**a) Monte-Carlo Calculations**

The following steps are performed to follow each photon:

i) The Compton mean free path $l_c$ and the two resonant
ones \( l_{h1} \) and \( l_{h2} \) (corresponding to the two solutions of eq. [18]) are computed using equations (17) and (23). A random number \( r_1 \) with \( 0 \leq r_1 \leq 1 \) is chosen and used to determine the type of interaction selected: if \( r_1 < l_{c-1}^{-1} / (l_{c}^{-1} + l_{h1}^{-1} + l_{h2}^{-1}) \), then the Compton interaction is chosen; otherwise, if \( r_1 < (l_{c}^{-1} + l_{h1}^{-1}) / (l_{c}^{-1} + l_{h1}^{-1} + l_{h2}^{-1}) \), then resonant scattering by an electron corresponding to the first solution of equation (18) is chosen; otherwise the second solution of equation (18) is taken.

ii) Once the type of interaction is known, the distance at which the scattering occurs is \( d = -l \ln \left( r_1 \right) \), where \( l \) is the mean free path and \( r_2 \) a second random number with \( 0 \leq r_2 \leq 1 \). If scattering would have to occur outside the layer, i.e., at optical depth either smaller than 0 or greater than \( t \) from equation (14), we consider the photon has escaped and is included in the emergent spectrum.

iii) Only in the case of Compton scattering, one has to determine the initial momentum \( p_0 \) of the scattering electron. It is obtained by solving the equation:

\[
r_3 = \frac{\int_{\epsilon_0}^{p_0} N J_0 (1 - \beta \cos \theta_0) \theta K_\Delta \theta \left( 1 - \beta \cos \theta_0 \right) \epsilon_0 \, d\epsilon_0}{\sigma_s (\epsilon, \theta)},
\]

where the notation are the same as in equation (17). The integral was performed using Chebyshev transforms, which allowed a precise, short, and easy computation for a large number of values of \( p_0 \) (in the case of resonant scattering, \( p_0 \) is determined by the resonance conditions, and has already been calculated in step [i]).

iv) One has now to choose the angle of the scattered photon with respect to the magnetic field. In the case of resonant diffusion, the transition rate is simply proportional to \( (1 + \cos^2 \theta_s) \), where \( \theta_s \) is the angle with respect to the magnetic field in the electron rest frame. Therefore, \( \theta_s \) is the solution of:

\[
\frac{3}{8} \int_{0}^{\theta_s} (1 + \cos^2 t) \sin t \, dt = r_4,
\]

where \( r_4 \) is a random number with \( 0 \leq r_4 \leq 1 \). The angle of the scattered photon with respect to the magnetic field is obtained by a Lorentz transform.

In the case of a Compton scattering, the angle \( \delta' \) between the direction of incident and scattered photons in the rest frame of the electron is obtained by solving:

\[
r_3 = \int_{0}^{\delta'} \frac{\epsilon' \, \epsilon' \left( e^{- \epsilon' / \epsilon_1} + \epsilon' / e^{- \epsilon' \sin^2 t} \right) \sin t \, dt}{\epsilon_1 \epsilon_1' \left( 1 + e^{- \epsilon' / \epsilon_1} - \sin^2 t \right) \sin t \, dt},
\]

\[
\epsilon_1 = \epsilon \left( 1 + \epsilon / \epsilon_1 - \cos t \right)^{-1},
\]

where \( r_3 \) is a random number with \( 0 \leq r_3 \leq 1 \). The other angle \( \psi' \) to complete the definition of the scattered direction is chosen randomly between 0 and \( 2\pi \); the angle \( \theta_s' \) with respect to the magnetic field is, in the rest frame of the electron, given by \( \cos \theta_s' = -\sin \delta' \sin \psi' \sin \theta_0' + \cos \delta' \cos \theta_0' \), \( \theta_0' \) being the angle of the incident photon with the magnetic field in the rest frame of the electron.

v) The energy of the scattered photon is obtained by performing a Lorentz transform from the electron rest frame to the laboratory rest frame; in the electron rest frame, it is given by \( \epsilon' = h \nu_0 \) for resonant scattering, and by equation (27) for Compton scattering.

An example of a Monte-Carlo simulation of the history of one photon during resonant scatterings is presented in Figure 3. One can clearly see that photons avoid too frequent "U-turns". For the case presented, it changes direction 155 times out of 500 scatterings. This is due to the one-dimensional electron distribution and relativistic effects in the resonance: when the energy of the photon is somewhat larger than the energy of the cyclotron resonance, then it can be scattered resonantly only by electrons moving in the same direction. This can be seen from equation (18), as if \( \nu e > h \nu_B \) then \( \beta \cos \theta > 0 \).

Finally, because of the beaming effect that has been mentioned in § IIb, the scattering in the laboratory frame is focused by a factor \( 1/\gamma^2 \).

b) Injected Spectrum

As mentioned in the previous paragraphs, the energy spectrum of the injected photons is the sum of a blackbody spectrum with temperature \( T_{bb} \) and a synchrotron spectrum with a temperature \( mc^2 / k \). For the sake of simplicity, the injected synchrotron flux \( F_{in, syn} \) is taken to be equal to half the backscattered flux \( F_{bs, bb} + F_{in, syn} \) of both blackbody and synchrotron photons (taking into account all the backscattered photons instead of only the high energy ones does not significantly affect the results). In this calculation, we assume that the other half of the backscattered flux, radiated toward the interior of the neutron star, goes to internal energy and is not reradiated during the burst. This assumption has some support from the observations (Laros et al. 1984), but here it was made for numerical convenience and seems to have not much influence on our final results. One can write:

\[
F_{in, syn} = \lambda_{syn} F_{in, syn},
\]

where \( \lambda_{syn} \) is, for given parameters of the layer, a constant of proportionality that is determined by injecting only synchrotron photons. It is then easily found that

\[
F_{in, syn} = \frac{F_{bs, bb}}{2 - \lambda_{syn}},
\]

and therefore

\[
\lambda_{syn} < 2.
\]

This condition puts another constraint on the temperature of the layer \( T \). If \( T \) is such that inequality (29) is satisfied, the injected spectrum is determined by equation (28); otherwise, there is no solution.

There therefore are only two free parameters: the value of the magnetic field and the temperature of the injected thermal photons (\( T \) being determined by eq. [13]). In principle, \( T_{bb} \) should be known from the physics of magnetococonversion, since part of the thermal component of radiation is produced by the dissipation in dense regions of the photosphere (Thompson optical depth greater than \( 10^3 \)) of sound waves that are generated by convective motions. Because so little is known about overstable magnetoconversion in extremely strong magnetic fields, \( T_{bb} \) is, in practice, a "free" parameter.

Figure 4 shows the value of \( T \) obtained by solving equation (13) as a function of \( T_{bb} \). It is seen that, for low values of \( T_{bb} \), \( T \) is nearly constant, as \( \lambda_{syn} \) is close to the critical value, 2. For large values of \( T_{bb} \), \( T \) becomes very small, and the numerical simulations show that the backscattered photons are too soft.
Fig. 3.—History of a photon undergoing resonant scattering, for $B = 5 \times 10^{12} \text{G}$, $kT = 2mc^2$, injected at an energy $h\nu_0 = 35 \text{keV}$, and with an angle $\theta_0 = 72.5^\circ$. Compton scattering was not taken into account. The upper figure shows the vertical coordinate $z$ as a function of the traveled distance $l$ (both in units of the Thomson mean free path). The lower figure shows the energy $E$ of the photon in keV.

Fig. 4.—Temperature of the reconnecting region (in units of $mc^2$), as a function of the temperature of the injected blackbody radiation $T_{\text{BB}}$, for a magnetic field of $10^{12} \text{G}$ (solid line), and $5 \times 10^{12} \text{G}$ (dashed line).

Five results

We present in Figure 2 some of the calculated spectra for the parameters listed in Table 1. Figure 2a shows a spectrum obtained for $\theta$, the angle between the line of sight and the magnetic field, equal to zero; it is seen that the flux increases with energy, reaches a maximum at about 500 keV, and then decreases rapidly. This behavior is characteristic of all the spectra obtained for $\theta = 0^\circ$ and is due to the focusing effect that we mentioned above, since most of the photons collected at $\theta = 0^\circ$ are soft photons having undergone resonant scattering. This can be understood by the following example. Let us assume that the reconnecting layer is optically thin, even in the resonance; the energy $E_1$ of a photon after a resonant scattering by an electron with a Lorentz factor $\gamma$ is $E_1 \approx \gamma E_{\text{B}}$, and $\gamma \approx E_{\text{B}}/E_0$, where $E_0$ is the energy of the photon before scattering. The emergent spectrum at $\theta = 0^\circ$, $n_1(E_1)$, is related
to the injected spectrum \(n_0(E_0)\) by \(n_1(E_1) = \tau(E_0) n_0(E_0) dE_0\), where \(\tau(E_0)\) is the optical depth, and the factor \(\tau^2\) results from the focusing effect. It is then readily seen that, since

\[ E_1 \approx \frac{E_0^2}{\tau^2}, \]

one has

\[ n_1(E_1) = \frac{\tau^2}{E_1} n_0(E_0) \left( \frac{E_0^2}{E_1} \right). \tag{30} \]

If \(\tau\) can be considered as constant, then the emergent spectrum has a maximum at \(\sim E_0^2/(2K_T)\).

The real story, however, is not so simple as that. Because the optical thickness is energy-dependent and greater than unity in the resonance, the emergent spectrum cannot be simply deduced from equation (30). From this simple instructive example, one can see, however, that the spectrum at \(\theta = 0^o\) depends strongly on the soft part of the injected spectrum. One must therefore be very cautious in drawing definite and precise conclusions on the exact shape of the spectra for \(\theta = 0^o\). In fact, the soft part of the injected spectrum should be flatter than a Planck spectrum (see, e.g., Nagel 1981). Moreover, transitions with \(\Delta n > 1\), which harden the emergent spectrum, have been neglected.

The emitted flux as a function of the angle between the line of sight and the magnetic field is presented in Figure 5 for the model \(a\). It is seen that at \(\theta = 0^o\), the observed flux per solid angle is greater by almost two orders of magnitude than the flux one would expect from isotropic emission. This shows that a significant fraction of the energy is emitted for values of \(\theta\) close to \(0^o\), and thus Figure 2a represents roughly the spectrum averaged over angles.

Observing spectra at an angle close to \(0^o\) should be exceptional; however, in that case, the inferred luminosity, obtained by multiplying the flux by \(4\pi D^2\), is an overestimate by orders of magnitude. This implies that the brightest \(\gamma\)-ray bursts need not be closest. This is an important result, since the low value of the ratio of persistent X-ray to burst \(\gamma\)-ray fluxes for GB 781119 has been used as an argument against the thermonuclear scenario (Pedersen et al. 1983). The thermonuclear model predicts a luminosity ratio \(L_\gamma/L_x\) ranging from 15 to 100, depending on whether hydrogen burns or not. The observed flux ratio for GB 781119 is 0.5, in apparent contradiction to the thermonuclear model. If, however, GB 781119 happens to be seen at an angle close to \(0^o\), then \(L_\gamma\) is much smaller than the

<table>
<thead>
<tr>
<th>Model</th>
<th>(B_{12}) (keV)</th>
<th>(T_{ms}) (msec)</th>
<th>(\tau)</th>
<th>(D) (kpc)</th>
<th>(\theta)</th>
<th>(\tau_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>5</td>
<td>13</td>
<td>3.15</td>
<td>(7.6 \times 10^{-2})</td>
<td>6</td>
<td>(0^o)</td>
</tr>
<tr>
<td>(b)</td>
<td>3</td>
<td>13</td>
<td>6.3</td>
<td>(2.3 \times 10^{-2})</td>
<td>0.4</td>
<td>(70^o)</td>
</tr>
<tr>
<td>(c)</td>
<td>5</td>
<td>5</td>
<td>270</td>
<td>(7.6 \times 10^{-2})</td>
<td>1</td>
<td>(38^o)</td>
</tr>
<tr>
<td>(d)</td>
<td>2</td>
<td>8</td>
<td>2</td>
<td>(2 \times 10^{-2})</td>
<td>1</td>
<td>(80^o)</td>
</tr>
<tr>
<td>(e)</td>
<td>2</td>
<td>5</td>
<td>18</td>
<td>(2 \times 10^{-2})</td>
<td>0.6</td>
<td>(42^o)</td>
</tr>
<tr>
<td>(f)</td>
<td>5</td>
<td>20</td>
<td>10</td>
<td>(1 \times 10^{-3})</td>
<td>8</td>
<td>(0^o)</td>
</tr>
</tbody>
</table>

Note.—The quantities \(\tau\) and \(\tau_s\) are respectively the Thomson and resonant optical depth computed for \(E = E_0\) and \(\theta\) as given in the table. For models \(a\) to \(e\), \(c_1 = c_2 = 1\); for model \(f\), \(c_1 = 0.013\), and \(c_2 = 1\). The distance \(D\) is chosen so that the maximum intensity of the differential spectrum is slightly less than 0.1 photon cm\(^{-2}\) s\(^{-1}\) keV\(^{-1}\), as in Mazets' catalog.

Figures 2b–2e show examples of \(\gamma\)-ray burst spectra. The spectrum presented in Figure 2b is rather typical of "normal" \(\gamma\)-ray bursts and resembles very closely the spectra of GB 790215 or GB 790329 (Mazets et al. 1981b). Figure 2c represents a hard spectrum, similar to that of GB 790412b or GB 791111; Figure 2d on the other hand shows a very soft spectrum that hardens for \(E > 100\) keV; such spectra have for instance been observed in GB 781102 or GB 78115b; and more soft \(\gamma\)-ray bursts could have such tails, which are not seen in the low statistics. Figure 2e shows a spectrum with a turnover at 40 keV, due to resonant scattering (see GB 781006a or GB 790402b for comparison).

All these spectra are essentially synchrotron spectra that are absorbed at low energy: at low energies, the resonant optical depth is of the order of 10, and photons with \(E < E_0\) are scattered to a direction close to \(\theta = 0^o\). At large enough energies (\(E > E_0\)), the reconnecting layer is optically thin, since equation (18) has no solution for \(n = 1\) and \(\theta = 0\).

We have also performed numerical simulations with \(c_1\) and \(c_2\) (which appear in eqs. [12] and [13]) different from unity. We found that, when the line of sight is not too close to the mag-
VI. CONCLUSION

We have shown that spectra of γ-ray bursts can be produced by the interaction of soft thermal X-ray photons with relativistic electrons. Those electrons are accelerated along the magnetic-field lines by short-scale magnetic-field reconnection, which results from the dissipation of Alfvén waves that are

This resemblance gives additional support to the idea that this burst is rather peculiar, and is not as close as one would expect from the high value of the observed flux.

netic field direction (θ greater than a few degrees), the emitted spectrum always has the same shape, except at low frequency (i.e., at energies smaller than $E_B$). The spectra obtained for θ = 0 are also very similar to the one shown in Figure 2a, except when the optical thickness τ is so small that the reconnection layer is optically thin to resonant absorption. Figure 2f shows such a thin spectrum obtained for $\theta = 0^\circ$ with $\gamma_s$ equal to 1/75 ($\tau = 10^{-3}$). This spectrum resembles very closely the one of GB 781119, although it is still too soft above 1 MeV (but remember that we have neglected transitions with $\Delta n = 2$).
created by oscillatory convection deep in a layer heated by a thermonuclear explosion. The physical parameters of the hot, reconnecting corona have been estimated in a self-consistent way; the optical depth for Thomson scattering is found to be of the order of $10^{-2}$; the corona is thus optically thick for resonant scattering (cyclotron absorption in the first Landau level, immediately followed by emission of a cyclotron photon), but, because the electrons are one-dimensional, the higher harmonics are optically thin.

Thermal photons of a few keV, resulting from the dissipation of sound waves that are also produced by magnetoconvection, are resonantly scattered to high energies by relativistic electrons, inside a narrow cone centered along the magnetic-field lines. Part of these photons with an average energy of the order of $mc^2$ escape, while those directed downward reach regions where the Thomson optical thickness is unity. There, reconnection is no longer efficient, and these regions are cool. Compton scattering of the high-energy photons on cold electrons produces a quasi-isotropic subpopulation of electrons with a temperature of the order of $mc^2/k$ that emits synchrotron radiation. We have treated by Monte-Carlo simulations the transfer of thermal and synchrotron radiation in the hot, reconnecting corona, and we have found that the spectra obtained are in good agreement with the observations.

This is a priori not very surprising, since it is known that the observed spectra can be fitted by synchrotron emission (Liang 1982; Liang, Jernigan, and Rodrigues 1983); however, because synchrotron emission results from a subpopulation of electrons and not from the whole layer, the calculated $\gamma$-ray luminosity is much lower than what has been inferred by Liang et al. and is compatible with the energy released by the thermonuclear explosion. We have also shown that emission is very strongly anisotropic and that, in a very few cases, very hard spectra should be observed when the line of sight coincides with the magnetic-field direction. Therefore the most intense $\gamma$-ray bursts are not necessarily the closest, and this may explain the very low ratio $L_\gamma/L_\nu$ observed for GB 781119.

To conclude, one should mention that our calculations represent idealized spectra, in the sense that the corona is assumed to be homogeneous, and that the Alfvén flux as well as the angle of the line of sight with respect to the magnetic field are taken to be constant. In reality, neither reconnection nor the Alfvén flux is constant in space and time. Therefore, this will lead to short time-scale variability, in addition to slower variations due to the rotation of the neutron star. Both effects seem to be present in the observed spectra (Vedrenne 1984), and the apparent lack of variability in some of them (Gilman 1980) could be more difficult to explain.

It is a pleasure to thank Gilbert Vedrenne for very helpful discussions on observations of $\gamma$-ray burst spectra. We would like to thank Edison Liang and John Kirk for useful criticism of an early version of this paper. We are grateful to participants in the Stanford 1984 $\gamma$-ray bursts workshop, and particularly to Peter Mészáros, for stimulating discussions. We thank the anonymous referee for his helpful comments.

**APPENDIX**

**SYNCHROTRON EMISSIVITY AT RELATIVISTIC TEMPERATURES**

Here we describe the method by which the synchrotron emissivity is computed. The results of this numerical calculation are compared with the approximate formulae given by Petrosian (1981), and their accuracy is estimated. The synchrotron emissivity of a
plasma at a temperature \( T \) in a magnetic field \( B \) is given by (Bekefi 1966):

\[
j_{\alpha}(\theta) = \frac{e^2\alpha^2}{2\pi c^2 B} \int_{1}^{\infty} \sum_{n=0}^{\infty} \left[ \frac{\cos \theta - \beta_{\|}^2}{\sin \theta} \right] J_n^2(x) + \beta_{\perp}^2 J_n^2(x) \right] dy 2\pi\beta_{\|} f(\beta_{\|}, \beta_{\perp}) d\beta_{\|} d\beta_{\perp},
\]

where \( y = n^2_{\alpha} \sqrt{\left[ 1 - \beta_{\|}^2 - \beta_{\perp}^2 - \omega(1 - \beta_{\perp}^2 - \cos \theta)^{1/2} \right.} \), and

\[
x = \omega \beta_{\perp} \sin \theta \sqrt{(1 - \beta_{\|}^2 - \beta_{\perp}^2)^{1/2}},
\]

\( J_n \) is the Bessel function of order \( n \), and \( J'_n \) its derivative, \( \omega_B = eB/mc \) is the cyclotron frequency, \( \beta_{\|} \) and \( \beta_{\perp} \) are the two components of the electron velocity, normalized to \( c \), parallel and perpendicular to the magnetic field, \( f(\beta_{\|}, \beta_{\perp}) \) is the electron distribution function, which, for a Maxwellian distribution, is given by:

\[
f(\beta_{\|}, \beta_{\perp}) = \frac{N}{4\pi TK_B(1/T)} y^7 e^{-y/T},
\]

where \( y \) is the Lorentz factor, \( K \) the Hankel function of second order, and \( N \) the number density.

Integrating equation (A1) over \( \beta_{\perp} \) gives:

\[
j_{\alpha}(\theta) = \frac{e^2\alpha^2}{2\pi c^2 B} \int_{1}^{\infty} \sum_{n=0}^{\infty} \left[ \frac{\cos \theta - \beta_{\|}^2}{\sin \theta} \right] J_n^2(x) + \beta_{\perp}^2 J_n^2(x) \right] \left[ 1 - \frac{\beta_{\|}^2 \cos \theta}{n^2} \right] f(\beta_{\|}, \beta_{\perp}) d\beta_{\|},
\]

\( \beta_{\perp} \) being the solution of \( y = 0 \). The Bessel functions can be approximated by the Wild and Hill (1971) formulae; for a given \( \omega, J_\alpha \) can then be simply computed.

However, for large values of \( \omega/\omega_B \), the function under the integral is nearly zero, except close to \( \beta_{\|} = 1 \); i.e., in an interval \( [\beta_{\|}, 1] \), \( \beta_\omega(\omega/\omega_B) \) increases with increasing values of \( \omega/\omega_B \); this can be used to reduce the computation time. We computed \( j_\alpha(\theta) \) for increasing values of \( \omega, \theta \) being fixed; \( \beta_\omega \) is determined by the condition that the function under the integral is \( 10^{-8} \) times its maximum value and is used for the next value of \( \omega \).

We found that in order to get 10% accuracy, for a temperature \( kT \geq mc^2 \), up to 10 harmonics had to be summed up for \( \omega/\omega_B = 2 \); for \( \omega/\omega_B = 9, 15, 200, 500 \), and 1000, the number of harmonics to be summed is 50, 100, 2000, 5000 respectively.

The comparison of the emissivity we obtain with the approximation given by Petrovskii (1981) is shown in Figure 6. It is seen that the steepest descent method used by Petrovskii (1981) gives accurate results for low values of the temperature of the emitting plasma; for \( kT \geq 0.5 mc^2 \), the error is moderate (less than 40%). The analytical approximation (Eq. [29] of Petrovskii's paper) leads to roughly the same precision; on the other hand, the simply asymptotic expression \( \sim \exp \left[-(\omega/\omega_B)^{1/3}\right] \) can be used only for high temperatures; for \( kT < 0.2 mc^2 \), the asymptotic formula converges very slowly and leads to order-of-magnitude errors at low frequency (even if \( \omega/\omega_B > mc^2/kT \)). Therefore, one must be very cautious when deriving the physical parameters of the emitting region if the simple asymptotic expression, \( \exp \left[-(\omega/\omega_B)^{1/3}\right] \) is used, especially at low temperature.

REFERENCES


Klein, O., and Nishina, Y. 1929, Zs. Phys., 52, 853.


Liang, E. P. 1984, private communication.


