A LEAKY MAGNETOHYDRODYNAMIC WAVEGUIDE MODEL FOR THE ACCELERATION OF HIGH-SPEED SOLAR WIND STREAMS IN CORONAL HOLES

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ABSTRACT

It is well established observationally that high-speed solar wind streams originate in coronal hole regions in the solar corona. Models of the solar wind flow based on this observation indicate that heat conduction alone cannot account for the observed properties of the wind and that other sources of heat and/or momentum must be sought. One suggested source for this additional momentum is "wave pressure" generated by magnetohydrodynamic (MHD) waves. Theories of wave-driven winds exist, but they are not consistent with the observed fact that high-speed streams originate in discrete magnetic structures in the solar corona. The waves assumed responsible for acceleration of the high-speed solar wind streams should have periods of approximately a hundred seconds if they are driven by photospheric turbulence. But MHD waves with periods this large have wavelengths $\lambda \gtrsim d$, where $d$ is the characteristic transverse size of the coronal hole. Current theories for the acceleration of the solar wind by MHD waves are valid only if the wavelength of the disturbance is much smaller than the characteristic transverse size of the coronal structure. This limit is not appropriate for the propagation of disturbances with periods $P \approx 100 \text{ s}$ in the acceleration region of the solar wind.

In this paper the effect of coronal hole magnetic structure on the propagation of MHD waves of all periods is considered. It is found that for the wave-period range discussed above the coronal hole structure acts as a "leaky" MHD waveguide, i.e., wave flux which enters at the base of the coronal hole is only weakly guided by the coronal hole structure. A significant amount of wave energy leaks through the side of the coronal hole into the surrounding corona. Dispersion relations are derived to describe the propagation of these leaky waves from both a geometric optics and an eigenmode approach. These two approaches are shown to yield equivalent results. The dispersion relation is then solved numerically to obtain both the real and the imaginary parts of the propagation constant. An analytic solution valid at high frequencies is also presented. It is found that for some frequencies the solutions of the dispersion relation indicate waves with negative, i.e., downward, group velocity. These waves cannot carry energy up from the photosphere into the corona, and therefore cannot contribute to the acceleration of the wind. Estimates for the periods of these waves give $P \sim 100 \text{ s}$.

The force on the coronal hole plasma due to the propagation of the leaky wave modes is calculated. It is found that the net force can be viewed as the sum of two terms: (1) magnetic wave pressure and (2) magnetic wave tension. The relative magnitude of these two forces is frequency-dependent. For very long wave periods or very short wave periods the pressure force is dominant. However, whenever the Alfvén wavelength is of the order of the transverse scale size of the coronal hole, the tensile force can completely dominate the wave-pressure term, resulting in a net downward force on the plasma within the coronal hole. The calculations presented here show that the wave tensile force can be important for waves with periods of the order of a few hundred seconds.

Taken together, these two results indicate that the solar wind plasma does not couple as efficiently to the 300 s oscillations as spherically symmetric theories of MHD wave-driven winds have indicated in the past. Numerical evaluation of this coupling efficiency must await a more detailed model incorporating the source terms at the base of the corona.

Subject headings: hydromagnetics — Sun: corona — Sun: solar wind

I. INTRODUCTION

The correlation between high-speed solar wind streams and coronal holes is now reasonably well established (Krieger, Timothy, and Roelof 1973; Neupert and Pizzo 1974; Wagner 1976; Nolte et al. 1976). This observational association has had two important consequences for solar wind theory: (1) it has narrowed the range of acceptable solar wind models by providing realistic estimates of the plasma pressure and density near the region where the wind is accelerated, and (2) it has emphasized the fact that at least one major component of the solar wind originates in a discrete structure within the solar corona. Recent theoretical efforts (Kopp and Holtzer 1976; Holtzer and Leer 1980; Leer and Holtzer 1980) have dealt almost exclusively with the implications of these two additional constraints for the various mechanisms proposed for acceleration of the flow. In particular, the effect of nonradial flow-tube geometry and the coronal plasma conditions on the steady state average flow within a high-speed wind stream has been evaluated. The basic conclusion is that thermal conduction alone is not sufficient to explain the observed particle flux and flow speed observed in the high-speed streams, and that there must be a significant addition of momentum and/or energy in the region of supersonic flow. At this point, the most likely source of this additional momentum seems to be magnetohydrodynamic (MHD) waves propagating up from the solar surface. (For an alternative to the wave hypothesis see Mullan...
Although existing theories for the acceleration of the solar wind by MHD waves have reached a considerable degree of sophistication, they are not yet completely self-consistent. They account for flow-tube geometry in the equations for the large-scale flow, but ignore the effect of flow-tube geometry on the propagation of the waves assumed responsible for the acceleration. In this paper the propagation of MHD waves in a structure of finite transverse size is considered, and the consequences for the acceleration of high-speed solar wind streams is discussed.

The notion that MHD "wave pressure" could provide a significant portion of the momentum required to accelerate the solar wind was proposed initially by Alazraki and Courtier (1971) and Belcher (1971) in the context of a spherically symmetric wind theory. They demonstrated that a wave-driven wind model could be made consistent with observations of the solar wind. This point has been confirmed by subsequent analyses (Hollweg 1973; Jacques 1977; Hollweg 1978a; McWhirter and Kopp 1979; Heinemann and Olbert 1980). The wave force used in these models is obtained from a spherically symmetric wave propagation model which neglects the boundaries of the coronal hole. Physically, one might expect that boundary effects would become important whenever the wavelength of the wave approached the diameter of the coronal hole, i.e., for \( d/(Pr_{\alpha}) < 1 \), where \( d \approx 10^{10} \text{ cm} \) is the characteristic transverse size of a coronal hole, \( \alpha_{\text{A}} \approx 10^{6} \text{ cm s}^{-1} \) is the local Alfvén speed in the corona, and \( P \) is the wave period of interest. Using this, one can estimate that boundary effects should be important for waves with periods greater than a few hundred seconds. This is almost precisely the period where the peak in the photospheric turbulence power spectrum due to the 5 minute oscillation is observed to occur. In the absence of nonlinear wave/wave interactions, one would expect similar periods in the corona. Alternatively, if one considers interplanetary oscillations to be a better indicator of the wave periods present in a coronal hole, the largest part of the power is at even longer periods \( (P \approx 10^{3}–10^{4} \text{ s}) \), making proper consideration of boundary effects even more critical. There is, however, some disagreement regarding the power and the spectral distribution of wave flux in the corona (Hollweg 1978b; Leer, Holtzer, and Fla 1982).

In a recent paper Fla et al. (1984) have proposed a model for the acceleration of solar wind streams by fast-mode MHD waves. They assume that the Alfvén speed within a coronal hole is smaller than the Alfvén speed of the surrounding medium, i.e., the lower observed density in the coronal hole is offset by a smaller magnetic field within the coronal hole, resulting in a lower Alfvén speed. In their view, coronal holes act as traps for fast-mode wave flux, which, in turn, accelerates the wind. However, such an equilibrium configuration is not self-consistent, since the magnetic pressure (which dominates in the corona) would be smaller inside the coronal hole than outside. This point is in fact made in their paper. In this paper, a self-consistent model is suggested which results in a qualitatively different picture of wave propagation in the vicinity of a coronal hole.

The fundamental assumption of this paper is that within a coronal hole the Alfvén speed is slightly larger than the Alfvén speed of the surrounding medium, i.e., there is a change of Alfvén speed at the boundary. It is this inhomogeneity which defines the boundaries of the flow tube for propagating waves. The Alfvén speed, \( \alpha_{\text{A}} = B/(4\pi \rho)^{1/2} \), is linearly proportional to the magnetic field strength, \( B \), and inversely proportional to the square root of the mass density, \( \rho \). Therefore, to know how the Alfvén speed varies, one must have some idea of the variation of \( B \) and \( \rho \). Observations indicate that the electron density in a coronal hole is essentially a factor of 3 smaller than the electron density in the normal quiet corona (Munro and Withbroe 1972). One would expect a similar reduction in the mass density. Furthermore, since the magnetic pressure dominates the thermal pressure throughout the corona, the magnetic field strength must be nearly equal inside and outside the coronal hole to preserve pressure balance. More detailed models of the coronal equilibrium tend to confirm this point (Steinolfson, Seuss, and Wu 1982). In addition, if the coronal hole were not in equilibrium, one would expect it to evolve on a time scale of the order of the Alfvén speed crossing time. Such rapid evolution is not observed; in fact, coronal holes have lifetimes of the order of a month. With these considerations in mind, the assumption of transverse gradients in the Alfvén speed does not seem unreasonable. A similar picture has been invoked in recent theory of coronal heating in closed magnetic loops with considerable success (Ionson 1982, 1984). It is also worthwhile to note that one should be able, at least in principle, to choose between the model described here and the model of Fla et al. (1984) by observing the magnetic field strength and density inside and outside of a coronal hole.

Given this assumption regarding the variation of the Alfvén speed, it is clear that for large wave periods, \( P \gtrsim 100 \text{ s} \), a waveguide description of MHD wave propagation is more appropriate than a spherically symmetric propagation theory. The central focus of this paper is to explore the implications of this waveguide picture for the acceleration of high-speed solar wind streams by MHD waves. Formally the MHD modes of a coronal hole are found to be almost completely analogous to the so-called leaky wave modes of the dielectric waveguide. In fact, many of the results obtained in this paper have close analogies in the extensive literature on the propagation of electromagnetic radiation in dielectric media (Marcuse 1972; Kapany and Burke 1972; Marcuse 1974). Basically it is found that coronal holes act as leaky waveguides for MHD waves in the solar corona. Wave flux which enters at the base of the corona is guided by the magnetic structure of the coronal hole from the photosphere out into the interplanetary medium. The boundaries of the coronal hole are not rigid, however, and a significant amount of wave energy can be radiated from the walls into the surrounding medium. The energy which leaks out of the coronal hole is transported across magnetic field lines by compressing the plasma in a direction transverse to the background magnetic field.

In spherically symmetric theories of MHD wave-driven winds, where there are no coronal hole boundaries, the waves are almost completely transverse. In the steady state, gradients in the background magnetic field are balanced by thermal pressure gradients. So the only force acting on the plasma is the wave pressure. In a finite flux tube, however, the waves generate a magnetic field component parallel to the direction of the bulk fluid flow, and this component when averaged over the period of the wave can result in a net tensile force on the coronal plasma which tends to counteract at least part of the wave-pressure force. The relative magnitude of the tensile force depends on the wavelength, the plasma number density, the plasma temperature, the magnetic field strength, the flow velocity, and the wave number. In these models, the presence of a magnetic field component parallel to the flow is essential to the production of a net force on the plasma. The relative magnitude of the tensile force depends on the wavelength (or, equivalently, on the period) of the disturbance. For example, if the wavelength is much smaller than the transverse scale size \( d \) of the coronal hole, wave propagation is not strongly affected by the external medium. In this case, one would expect the tensile force to be...
negligible when compared with the pressure force. Similarly, when the wavelength is much larger than \( \delta \), the characteristics of wave propagation are determined primarily by conditions in the external medium. Again in this case the coronal hole boundaries are relatively insignificant, so in both of these limits the waves which propagate are primarily incompressible and the resulting tensile force on the coronal plasma is relatively small. In the intermediate frequency range where \( \omega \delta/\nu_\alpha \approx 1 \), the tensile force can dominate the wave-pressure force entirely. In all cases the leaky-waveguide model predicts a smaller (or sometimes even a downward) net wave force on the coronal hole plasma.

The remainder of this paper will be devoted to the task of amplifying and quantifying the physical picture presented above. In § IIa the equations describing wave propagation are derived, and the solution of the wave equation for the propagation of plane MHD waves in a simplified coronal hole waveguide with slab symmetry is examined using a geometric or ray theory. Expressions for the reflection and transmission coefficients at the boundary of the coronal hole waveguide (analogous to the Fresnel coefficients in electromagnetic theory) are obtained, and the eigenvalue equations for wave propagation are derived from the geometric theory. In § IIb the eigenvalue equation is derived from the more complete mode theory for the slab coronal hole. The eigenvalue equation is then solved numerically for the complex propagation constant. The imaginary part of the propagation constant is a measure of the leakage rate and hence is a measure of the strength of the tensile force on the plasma. In § IIc an analytic solution of the dispersion relation, valid for high frequencies, is presented. Finally, in § IId, the implications of this mechanism for acceleration of the solar wind are discussed.

II. WAVEGUIDE SOLUTION FOR A MODEL CORONAL HOLE

Assume a cylindrical, though not necessarily circular, coronal hole model, and choose a coordinate system such that the gradient in the density, \( \rho_0 \), is perpendicular to the \( z \)-axis. Then Fourier-transform the velocity, using

\[
\tilde{v}(x, z, t) = \int \frac{d\omega}{2\pi} \int \frac{dk_z}{2\pi} v(x, k_z, t)e^{i(k_zz - \omega t)}
\]

and its corresponding inverse,

\[
v(x, k_z, \omega) = \int dt \int dz \tilde{v}(x, z, t)e^{-i(k_zz - \omega t)}.
\]

The magnetic field \( \tilde{B} \) can be similarly transformed. Using these, the linearized MHD equations, equations (A13)-(A16) in the Appendix, can be combined to yield a single scalar equation for the quantity \( \phi(x, k_z, \omega) = \nabla \cdot \tilde{v}(x, k_z, \omega) \) given by

\[
\nabla \cdot \left( \frac{1}{\kappa^2} \nabla \phi \right) + \phi = 0,
\]

where \( \kappa^2 = \omega^2/c_N^2 - k_z^2 \). The components of the velocity are related to the gradient of \( \phi \) by

\[
v(x, k_z, \omega) = -\frac{1}{\kappa^2(k_z, \omega)} \nabla \phi.
\]

Similar equations have been obtained previously by Ionson (1978), Wentzel (1979), and Wentzel (1980) for the analysis of Alfvénic surface waves. In this paper a simplified slab coronal hole model is considered (see Fig. 1). This model is simple enough to be mathematically tractable, yet complex enough to illustrate the physics of the propagation of MHD waves in the coronal hole waveguide. Only cases where \( \kappa^2 \) is constant (except at the interface between two media) and the \( y \)-dependence of \( \phi \) and \( \tilde{B} \) is negligible are considered. For these cases equation (3) reduces to the relatively simple form

\[
\frac{d^2 \phi}{dx^2} + \kappa^2 \phi = 0.
\]

Solutions of this equation are, of course, plane waves proportional to \( e^{i\kappa x} \) and \( e^{-i\kappa x} \).

a) Geometric or Ray Analysis of the Slab Waveguide

Consider an MHD wave incident on the interface between two media as shown in Figure 2. Such a situation would be relevant at the boundary of a coronal hole, for instance. Each medium is assumed to be characterized by different values of the propagation constant \( \kappa \). This variation is the result of density variations in the model presented here. As one might expect, a portion of the incident wave energy is reflected from the interface, while the rest is transmitted. The fraction of reflected and transmitted energy can be calculated by using the
solutions to equation (5) above, along with the boundary conditions

\[ \phi_1 = \phi_2 , \]  
\[ \frac{1}{\kappa_1^2} \frac{\partial \phi_1}{\partial x} = \frac{1}{\kappa_2^2} \frac{\partial \phi_2}{\partial x} . \]  

These are obtained by requiring the continuity of the normal component of the velocity and continuity of the pressure. The reflection and transmission coefficients obtained in this way are completely analogous to the well-known Fresnel coefficients of electromagnetic theory. Refraction of fast-mode MHD waves was discussed previously by Barnes (1969).

The wave solutions in media 1 and 2 of Figure 2 can be written as

\[ \phi_1 = A_u e^{i(k_1 x + \kappa_1 x - \omega t)} + A_r e^{i(k_2 x - \kappa_2 x - \omega t)} \]  
\[ \phi_2 = A_t e^{i(k_2 x + \kappa_2 x - \omega t)} , \]

where \( A_u, A_r, \) and \( A_t \) represent the incident, reflected, and transmitted wave amplitudes. Using these solutions along with the boundary conditions, one can obtain the reflection coefficient,

\[ \frac{A_r}{A_i} = \frac{\kappa_1 - \kappa_2}{\kappa_1 + \kappa_2} = -\frac{\sin \theta_1 - (N^2 - \cos^2 \theta_1)^{1/2}}{\sin \theta_1 + (N^2 - \cos^2 \theta_1)^{1/2}} , \]

where \( \theta_1 \) is the grazing angle defined in Figure 2 (note that \( \theta_1 \) is not the usual angle of incidence measured from the surface normal) and \( N = \frac{v_{A1}}{v_{A2}} \) is the ratio of the Alfven speed of medium 1 to that of medium 2. This ratio, \( N \), plays the same role as the ratio of refractive indices in the Fresnel equations. The transmission coefficient can also be calculated:

\[ \frac{A_t}{A_i} = \frac{2\kappa_2}{\kappa_1 + \kappa_2} . \]

Graphs of the reflection coefficient are shown in Figure 3a (3b) for cases where \( N^2 \) is smaller (larger) than unity. The types of wave propagation implied by these two cases are qualitatively and quantitatively different. To see this, consider first the case where \( N^2 < 1 \).

In this case, when \( \cos^2 \theta_1 > N^2 \) the incident wave is totally reflected. This is shown in Figure 3a. This phenomenon results in the fully guided modes of the dielectric waveguide. For these waves energy is trapped entirely within the guiding structure. Ionson (1982) has emphasized the importance of this phenomenon for heating closed loops within the solar corona. However, for coronal holes \( N^2 > 1 \), so that there is no possibility of total internal reflection at the interface between the plasma inside and outside of the coronal hole. Nevertheless, as shown in Figure 3b the reflection coefficient can become relatively large, approaching unity, when the grazing angle becomes small. The high reflectivity at grazing incidence allows the propagation of nearly guided leaky wave modes in the coronal hole flux tube. Mathematically these waves are described by the same dispersion relation as the fully guided modes. The difference is that the propagation constant, \( k_z \), within the flux tube is allowed to be complex. It is worthwhile to emphasize here that the imaginary part of the propagation constant is not due to damping of the wave, since only dissipationless processes have been considered in this analysis. Instead, the imaginary part of \( k_z \) is a measure of the rate at which wave flux leaks through the boundary. Formally these leaky solutions are obtained as the complex generalization of the dispersion relation for fully guided modes.

It would seem at first glance that waves of any frequency and wavelength could propagate within the waveguide as long as the reflection coefficient was sufficiently large for a significant amount of the power to be retained in the flux tube. This is not in fact the case, however. Consider the waveguide shown in Figure 4. For the disturbance to be a normal mode of the waveguide, the wave which travels along the ray from point C to point D with two reflections must remain in phase with the ray which travels from point A to point B. Mathematically this is expressed as

\[ \frac{e^{i\omega \tau}}{\tau} (CD - AB) + 2\zeta = 2\pi n , \]  

Graphs of the reflection coefficient as a function of the angle of incidence for a coronal loop where \( v_{A1} < v_{A2} \). The numerical values labeling the curves give the ratio of the Alfven speed outside the coronal hole to the Alfven speed inside, \( v_{A2}/v_{A1} \). Angles where \( \theta_1/\theta_1^* = -1 \) experience total internal reflection. (b) The reflection coefficient as a function of the angle of incidence for a coronal hole where \( v_{A1} > v_{A2} \). Again the numerical values labeling the curves indicate the ratio of the Alfven speeds.
where \( m \) is an integer, \( \zeta = \pi - 2\psi \) is the phase change for each reflection, and \( \psi \) is calculated from the expression for the reflection coefficient above as

\[
\tan \psi = \frac{\gamma}{\kappa} = \frac{(\cos^2 \theta_1 - N^2)^{1/2}}{\sin \theta_1}.
\]

In this expression the substitution \( \kappa_1 = i\gamma \) has been used, and the subscript on \( \kappa_1 \) has been dropped for convenience. This notation will be employed throughout the remainder of this paper. Equation (11) can be solved. The result is two eigenvalue equations, i.e., dispersion relations, for the waves in the slab waveguide given by

\[
\tan \kappa d = \frac{\gamma d}{\kappa d}
\]

(13)

and

\[
\cot \kappa d = -\frac{\gamma d}{\kappa d}.
\]

(14)

The first (second) expression is the dispersion relation for the symmetric (antisymmetric) wave mode of the coronal hole. This identification, which is not completely apparent from the geometric theory presented here, becomes obvious in the normal mode analysis presented below. The primary virtue of the simple ray theory presented here is that it clearly demonstrates the physical mechanism responsible for the eigenvalue condition and therefore provides considerable insight into the wave propagation process within a magnetic flux tube.

**b) Wave Mode Analysis**

In the previous paragraph the solutions of the wave equation were discussed from the point of view of geometric optics, using complex exponentials as eigenfunctions of the wave equation. This choice of eigenfunction could also be made here. However, the algebra of the eigenvalue solution is simplified considerably if one considers the symmetric and antisymmetric wave modes instead. In this paper, the terms symmetric and antisymmetric will be used to describe the symmetry properties of the fluid displacement. This terminology is consistent with standard practice. The reader should note, however, that since the displacement is related to the derivative of \( \phi \), the functions \( \phi \) will have the opposite symmetry. In the paragraphs which follow, the symmetric modes are discussed in considerable detail. Solutions of the dispersion relation for the antisymmetric mode are presented without detailed derivation, since much of the algebra is similar.

The symmetric solution inside the coronal hole flux tube is of the form

\[
\phi(x, \kappa_2, \omega) = A \sin \kappa_d e^{(x + d)}, \quad x \leq d
\]

(15)

\[
= A \sin \kappa, \quad -d \leq x \leq d
\]

(16)

\[
= A \sin \kappa d e^{-\gamma(x - d)}, \quad x \geq d
\]

(17)

The analogous solution for the antisymmetric case is obtained by replacing the sine with the cosine in the equations above. The dispersion relations for the symmetric mode as discussed above in the context of geometric theory can be easily obtained from the solutions and the boundary conditions. Dispersion relations for symmetric and antisymmetric wave modes have been obtained previously by Edwin and Roberts (1982). They discuss solutions for which \( \kappa \) is real. Such solutions correspond to the fully guided models of the slab waveguide and are appropriate for coronal loops where \( v_A < v_A^* \). However, for coronal holes \( v_A > v_A^* \), and there are no solutions with \( \kappa \) purely real. The solution of the dispersion relation for complex \( \kappa \) is developed below.

The dispersion relation for the symmetric mode can be written as

\[
\frac{\kappa^2 d^2}{\cos^2 \kappa d} = -W^2,
\]

(18)

where the relation \( W^2 = -(\gamma^2 d^2 + \kappa^2 d^2) = (\omega^2 d^2/v_A^2) (N^2 - 1) \) was used. From this single complex-valued expression, one can obtain two equations by letting \( \kappa d = u_1 + iu_2 \) and then equating the magnitude and phase of equation (18) to obtain

\[
u_1 \cot u_1 = u_2 \tan u_2
\]

(19)

and

\[
W^2 = \frac{2(u_1^2 + u_2^2)}{\cos 2u_1 + \cosh 2u_2}.
\]

(20)

In these expressions, \( W \) is a parameter which essentially measures the frequency of the disturbance, so that equations (19) and (20) form a system of two equations from which the numerical values of \( u_1 \) and \( u_2 \) can be uniquely determined for any given value of the frequency parameter \( W \). The propagation constant is obtained from the relation defining \( \kappa \) as

\[
k_d^2 d^2 = W^2 - \kappa^2 d^2
\]

(21)

These equations were solved numerically, and the resulting values for the real and imaginary parts of \( \kappa \) are shown in Figures 5 and 6 for the lowest order symmetric wave mode. The order of the solution is determined by order of the branch of the solution chosen in equation (19).

Some interesting physical insights into wave propagation in coronal holes can be gained by considering the solution for the real part of \( \kappa \) shown in Figure 5. Physically one would expect...
that whenever the wavelength becomes small enough, or equivalently when the frequency becomes large enough, the effect of the coronal hole boundaries should become negligible. This behavior is clearly seen in Figure 5, where the dispersion relation tends to approach the Alfvén wave relationship \( \omega = k_z v_{A1} \) in the limit where \( W \to \infty \). In the opposite limit, i.e., very large wavelength, one would expect that the physical properties within the coronal hole would be less important in the dispersion relation. This effect can be clearly seen in the low-frequency portion of Figure 5, where the dispersion relation approaches \( \omega = k_z v_{A2} \), the dispersion relation for Alfvén waves in the external medium.

Another interesting feature of this wave solution is the region of negative group velocity in the region near \( W \approx 1 \). For the specific model calculated the group velocity is negative in the range \( 0.65 \leq W \leq 3.1 \). Waves driven at these frequencies near the base of the corona cannot conduct energy upward in the coronal hole. For typical parameters \( N = 3, v_{A1} = 10^8 \), and \( d = 10^{10} \) this range corresponds to periods of the order of \( 550 \leq P \leq 2700 \) s. In view of the approximations inherent in this analysis, one should not take this range too literally. Nevertheless it indicates that the solar wind in coronal holes may not couple to \( p \)-mode oscillations in the convection zone as efficiently as spherically symmetric models have indicated in the past. In addition, there is another factor which tends to reduce the effect of wave pressure at these periods, the magnetic tension force. This point is discussed in more detail below.

Finally, in Figures 7 and 8 the solution of the dispersion relation for the lowest order antisymmetric mode is presented. The overall features of the solution are similar to those discussed above for the symmetric mode. The primary difference is that there is no low-frequency antisymmetric mode with positive group velocity.

It is interesting to note from equation (A16) that even though the parallel velocity component vanishes, the parallel component of the magnetic field, \( B_z \), does not. This is because the compressibility of the plasma has been retained in these equations. It is of course well known that for classical Alfvén waves \( B_z = 0 \). The fact that \( B_z \neq 0 \) for these waves indicates that they may be subject to transit-time damping by the thermal plasma in the region of subsonic flow. This damping could serve as a significant source of heating for the plasma in the coronal hole. Spherically symmetric models incorporating wave heating by MHD fast modes have been considered by Barnes (1969) and Barnes and Hung (1972, 1973). Only momentum additions are considered in the formalism discussed in the Appendix. Further, a single-fluid description of
the plasma has been used, so that kinetic processes such as transit-time damping are not included. Therefore, the question of heating by these waves must be left as a subject for future investigation.

**c) Analytic Solution of the Dispersion Relation for High-Frequency Waves**

An approximate solution for the dispersion relation describing the propagation of symmetric wave modes can be derived for the limit in which the wavelength of the disturbance within the coronal hole waveguide is small. These are the so-called nearly guided modes. Presumably a similar solution could be developed for the asymmetric modes as well, since the analogous solution for dielectric waveguides has been obtained (Marcuse 1972). The importance of this approximate solution is twofold; (1) it allows analytic description of the wave acceleration process, at least in the small-wavelength regime, and (2) it provides a valuable check on the numerical solution presented above.

The dispersion relation can be separated into real and imaginary parts by again substituting $x_d = u_1 + iu_2$ and $y_d = s_1 + is_2$ into the dispersion relation. It is assumed that $u_1 \gg u_2$, $s_2 \gg s_1$, and $s_2 \gg s_1 u_1$. With these the dispersion relation reduces to

$$\tan u_1(1 - \tanh u_2) = \frac{s_2 u_2}{1 + \tan^2 u_1 \tanh^2 u_2} \tag{22}$$

and

$$\tanh u_2(1 + \tan^2 u_1) = \frac{s_2}{1 + \tan^2 u_1 \tanh^2 u_2} \tag{23}.$$ 

The right-hand side of equation (22) is very large. To make the left-hand side large also, one must require that $\tan u_1 \gg 1$ and $u_2 < 1$. If we let $u_1$ equal $M\pi/2 - \eta$, where $M$ is an odd integer, the equations reduce to

$$\eta = \frac{s_2 u_2^3}{u_1^2} \tag{24}$$

and

$$u_1 = s_2 u_2. \tag{25}$$

Now use the relation $\gamma^2 d^2 + \kappa^2 d^2 = -W^2$, with $W^2 = (\omega^2 d^2/v_A^2)(K^2 - 1)$, to obtain the following approximate solutions:

$$\eta = \frac{(M\pi/2)}{W^2 + (M\pi/2)^2} \tag{26}$$

and

$$u_2 = \frac{M\pi/2}{[W^2 + (M\pi/2)^2]^{1/2}}. \tag{27}$$

From equation (26) we see that this solution is valid for $W \gg M\pi/2$. The real and imaginary parts of $k$ can be obtained from the definition of $\kappa$. These results are in complete agreement with the numerical calculations represented in Figures 5 and 6 in the high-frequency range.

**d) Calculation of the Time-averaged Wave Force**

In the previous paragraphs, the propagation characteristics of MHD waves in the coronal hole waveguide were discussed. In this section the time-averaged force on the coronal hole plasma due to these waves is calculated. In frequency regimes where the group velocity is negative the time-averaged force is assumed to vanish, since only a model with sources at the base of the coronal hole is being considered.

The general expression for the time-averaged wave force is given in equation (A7) of the Appendix. Using this expression and the coronal hole model shown in Figure 1, one can write the $z$-component of the time-averaged force as

$$\langle F_z \rangle = -\frac{\partial}{\partial z} \langle B_x^2 \rangle + \frac{1}{4\pi} \left( \frac{\partial B_x}{\partial x} \right), \tag{28}$$

where the angular brackets indicate a time average over the wave period. Using this expression and equations (A15) and (A16), one can write the force as

$$\langle F_z \rangle = -\frac{\partial}{\partial z} \langle B_x^2 \rangle - 2\kappa^2 \frac{\langle B_x^2 \rangle}{8\pi k_z}. \tag{29}$$

To estimate the relative magnitude of these two terms, define a gradient scale length $L$ by

$$\frac{\langle B_x^2 \rangle}{8\pi L} = -\frac{\partial}{\partial z} \frac{\langle B_x^2 \rangle}{8\pi}. \tag{30}$$

Then the force can be written in a form which is more easily evaluated:

$$\langle F_z \rangle = \left[ 1 - 2k_{zi} L \left( 1 + \frac{\omega^2}{(v_1^2)^2} \right) \right] \frac{\langle B_z^2 \rangle}{8\pi L}. \tag{31}$$

where $k_{zi}$ is the imaginary part of $k_z$. From this expression, one can see that the force induced on the plasma by the waves consists of two competing terms. The first term is simply the wave-pressure force due to the expansion of the coronal hole. For a perfectly cylindrical coronal hole this term would of course vanish, since $L \to \infty$; however, real coronal holes do show expansion. In the context of the theory presented here, we assume this expansion to be slow, so that the coronal hole can be considered at least locally as cylindrical. The second term represents the effect of flux leakage. Whenever $k_{zi} L$ is small, the effect of leakage is negligible and the normal expression for the wave-pressure force is recovered. The competition between these two forces determines the net force on the plasma.
LEAKY MHD WAVEGUIDE MODEL

To examine some general characteristics of this expression, let us assume that \( L = d \). This is a reasonable assumption in the acceleration region of the solar wind where the gradient scale length is of the order of a solar radius. At heights of a few solar radii above the surface of the Sun, the radius of the coronal hole flux tube is also about a solar radius. At other heights within the coronal hole this assumption may not be as well justified, and details of the solution may change. However, one would anticipate that the overall properties of the solution would remain. In Figure 9 the normalized wave force \( \epsilon_f \), i.e., the term in brackets in equation (31), is plotted as a function of normalized frequency, \( W \).

The most striking feature of the results presented in Figure 9 is that for some frequencies the net force on the coronal plasma can be downward. The effect of wave pressure is most severely reduced in the range of periods of the order of a few hundred seconds. This is of course precisely the frequency where the majority of the turbulent power in the photosphere is observed to exist. The model is relatively simple. Nevertheless, these calculations indicate that the coupling of solar \( p \)-modes (300 s oscillations) is almost certainly not as efficient as existing theories have indicated.

The idea that waves can produce a downward force in the corona is unfamiliar and deserves some further consideration. This force can be understood in a qualitative way by considering the forces that a magnetic field can impose on a plasma. These are (1) magnetic tension along \( B \) and (2) magnetic pressure in a direction perpendicular to \( B \). Because the leaky-waveguide modes derived here have \( B_z \neq 0 \), the waves can exert both of these forces on the coronal plasma. To see this, use the divergence condition on \( B \) to rewrite the force, and then average over \( x \) to obtain

\[
\langle f_z \rangle = -\frac{\partial}{\partial z} \left( \frac{\langle B_z^2 \rangle}{8\pi} - \frac{\langle B^2 \rangle}{8\pi} \right) + \frac{1}{4\pi} B_x B_y \bigg|^{+d}_{-d}.
\]

If the final term, the surface stress, is small, the force on the plasma is determined primarily by a balance of the first two terms. If one assumes that waves within the coronal hole are driven only by sources at the base, then the energy density of the waves must decrease with \( z \). Then the transverse component of the wave magnetic field, \( B_x \), causes a magnetic pressure to be exerted on the plasma in the positive \( z \)-direction. In spherically symmetric incompressible theory this is the only force that the wave can exert on the plasma. But when the wave is compressive, \( B_x \neq 0 \), there is another force, magnetic tension, which acts in the negative \( z \)-direction. The balance between these two forces determines the net force on the plasma. For some frequencies, \( W \approx 1 \), the tension term can dominate the wave-pressure term. The calculations presented in this paper can be used only to estimate the relative magnitude of these two contributions to the wave force. The absolute magnitude of the force depends on the amplitude of the wave in the corona, which is of course in general not known.

A second important feature evident from Figure 9 is the presence of long-period, symmetric guided wave modes. For typical solar coronal parameters, these waves have periods of the order of a few hours corresponding to the \( g \)-mode period of the Sun. However, recent observations of the amplitude of these oscillations by Delache and Sherrer (1983) show that they probably do not have enough power to be energetically significant for powering the solar wind.

III. SUMMARY AND CONCLUSIONS

High-speed solar wind streams are now known to originate in discrete open-field magnetic structures within the solar corona, called coronal holes. Observations of the plasma state within coronal holes, when combined with \textit{in situ} measurements of solar wind properties by spacecraft in the interplanetary medium, can provide enough information to allow a valuable test of solar wind theory. Results so far have indicated that classical solar wind theory with heat conduction as the sole energy transport mechanism cannot account for the observed properties of high-speed solar wind streams. Another mechanism is needed to transport energy from the base of the corona out to the wind acceleration region. MHD waves, driven by large-scale convective motions of the photosphere, have been suggested as a possible source of additional acceleration for the wind. Most of the turbulent power in a coronal hole is carried by MHD waves with periods of a few hundred seconds or longer. This is evident from direct observations of turbulence in the solar photosphere, as well as from \textit{in situ} observations of turbulence in the solar wind. Waves with periods this long have wavelengths which are typically as large as the transverse scale of the coronal hole flux tube itself. For these waves boundary effects are important, and the coronal hole must be treated as a waveguide. The basic purpose of this paper has been to examine the propagation of MHD waves using this waveguide approach.

It was found that wave propagation within a coronal hole waveguide bears many similarities to the propagation of electromagnetic radiation in a leaky dielectric waveguide. Wave energy which enters the coronal hole at the base is only partly guided by the structure. Many of the characteristics of propagation can be obtained from a relatively simple ray theory. The relative confinement of wave energy within the coronal hole was interpreted in terms of the reflection and refraction of plane waves at the boundary of the coronal hole. In addition, the dispersion relation was obtained from the ray theory by considering the interference of plane waves within the waveguide.

A more rigorous waveguide mode theory was presented as well. Solutions of the wave equation were developed for a...
model coronal hole, and the dispersion relation for the wave modes was obtained. Numerical solutions of the dispersion relation were presented. Results of this solution can be summarized as follows. High-frequency, short-wavelength waves propagate inside the coronal hole at essentially the Alfvén speed of the interior. The boundaries are not important for these small-scale disturbances. Very low frequency, long-wavelength, symmetric waves can propagate at essentially the Alfvén speed of the external medium. These waves have scales large enough so that the coronal hole has little effect on their propagation characteristics. These waves are essentially the "infinite medium" modes of the external plasma. As one might have expected, when the wavelength is of the order of the transverse size of the coronal hole, the waves propagate at a phase speed intermediate between the phase speed of the interior and that of the exterior medium and the leakage rate is maximized. Finally, detailed examination of the dispersion relation shows that for a band of wave periods near 100 s the group velocity is downward. Oscillations at the base of the corona with these periods cannot transfer energy upward into the corona. This is important, since the turbulent power spectrum in the photosphere seems to peak very near this period because of the 300 s p-mode oscillations. The model used in these calculations is a rather simplified one, so that more detailed calculations will have to be done before this effect can be quantified.

Finally, the time-averaged force on the plasma due to the propagation of these leaky wave modes in the coronal hole waveguide was evaluated. It was shown that the force can be thought of as consisting of two terms, a magnetic wave pressure and magnetic wave tension. The balance between these two forces determines the direction and magnitude of the net force on the plasma. Calculations were presented which show that when the leakage rate becomes large the tensile force can become significant. For some frequencies the tensile force can dominate the wave-pressure force, resulting in a net downward wave force.

These two results, taken together, imply that the coupling of the solar wind plasma to the turbulence in the photosphere may not be as efficient as previous theories of wave propagation in the corona have indicated. Additional calculations incorporating a more detailed coronal hole model and source terms at the base of the corona must be done before this effect can be realistically quantified.

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APPENDIX

A TWO-TIME-SCALE MODEL FOR THE ACCELERATION OF HIGH-SPEED SOLAR WIND STREAMS BY MHD WAVES

Fluid motions within solar coronal holes are observed to change on two vastly differently time scales. First, there is the relatively long time scale required for the evolution of the large-scale flow. This time scale is by definition of the order of the lifetime of the coronal hole itself, i.e., 10–100 days. Superposed on this average motion are fluctuations which have a much shorter time scale. These fluctuations are expected to have periods in the range of a few minutes to a few hours. Evolution of the averaged coronal hole properties is the result of the cumulative, averaged effect of these relatively rapid processes. (In the context of the linearized theory developed in this paper these short time scale processes are thought of as being composed of the normal modes of the coronal hole structure.) In this appendix this two-time-scale picture is used to obtain a self-consistent description of both the average and the rapid fluid motions from the equations of magnetohydrodynamics.

I. EQUATIONS FOR THE AVERAGED FLOW

To begin, consider the MHD equation of continuity,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

(A1)

the momentum equation,

$$\frac{\partial \mathbf{v}}{\partial t} + (\rho \mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla P + \frac{1}{4\pi} (\mathbf{B} \cdot \nabla) \mathbf{B} + \rho \mathbf{g},$$

(A2)

and the induction equation describing the evolution of the magnetic field,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}).$$

(A3)

In equation (A2) $P$ is the total (thermal and magnetic) pressure. The remainder of the notation is standard. One then averages these equations over a time interval which is long compared with the characteristic time for rapid processes, but short compared with the characteristic time for evolution of the average flow. Assume that each fluid property, e.g., density or velocity, can be decomposed

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into a slowly evolving average part and a rapidly varying part. If one defines \( \langle f \rangle = f_0 \) and \( f_1 = f - \langle f \rangle \), where the angular brackets indicate the time average, the equations can be written as

\[
\frac{\partial \rho_0}{\partial t} + \nabla \cdot (\rho_0 v_0) = -\langle \nabla \cdot (\rho_1 v_1) \rangle ,
\]

(A4)

\[
\rho_0 \frac{\partial v_0}{\partial t} + \rho_0 (v_0 \cdot \nabla) v_0 = -\nabla P + \frac{1}{4\pi} (B_0 \cdot \nabla) B_0 + \rho_0 g + F ,
\]

(A5)

and

\[
\frac{\partial B_0}{\partial t} = \langle \nabla \times (v_0 \times B_0) \rangle .
\]

(A6)

These equations describe the average properties of the fluid flow in a high-speed wind stream. These equations, describing the average flow, are coupled to the equations describing the relatively rapid fluctuations by the wave force, \( F \). This force is given by

\[
F = \frac{\langle D_1(B) \rangle}{4\pi} - \rho_0 \langle D_2(v) \rangle - \langle \rho_1 D_1(v) \rangle - \langle \rho_1 \frac{\partial v_1}{\partial t} \rangle - \nabla \langle P_2 \rangle ,
\]

(A7)

where \( D_1(q) \) and \( D_2(q) \) represent the first-order and second-order contributions to the \( (q \cdot \nabla)q \) terms in the momentum equation. The specific form for these terms is determined by the choice of coordinate system. In equation (A7) averages of third-order terms were neglected.

Several of the terms in equation (A7) have relatively simple physical interpretations in spite of their algebraic complexity. The first term describes body force due to magnetic tension. It has two components: (1) the magnetic tension due to the transverse component of the wave magnetic field and (2) the magnetic tension due to the parallel component of the wave magnetic field. The second term describes the centrifugal force due to the oscillatory motion of the plasma. In the spherically symmetric Alfvén wave-driven wind theory the magnetic tension force due to compression vanishes, and the centrifugal force is just balanced by the magnetic tension caused by the transverse magnetic field of the wave, so that these first two terms sum to zero. This is not the case in a coronal hole flux tube, where magnetic tension which results from compression of the fluid can be important. The third and fourth terms are proportional to the first-order density perturbation. Again in the spherically symmetric Alfvén wave theory these terms vanish, since density changes are of second order. For the specific case considered in this paper these terms are also negligible. It is possible, however, to envision circumstances where they could provide a significant contribution to the wave force. Finally, the last term is just the gradient in the wave pressure, which is the force obtained in the spherically symmetric theory of Alfvén wave-driven winds.

II. EQUATIONS FOR THE RAPIDLY FLUCTUATING FLUID MOTIONS

To obtain equations describing the rapidly fluctuating components, subtract these time-averaged expressions from the exact equations. Neglecting terms of second order or greater, one obtains

\[
\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_1 v_0 + \rho_0 v_1) = 0
\]

(A8)

and

\[
\rho_1 \frac{\partial v_0}{\partial t} + \rho_0 \frac{\partial v_1}{\partial t} + \rho_1 (v_0 \cdot \nabla) v_0 + \rho_0 D_1(v) = -\nabla P_1 + \frac{1}{4\pi} D_1(B) + \rho_1 g ,
\]

(A9)

\[
\frac{\partial B_1}{\partial t} = \nabla \times (v_1 \times B_0) + \nabla \times (v_0 \times B_1) .
\]

(A10)

Equations (A4)-(A10) are the general relationships for the two-time-scale description of solar wind acceleration used in this paper. Included in this linear description are the effects of gravity, the geometry of the medium supporting the wave, and the effects of longitudinal and transverse plasma gradients. Naturally a solution incorporating all of these effects would be quite complicated and, at least for the present, is not available. This paper instead focuses on the role of transverse gradients for the acceleration of the coronal hole plasma. Fortunately, considerable physical insight into this process can be obtained with a minimum of mathematical complexity by considering a very simplified model for the coronal hole waveguide. In the following paragraphs the assumptions of this simple model are described.

Thermal pressure is neglected in the first-order equation. This is easily justified for the solar corona, where observations indicate that thermal pressure is orders of magnitude smaller than the magnetic pressure. In addition, this means that gradients of \( B_0 \) can be neglected in the first-order equations, since pressure balance implies that the magnetic field strength must be nearly constant across the coronal hole boundary. Thermal pressure is, of course, an important term in the zero-order equation, since the magnetic forces \( J_0 \times B_0 \) are presumed small. Gravity will also be neglected in the first-order equation. Physically this approximation is justified whenever the wavelength is smaller than the pressure scale height. In the zero-order equation gravity is an important force. Only flow in the region where \( v_0 \ll v_A \) will be considered. This assumption is reasonably well satisfied in the acceleration region, and it
simplifies the problem considerably by essentially decoupling the first-order equation from the average flow. The coronal hole magnetic field is assumed to be described by

$$B(x, x, t) = B_0(x) \hat{x} + B_1(x, x, t), \quad (A11)$$

and the velocity is assumed to be given by a function of the form

$$v(x, x, t) = V_0(x) \hat{x} + v_1(x, x, t). \quad (A12)$$

The subscripts refer to the direction defined by the zero-order magnetic field.

With these assumptions one can derive equations for the components of the first-order fluid velocity and magnetic field. The parallel component vanishes in first order. The waves are therefore transverse with a velocity described by

$$\rho_0 \frac{\partial v_\parallel}{\partial t} = -\nabla \cdot \left( B_0 \hat{v} \right) + \frac{B_0}{4\pi} \hat{D} \cdot B, \quad (A13)$$

where

$$\hat{D} = \begin{cases} \frac{1}{r} \frac{\partial}{\partial r} r & \text{for spherical coordinates} \\ \frac{\partial}{\partial z} & \text{for cylindrical or Cartesian coordinates} \end{cases} \quad (A14)$$

The magnetic field is obtained from the induction equation as

$$\frac{\partial B_\parallel}{\partial t} = -\nabla \cdot (\rho_0 \hat{v}) \quad (A15)$$

for the parallel components and

$$\frac{\partial B_\perp}{\partial t} = B_0 \hat{D} \cdot v_\perp \quad (A16)$$

for the perpendicular components. The equation for the density perturbation is obtained from equation (A8):

$$\frac{\partial \rho_1}{\partial t} = -\nabla \cdot (\rho_0 \hat{v}) \quad (A17)$$

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