CENTER-TO-LIMB VARIATION
OF THE CONVECTIVE LINE SHIFT

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ABSTRACT

Full disk Fraunhofer line core shifts have been analysed for
evidence of a latitude dependence to the residual convective blue
shift, and temporal variability greater than what’s expected from
the low-\( \lambda \) 5 min oscillations. We find no evidence for a latitude
dependence to the mean convective shift at the two sigma level of
105 m s\(^{-1}\) . The global symmetric lineshift profile shows temporal
variability in the 5-min band which is larger than expected based
on estimates of the 5-min mode amplitudes.

1. INTRODUCTION

The convective lineshift seems to be well described, at least
qualitatively, by Beckers and Nelson (1978). They were able to
model the form (but not necessarily the amplitude) of the center-
to-limb variation of a photospheric line from a granulation model.
Their results were particularly sensitive to both the horizontal and
vertical convective cell velocity amplitudes and cell sizes. For
example, Figure 1 plots two of their models against corresponding
observations of the Fe I line at 630.251 nm (Kuhn, 1980). It is
evident that the center-to-limb line shift variation is sensitive to
the granule convection.

Beckers and Taylor (1980) and Brandt and Schröter (1982) have
claimed a latitude variation in the convective line shift. On the
other hand Labonte and Howard (1983) did not observe such an effect.
Since such a variation would imply an important difference in the
granule convection from pole to equator and since the observations
are inconclusive, it would be useful to have more data. The follow-
ing paper follows from data that were obtained to study global scale
5-min oscillations.
We may expect the 5-min oscillations to appear as an additive line shift contribution to the convective shift. If this is so then the fluctuations in the global lineshift curve (center-to-limb) should be consistent with the low-$\ell$ 5-min mode amplitudes. In this paper we will check this assumption.

2. DATA

The data collection procedures have been reported elsewhere (Kuhn & O'hanlon 1983). The line shift of Fe I 630.25nm, along a slit oriented E-W and N-S from limb to limb, is obtained once a minute, with spatial resolution of about $1.7 \times 10^4$km. Line core positions are found by quadratic interpolation and line shifts are determined from telluric references within .1 nm in wavelength of the Fe I line.

3. ANALYSIS AND RESULTS

Figure 2 shows the mean line shift as a function of heliocentric angle $\theta$ along the N-S and E-W directions. The lineshift represents an average over the 5 hours of observation. A least squares best fit linear function of the form $a + b \sin \theta$ has been subtracted from each observation before averaging. The scatter in the difference between E-W and N-S of the points in Figure 2 is 52 ms$^{-1}$. Most of this is due to real surface velocity noise, since the expected mean velocity measurement noise in each pixel is only about 4 ms$^{-1}$. The mean velocity offset that comes from the fit to each observation is essen-
tially the same for the E-W and N-S data. The gaussian width of the
distribution of differences between the E-W and N-S constant term is
32 m s\(^{-1}\).

![Figure 2: Mean Center-to-Limb Line Shift Variation, Circles: N-S, Crosses, E-W.](image)

If we assume the curves of Fig. 2 represent the mean center-to-
limb variation of the convective shift then we can fit these curves
(to N-S and E-W data respectively) to the individual observations
taken over 5 hours. Figure 3 plots the fit coefficient as a function
of time for both slit orientations. The vertical axis units are
such that unity represents the mean profile of Fig. 2. It's clear
that there are 20 percent fluctuations in the center-to-limb curve
derived in this fashion. Figure 4 plots the power spectrum and, as
expected, shows that much of the fluctuation power is in the 5-min
oscillation band between \(\omega = 0.015\) to \(\omega = 0.025\) s\(^{-1}\). Notably there
is more power in the E-W slit orientation.
Figure 3: Temporal Variation of Line Shift Fits, Dotted: E-W, Solid: N-S.

Figure 4: Power Spectrum of Convective Shift Coefficients.
4. DISCUSSION

The Beckers and Nelson convective shift model seems to qualitatively describe the observations, but their velocity amplitude calculation was not done realistically -- they did not integrate the line profile over a granule, find the resulting line core, and thus the line shift. We have done this with a modified LTE routine (Source function has been scaled at each depth by the ratio of NLTE to LTE Fe line source functions calculated by Lites [1972]) and find line shifts that are too small by nearly a factor of 10 using the Nelson and Musman (1976) granule model. It seems likely that the vertical and horizontal velocities are too low near the temperature minimum (near the velocity formation height of this line).

The points in Fig. 2 suggest that there is no systematic variation in the convective shift with latitude. Taking the observed scatter in the difference between E-W and N-S points gives a, perhaps conservative, two sigma upper limit to the variation of 104 m s\(^{-1}\). A smooth difference in the variation with sin \(\theta\) between slit orientations could be expected to appear in the constant fit terms. This yields a two sigma limit of 62 m s\(^{-1}\).

A rough calculation gives an estimate of the expected contribution of the low-\(\lambda\) 5-min oscillation modes to the fluctuation power observed in Fig. 4. We assume that the amplitude of each spherical harmonic mode is 0.30 m s\(^{-1}\) (see Kuhn, 1983 and references). Along the equator essentially the \(m=2\) modes are correlated with the smooth functional form of the convective shift indicated in Figure 2. Due to the pixel size and slit width we take \(\lambda = 250\) as a high spatial frequency cut off. If we now assume that the power in each \(m=2\) mode from \(\lambda = 2\) to 250 contributes with unity weight to the fluctuation in the limb curve we find that we expect \(\sigma \sim 1\) to 2 percent in units of the mean center-to-limb convective shift curve. This number should be contrasted with \(\sigma_{E-W} = 5\) percent and \(\sigma_{N-S} = 10\) percent. The observed fluctuations are only factors of \(\lambda\) order unity above a rough expectation so it's hard to argue forcibly that something is wrong with our assumptions (although I do believe the expected 5-min contribution is a conservative high estimate).

A clear characteristic of the convective shift is that it's symmetric in \(\theta\), whereas the 5-min oscillation velocity field is not. Since cos and sin spatial transforms sample the symmetric and antisymmetric (in \(\theta\)) velocity fields it is interesting to look for significant differences between symmetric and antisymmetric terms. A mean convective shift profile and mean linear function are subtracted from each observation. Spatial least squares fits to \{cos2\(\theta\), sin2\(\theta\), cos4, sin4\(\theta\), ... cos10\(\theta\), sin10\(\theta\)\} were performed at each time. Temporal Fourier transforms of each of these 10 coefficients were calculated and the mean power in the band \(0.015 < \omega < 0.025\) s\(^{-1}\) was evaluated. The mean noise power of \(0.03 < \omega < 0.04\) has been subtracted from the 5-min band power to get the data displayed in Figure 5.
Figure 5: 5-min Band Power in Spatial Cosine and Sine Fits.

Each set of two points in Fig. 5 represents the N-S and E-W power in \( \cos 2n\theta, \sin 2n\theta \) for \( n = 1,5 \) pairs. Diamonds show the power for the E-W slit and squares indicate the N-S power. It's clear that for both slit orientations the cosine power is notably greater than the antisymmetric sin coefficient power at \( \{\sin\}(2\theta) \) and \( \{\cos\}(4\theta) \). At large wavenumbers this difference vanishes. In other words at low wavenumbers there is significantly more 5-min fluctuation power in the symmetric velocity contribution than in the antisymmetric.

One concern with this result is that the 5-min oscillations are primarily radial vector velocities. Thus a factor of \( \cos \theta \) weights their contribution to the line-of-sight velocity. Yet this does not explain the enhanced symmetric power in Figure 5. Restricting the domain to \( |\theta| < 30^\circ \) and weighting the data by \( (\cos \theta)^{-1} \) does not change these results.

5. CONCLUSIONS

There are two principal conclusions:
1) There is no latitude dependence to the convective shift of 630.251 larger than 104 m s\(^{-1}\).
2) The symmetric large scale velocity field has fluctuation power significantly above the antisymmetric power in the 5-min oscillation band.

We notice also, as has been pointed out at various times in this conference, that the modeled velocity amplitudes of the convective shift are too low, suggesting that the convection penetrates to greater heights.
REFERENCES

8. ACCURACY OF MODELS OBTAINED FROM
LOW-RESOLUTION OBSERVATIONS