PROBLEMS OF FLUX TUBE FORMATION

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ABSTRACT

Recent theoretical studies of magnetoconvection predict that strong fields will be formed between granules but also suggest that much of the magnetic flux may remain near the center of a granule.

1. INTRODUCTION

Over the last few years we have gradually attained a better understanding of nonlinear magnetoconvection and of the processes that confine magnetic flux to isolated tubes between convection cells. The correspondence between theoretical results and the best high-resolution observations of solar magnetic fields is close enough to seem convincing but several complications still remain. In particular, it is still uncertain whether flux tubes should be formed at the centers or peripheries of convection cells.

The Lockheed observations, described by Title (1983) in these Proceedings, show that small-scale magnetic fields lie between granules in the solar photosphere. These fields presumably coincide with the filigree and with facular points (Muller 1983) whose radii are typically 80Km (Muller and Keil 1983). Title also finds larger magnetic features, filling entire granules, which probably correspond to magnetic knots (Spruit and Zwaan 1981) and facular granules (Muller 1977).

Although theoretical arguments generally favor the production of intense magnetic fields at cell boundaries (Weiss 1978) some recent calculations suggest that more flux may end up at cell centers (Proctor and Weiss 1982; Galloway and Proctor 1983). In this note I shall discuss some of the problems raised by these theoretical investigations.
2. KINEMATIC CALCULATIONS

The simplest set of results are those obtained by solving the induction equation
\[
\frac{\partial \mathbf{B}}{\partial t} = \text{curl}(\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}
\] (1)

for the magnetic field \( \mathbf{B} \), with a prescribed steady velocity \( \mathbf{u} \) and a uniform magnetic diffusivity \( \eta \). We are interested in cases where the magnetic Reynolds number, \( R_m = UL/\eta \), with \( U \) and \( L \) a typical velocity and length, is very large. So far, most calculations have been two-dimensional. Then it can be shown that the steady solution is unique; in the classical (symmetrical) problem equal amounts of flux are concentrated into sheets between upward and downward moving fluid. The behavior of time-dependent calculations depends on the choice of initial conditions and two time-scales are involved: the turnover time \( \tau_0 = L/U \) and the diffusive time \( \tau_\eta = L^2/\eta \). When the magnetic field is initially uniform, flux sheets are formed in a time of order \( \tau_0 \). With symmetry, flux expulsion occurs after a time \( T \approx R_m^{-1/3} \tau_0 < \tau_\eta \) (Rhines and Young 1983). If, on the other hand, flux is initially concentrated into isolated sheets, described by \( \delta \)-functions, the field spreads out to fill flux sheets after a time \( \tau_0 \) while the final, symmetrical, steady solution is only reached after a time \( \tau_\eta = R_m \tau_0 \).

Galloway and Proctor (1983) have investigated three-dimensional behavior in hexagonal cells with updrafts along their central axes. They find that strong fields appear at cell corners after a time \( \tau_0 \); the final steady state is only reached after a time \( \tau_\eta \) but eventually most of the magnetic flux remains near the center of the cell. This central flux rope resembles that found in axisymmetric calculations (cf. Proctor and Weiss 1982): at the base of the cell, the inflow concentrates flux near the axis but at the top the central field is weak compared with that found at the corners.

3. DYNAMICAL CALCULATIONS

Most studies of magnetoconvection have relied on the Boussinesq approximation (Proctor and Weiss 1982). In addition to equation (1) we have to solve the equation of motion
\[
\rho_0 \frac{\partial \mathbf{u}}{\partial t} = \rho \mathbf{g} - \nabla p + \mathbf{j} \times \mathbf{B} + \rho_0 \nu \nabla^2 \mathbf{u}
\] (2)

and the heat equation
\[
\frac{\partial T}{\partial t} = \kappa \nabla^2 T.
\] (3)

Here \( \rho_0 \) is a reference value of the density \( \rho \), \( p \) is the pressure, \( \mathbf{j} \) the electric current, \( \nu \) the viscous diffusivity and \( T \) the temperature.
The resulting behavior depends critically on the ratio, \( \zeta = \eta / \kappa \), of the magnetic to the thermal diffusivity. In typical stellar conditions, \( \zeta \ll 1 \).

Two-dimensional computations show that flux is confined to isolated tubes from which the motion is excluded (Weiss 1981a,b). Convection cells are no longer symmetrical; asymmetry is preferred, with most of the flux concentrated (say) in regions of downward-moving fluid. The preferred cell-width is wider than that in the absence of a magnetic field and the solutions are not unique. In particular, magnetic flux may be gathered into a few massive sheets instead of being distributed equally between adjacent cells.

The behavior of three-dimensional cells is not yet clear. We might, however, expect to find a preference for axial flux concentrations, as in the axisymmetric calculations of Galloway and Moore (1979). It is known that axisymmetric cells are unstable and liable to fragmentation. If this instability is encouraged (as seems likely) by axial magnetic fields then lifetimes of large granules would be reduced in active regions.

Only a few non-Boussinesq effects have so far been investigated. Compressibility leads to the evacuation of intense magnetic flux tubes (Cattaneo 1984). The density stratification also strengthens the downward motion (Nordlund 1983; Hurlbut, Toomre and Massaguer 1983) and may thereby reduce the central concentration. Another important effect is the variation of \( \zeta \) with depth. In the Sun, the opacity increases owing to ionization of hydrogen and the thermal diffusivity \( \kappa \) consequently decreases with depth so that the ratio \( \zeta \) increases downwards. As a result we might expect the inflow at the base of a granule to be less effective, with a further reduction in the central concentration. Moreover, since \( \zeta > 1 \) for depths between 2000 km and 20,000 km below the photosphere, we should not expect permanently isolated flux tubes to be formed locally in this region immediately below the granules (cf. Knobloch and Weiss 1984).

4. CONCLUSION

On the basis of the theoretical investigations outlined above, it would be hard to predict whether magnetic flux ends up at the centers of granules or in the intergranular lanes. This is because there is a horizontal inflow near the base of a granule, which competes with the outflow observed at the photosphere. Schmidt, Simon and Weiss (1984) have modeled the motion of buoyant flux tubes in granules and find that the inflow has more effect on larger flux tubes. Thus tubes with fluxes greater than \( 10^{18} \) Mx may be carried to the center and form facular granules, while tubes with smaller fluxes are swept towards the corners and appear as facular points.
References

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