THE EFFECT OF REYNOLDS STRESS IN THE SOLAR CONVECTIVE ZONE ON THE VERTICAL STRUCTURE OF FLUX TUBES, AND ON THEIR CONVECTIVE INSTABILITY

A. A. van Ballegooijen
Dept. 52-13, Bldg. 202
Lockheed Research Laboratory
3251 Hanover Street
Palo Alto, California 94304

ABSTRACT

A model of the fieldstrength in slender magnetic flux tubes, as function of depth in the convective zone, is described. The tubes are assumed to be vertical, and in thermal equilibrium with the surrounding medium. Deviations from hydrostatic equilibrium, due to Reynolds stresses in the convective zone, are taken into account. The convective instability of the flux tubes is briefly discussed.

1. Introduction

The structure and evolution of the magnetic field at the solar surface is determined to a large extent by processes that occur deeper down in the convective zone. For example, the differential rotation and convection in the deep layers of the convective zone are responsible for the dynamo process that generates the magnetic field from which active regions are formed. The formation of a network and the motion of magnetic features away from decaying sunspots are due to the interaction of magnetic flux tubes with the supergranular flow (Meyer et al., 1979). The details of these processes depend on the degree of fragmentation of the sub-surface field, and on the fieldstrength in the individual magnetic elements: this determines how fast the flux tubes can move with respect to the surrounding, field-free medium and thus, to what extent the flux tubes can be deformed by the convection.

In this paper the fieldstrength of slender magnetic flux tubes is discussed. For simplicity I assume that the tubes are vertical to a large depth in the convective zone before they turn horizontal. The equilibrium of such a vertical flux tube is considered, taking into account the Reynolds stress in the ambient convective zone on the fieldstrength in the tube. The convective stability of this equilibrium, i.e., the stability against vertical displacements along the tube, is also discussed.
2. Equilibrium of a vertical flux tube

The field strength $B(z)$ of a slender magnetic flux tube, as function of depth $z$ below the surface, is determined by the condition of pressure balance:

$$\frac{B^2}{8\pi} = p_e - p_l$$

(1)

where $p_e$ and $p_l$ are the external and internal gas pressure, respectively. Assuming that the gas inside the vertical tube is stationary, the internal gas pressure $p_l(z)$ is determined by the condition of hydrostatic equilibrium:

$$\frac{dp_l}{dz} = \rho_l g$$

(2)

where $\rho_l$ is the internal density and $g$ is the gravitational acceleration. The external gas pressure $p_e(z)$, however, is affected by deviations from hydrostatic equilibrium due to the vertical component of the Reynolds stress $\langle \rho v_z^2 \rangle$ in the convective zone:

$$\frac{d}{dz} (p_e + \langle \rho v_z^2 \rangle) = \rho_e g$$

(3)

Here, $\rho_e$ is the external density, $v_z$ is the vertical velocity and $\langle \ldots \rangle$ denotes an average over horizontal surfaces in the convective zone. Using the gas law ($p = \rho T$) and assuming that the flux tubes are in temperature equilibrium with the surrounding medium ($T_l = T_e$), the following expression for the ratio of magnetic pressure $B^2/8\pi$ and gas pressure $p_e$ may be derived:

$$\frac{d}{dz} \left( \frac{B^2}{8\pi p_e} \right) = -\frac{1}{\rho_e} \frac{d}{dz} \langle \rho v_z^2 \rangle$$

(4)

which is valid provided $B^2/8\pi p_e \ll 1$.

We conclude that an increase of $\langle \rho v_z^2 \rangle$ with depth $z$ results in a decrease of $B^2/8\pi p_e$. This may be understood as follows: the increase of $\langle \rho v_z^2 \rangle$ with depth corresponds to an outward decrease of turbulent pressure in the convective zone. Due to this turbulent pressure gradient the outer layers of the convective zone are slightly uplifted, causing the gas pressure to be somewhat higher than in the case of hydrostatic equilibrium. Since this effect does not occur within the flux tubes, the magnetic field of the tubes can be confined by the increase of external gas pressure.

Using the mixing-length model of Spruit (1977), I computed $B^2/8\pi p_e$ as function of depth. The Reynolds stress was estimated
assuming an isotropic velocity distribution:

\[ \langle \rho v_z^2 \rangle = \frac{1}{3} \rho e v^2, \]

where \( v(z) \) is the mean convective velocity. The results are shown in Fig. 1 for three values of \( B(z_0) \), the assumed field strength at the base of the convective zone \( (z_0 = 200,000 \text{ km}) \). The ratio \( B^2 / 8\pi \rho e \) calculated with the equipartition field strength \( (B_{eq}^2 / 8\pi = 1/2 \rho e v^2) \) is also shown.

![MODELS OF FLUX TUBES IN THERMAL EQUILIBRIUM](image)

**Fig. 1.** The ratio \( B^2 / 8\pi \rho e \) as function of depth \( z \) below the solar surface. Full line: field strength \( B(z) \) in vertical flux tubes, for three values of \( B(z_0) \), the field strength at the base of the convective zone \( (z_0 = 200,000 \text{ km}) \); dashed line: equipartition field strength \( B_{eq}(z) \).

Figure 1 shows that, for \( B(z_0) > 4 \times 10^4 \) Gauss, there is a region in the deep layers of the convective zone where \( B > B_{eq} \) and where \( B^2 / 8\pi \rho e \) is approximately constant with depth. In this region the Reynolds stress has little effect on the field strength.
of the flux tubes. Higher up in the convective zone $B(z)$ is nearly independent of $B(z_0)$, and the field strength follows approximately the equipartition value: $B(z) = 0.6 B_{eq}(z)$ for $1000 < z < 10,000$ km. For depths $z < 1000$ km the quantity $B^2/8\pi p_e$ reaches a maximum of about $10^{-2}$. Thus the high field strengths observed at the solar surface, which require $B^2/8\pi p_e \sim 1$ at $z = 0$, cannot be explained with the present model.

3. Convective instability

Presumably the decrease from $B^2/8\pi p_e \sim 1$ at the surface to $B^2/8\pi p_e \sim 10^{-2}$ at depth $z = 1000$ km is due to a temperature deficit of the gas within the flux tubes. It has been suggested that a temperature deficit may be produced by a process of convective instability and collapse (Webb and Roberts, 1978; Spruit and Zweibel, 1979; Spruit, 1979). This instability is driven by the superadiabatic stratification of the convective zone: an adiabatic downflow within a flux tube that is initially in thermal equilibrium ($T_i = T_e$) will cause the internal temperature to become lower than the external temperature, thereby accelerating the flow. In previous studies of the instability the effects of Reynolds stress on the equilibrium state of the tube were neglected. Consequently, the ratio $B^2/8\pi p_e$ was constant with depth in these models. Since $B^2/8\pi p_e$ actually decreases with depth for $z > 1000$ km (see Fig. 1), the stabilizing effect of the deeper layers may have been seriously overestimated in the earlier calculations.

The stability analysis was repeated for the present model of flux tubes, using a simplified model of the convective zone. The ratio $\gamma$ of specific heat coefficients $c_p$ and $c_v$ is taken constant with depth. A constant, dimensionless parameter $q$ is defined that relates the entropy gradient in the external medium to the mean convective velocity:

$$\frac{dS_e}{dz} = q \frac{v^2(z)}{3zT_e(z)}$$

According to mixing-length theory, $q$ is of order unity. I found that in the region where $B(z)$ follows approximately $B_{eq}(z)$ ($10^3 \leq z \leq 10^4$ km in Fig. 1), convective instability occurs if $q$ is larger than a critical value:

$$q > \frac{9\gamma^2 - 5\gamma + 11/4}{6(3\gamma-1)(\gamma-1)^2}$$

Unfortunately, mixing-length theory cannot accurately predict the value of $q$, and since both sides of Eq. (7) are of order unity, it is unclear whether flux tubes are convectively stable or unstable for $z > 1000$ km. However, the tubes are much less stable than in
the unrealistic case where $B^2/8\pi p_e$ is of order unity and constant with depth.

4. Conclusion

From this analysis I conclude that deviations from hydrostatic equilibrium should be taken into account in models of magnetic flux tubes below the solar surface. The field strength of these flux tubes is of the order of the equipartition field strength for $z > 1000$ km, except perhaps in the deep layers of the convective zone. This affects the convective stability of the tubes and may also be important for the dynamics of the sub-surface magnetic field since individual, slender flux tubes with $B \sim B_{eq}$ are not strong enough to withstand the deformation by turbulent convective motions. Consequently, the magnetic field deep inside the convective zone may be strongly tangled up by the convective flow.

Acknowledgement

The author gratefully acknowledges financial support from the Lockheed Independent Research Program.

References


