The 160-minute solar pulsations are not excited by gravitational waves

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In light of the recent proposal that the 160-min solar pulsations might be triggered by gravitational waves arriving from the nearby Gemima γ-ray binary system, the resonance amplitude of the corresponding forced oscillations is calculated. The maximum possible amplitude is at least 10³ times lower than observed.

In a recent letter Waghete ¹ reports the suggestion by a group of European scientists (G. Issak, G. F. Bignami, P. Delache, J. Paulon) that the sun's 160-min pulsations might be triggered by gravitational waves being emitted by the object Gemima (2CG 198 + 04), the brightest unidentified γ-ray source (discovered² in 1975). The γ-ray intensity of Gemima seems to fluctuate with a period of about 160 min, indicating that the object could be a binary system. This source is thought to be located between 3 and 300 light years from the sun. As its galactic coordinates are l ≈ 139°, b ≈ 4°, and we receive its radiation at a small angle to the ecliptic plane: β ≈ −5°, δ.

The solar oscillations of 160.01 ± 0.003 min period, discovered at the Crimean Astrophysical Observatory by Severny et al.,⁴ have been kept under observation for the past decade and show high stability in phase. Their amplitude is several times that of other long-period solar oscillations and has a value ΔR/Δc = 10⁵.

This letter presents theoretical calculations of the oscillations that would be excited in the sun by gravitational waves from a binary system, and compares the results with the observational evidence. Full details of the solution will be published in the "Izvestiya" of the Crimean Astrophysical Observatory.

As they pass through the sun, gravitational waves will raise tides on it. The three-dimensional force F exerted on unit mass may be expressed in terms of the potential Φ:

\[ F = -\nabla \Phi, \quad \Phi(r) = \sum_{\nu} \frac{1}{r^2} R_{\nu} \delta^2 x^2, \]

where the x are the components of the radius vector r from the center of the sun, and the R are the fluctuations of the curvature tensors due to the gravitational waves. For a plane gravitational wave traveling in the z direction,

\[ R_{\nu \mu} = -R_{\nu \rho} = -\frac{1}{2} \hat{h}_e(t - z/c), \]

\[ R_{\nu \rho} = R_{\rho \nu} = -\frac{1}{2} \hat{h}_e(t - z/c). \]

All the other curvature-tensor components will vanish. Here, the R are dimensionless functions of the argument t - z/c, describing the instantaneous wave amplitude in two mutually orthogonal polarizations (±, ±); the choice of these directions is arbitrary. Then

\[ \Phi(r) = -\frac{1}{4} \hat{h}_e(z^2 - y^2) - \frac{1}{2} \hat{h}_e x y. \]

or in spherical coordinates (ρ, φ, θ) with polar axis directed along the z axis:

\[ \Phi(\rho, \phi, \theta) = -\frac{\gamma}{2} \left[ \hat{h}_e Y_2^2(\theta, \phi) + \hat{h}_e Y_2^0(\theta, \phi) \right], \]

where γ = 2(2π/15)²/2 and the Y²(θ, φ) are the real spherical harmonics.

The amplitudes h, h of a gravitational wave emitted by a binary system in the direction z toward the sun will at distance R from the system be determined by the reduced quadrupole moment tensor evaluated at the delayed time t - z/c:

\[ h_e(t - z/c) = \frac{\gamma}{c^2 R} \left( \Omega_{\alpha \beta} - \Omega_{\rho \sigma} \right)_{(t - z/c)}, \]

\[ h_e(t - z/c) = \frac{\gamma}{c^2 R} \left( \Omega_{\rho \sigma} \right)_{(t - z/c)}, \]

where \( \Omega_{\alpha \beta} = \sum_{\mu} m_{\mu} \left( x_{\alpha} x_{\beta} - \frac{1}{3} \delta_{\alpha \beta} \right), \) with the m, denoting the masses of the binary components and their coordinates in an (x, y, z) Cartesian system referred to the center of mass. For a binary consisting of mass m, ms, revolving in a circular orbit of radius a at angular velocity ω, we have

\[ h_e(t - z/c) = \frac{4Gm_a^2 m_s^2}{c^2 R} \cos \frac{(1 + \cos^2 \delta)}{2} \cos [2\omega (t - z/c)], \]

\[ h_e(t - z/c) = \frac{4Gm_a^2 m_s^2}{c^2 R} \cos \delta \sin [2\omega (t - z/c)], \]

where μ = m, ms/(m, + ms) is the reduced mass, while ω and the angle between the ray direction (the z axis) and the normal to the orbit plane.

Substituting the expressions (2), (3) into Eq. (1), we obtain

\[ \Phi(r, \theta, \phi, \psi) = -\frac{\gamma}{c^2 R} \left( \sum_{\mu = 0} A_{\mu 0} (\theta, \phi) \exp (\pm 2i\omega t) \right), \]

where

\[ A_{\mu 0} (\theta, \phi) = \frac{16Gm_a^2 m_s^2}{c^2 R} \sin^2 \theta \left( \frac{1 + \cos^2 \delta}{2} \right), \]

while A(0) = 0 if m, = 2; and the Y²(θ, φ) are the complex spherical harmonics. Since the sun's radius is much shorter than the wavelength, we have set z = 0 in the exponent in Eq. (4).
Now let us introduce a spherical coordinate system \((r', \phi', \psi')\) whose polar axis \(z'\) is normal to the ecliptic plane. Let \(\alpha\) denote the angle between the direction of the gravitational wave (the \(z\) axis) and the \(z'\) axis. Then in the new coordinate system the potential will be expressed by

\[
\Phi (r', \phi', \psi, t) = \text{Re} \left\{ -\frac{\gamma B\rho a^2}{2} \sum_{m=-2, -1, \ldots, 2} b^{(m)}(a, \delta) \times Y_{2m}(\phi', \psi) \exp \left( -i2\omega t \right) \right\},
\]

where \(b^{(m)}(a, \delta) = \sum_{n=-2, -1, \ldots, 2} a^{2m} c^{(m)}(a) d^{(m)}(\delta);\)

with the \(d^{(m)}(\delta)\) representing generalized spherical harmonics. Written out explicitly, the coefficients \(b^{(m)}(a, \delta)\) are:

\[
b^{(m)}(a, \delta) = \left( \frac{1 + \cos \delta}{2} \right)^{1/2} \left( \frac{1 + \cos \alpha}{2} \right)^{1/2} \\
+ \left( \frac{1 - \cos \delta}{2} \right)^{1/2} \left( \frac{1 - \cos \alpha}{2} \right)^{1/2},
\]

\[
b^{(1)}(a, \delta) = \frac{i}{2} \sin \alpha \left[ \cos \alpha (1 + \cos \delta) \pm 2 \cos \delta \right],
\]

\[
b^{(0)}(a, \delta) = -\frac{1}{4} \sqrt{\frac{3}{2}} (1 - \cos \alpha) (1 + \cos \delta).
\]

Since \(\alpha = 90^\circ + \beta \approx 90^\circ + \beta\), all the components of \(b^{(m)}\) will be nonzero. Henceforth we shall omit the primes on the coordinates \((r', \phi', \psi')\).

The equation of forced adiabatic oscillation of the sun is:

\[
\frac{\partial^2 \xi}{\partial t^2} + L\xi = -\nabla \Phi,
\]

where \(\xi\) denotes the displacement vector and \(L\) is the linear self-adjoint operator. We seek a solution to Eq. (6) of the form

\[
\xi(r, \theta, \psi, t) = \sum_{m,n} X_n^{(m)}(r) Y_n^{(m)}(\theta, \psi),
\]

where the \(Y_n^{(m)}(r, \theta, \psi) = \left( \xi_{r, n}(r) \xi_{\theta, n}(r) \frac{\partial}{\partial \theta} \xi_{\psi, n}(r) \right) Y_{n, m}(\theta, \psi)\) are the eigenfunctions of the operator \(L\), that is, \(L\xi_n^{(m)} = \omega_n^{2} \xi_n^{(m)}\), with \(\omega_n\) the frequency of the \(n\)-order multipole mode. (We neglect the frequency splitting due to the sun's rotation.)

The equation for the oscillation amplitude \(X_n^{(m)}(t)\) will be

\[
\frac{d^2 X_n^{(m)}(t)}{dt^2} + \omega_n^2 X_n^{(m)}(t) = B_n^{(m)}(t),
\]

where

\[
B_n^{(m)}(t) = \frac{\left< \xi_n^{(m)}(r) \nabla \Phi \right>}{\| \xi_n^{(m)}(r) \|^2};
\]

here the angle brackets signify a scalar product, and \(\| \xi_n^{(m)}(r) \|^2 = \left< \xi_n^{(m)}(r), \xi_n^{(m)}(r) \right>\). Then

\[
B_n^{(m)}(t) = \gamma R_0 Q_0 \exp \left( -i2\omega t \right),
\]

where

\[
Q_0 = \frac{2}{\pi} \frac{1}{(s_n + 3\xi_n \rho \rho^2 d^2)}
\]

\[
Q_0 = \frac{1}{6} \frac{1}{(s_n + 6\xi_n \rho \rho^2 d^2)}
\]

and \(R_0\) is the radius of the sun. If the gravitational-wave frequency \(2\omega\) is close to the natural oscillation frequency \(\omega_n\) (resonance), then we must add to Eq. (7) a term \((2/\tau d) X_n^{(m)}\) describing the decay of this mode due to dissipation of the pulsational energy inside the sun:

\[
\dot{X}_n^{(m)} + \frac{2}{\tau d} \dot{X}_n^{(m)} + \omega_n^2 X_n^{(m)} = B_n^{(m)}(t)
\]

(\(\tau d\) denotes the characteristic damping time).

One should also recognize that the binary orbit parameters will change with time as energy is lost to gravitational radiation. In particular, the orbit radius will diminish according to the law \(a = a_0 (1 - t/\tau_0)^{1/2}\), and \(\alpha = \alpha_0 (1 - t/\tau_0)^{1/2}\). Since the orbit parameters are related by Kepler's law \((\omega_0^2 a^3 = GM)\), we may write the \(B_n^{(m)}(t)\) in the form

\[
B_n^{(m)}(t) = \frac{5}{64} \gamma Q_0 \omega_n^2 \left( \frac{a}{\rho} \right) \rho R_0 \frac{1}{(1 - t/\tau_0)} \exp \left( -i2\omega (1 - t/\tau_0)^{-1/2} \right).
\]

Without loss of generality we may regard exact resonance as occurring at time \(t = 0\).

We now introduce the dimensionless variables \(x = X_n^{(m)}(r)/R_0\), \(\tau = \omega_n\), and set

\[
e = \omega_0 \tau_0, \quad k = \tau_0 \tau, \quad \lambda = \omega_0 / \rho R_0, \quad \tau_0 = \frac{5}{64} \gamma Q_0 \omega_n^2 \left( \frac{a}{\rho} \right) \rho R_0 \frac{1}{(1 - t/\tau_0)} \exp \left( -i2\omega (1 - t/\tau_0)^{-1/2} \right),
\]

and take \(T = \varepsilon \tau\) as a *slow* time scale. Then Eq. (9) will become

\[
x + 2\tau_0 x + x = ef(\tau) \exp \left[ -i2\omega (\tau - \varepsilon \tau) \right] F(\tau),
\]

where \(f(\tau) = k \tau_0/(1 - k\tau), \nu(\tau) = (1 - k\tau)^{-1/2}\), the amplification and frequency of the external force, are slowly varying functions of time.

The parameters \(\tau_0, Q_0, \varepsilon\) entering into Eq. (10) depend on the model adopted for the internal structure of the sun. Table I gives these quantities for natural quadrupole oscillations of \(\approx 160\)-min period, as computed for four structure models: the standard model, a model with a low initial heavy-element abundance (model C of Christensen-Dalsgaard et al.), and two models with turbulent diffusion, for Reynolds numbers \(Re = 100\) and 200. We are describing the calculation procedure and the detailed properties of the models in a forthcoming paper.

Since the three parameters depend little on which model is used for the internal structure, let us take the numerical estimates the values \(\tau_0 = 5 \cdot 10^5 \text{yr}, Q_0 = 3 \cdot 10^{-3}, \varepsilon = 1 \cdot 10^{-4}\). For the other parameters involved, we adopt the values \(\omega_n = 6.55 \cdot 10^{-4} \text{sec}^{-1}, K = 4.6 \cdot 10^{14} \text{cm} = 4.8 \cdot 10^{-5} \text{light yr},

\[
\frac{1}{\tau_0} = 1.2 \cdot 10^{-2} \left( \frac{M}{M_\odot} \right)^{1/3} \left( \frac{\mu}{M_\odot} \right)^{1/3} \tau^{-1},
\]

\[
k = \frac{\tau_0}{\tau} = 6.10^{14} \left( \frac{M}{M_\odot} \right)^{1/3} \left( \frac{\mu}{M_\odot} \right)^{1/3},
\]

\[
\nu = \frac{5}{64} \gamma Q_0 \omega_n^2 \left( \frac{a}{\rho} \right) \frac{1}{\rho R_0} = 0.3 \cdot 10^{-12} \left( \frac{a}{\rho} \right) \frac{1}{\rho R_0}
\]

191 Sov. Astron. Lett. 10(3), May-June 1984

A. G. Kosovichev
TABLE I. Parameters in Forced-Oscillation Equation

<table>
<thead>
<tr>
<th>Solar structure model</th>
<th>Oscillation mode</th>
<th>Period, min</th>
<th>( \tau_3 \times 10^2 )</th>
<th>( Q_\text{max} \times 10^{-2} )</th>
<th>( \varepsilon \times 10^{-5} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard model</td>
<td>( \delta )</td>
<td>162.5</td>
<td>5.15</td>
<td>1.95</td>
<td>0.96</td>
</tr>
<tr>
<td>model C</td>
<td>( \delta )</td>
<td>156.4</td>
<td>9.96</td>
<td>3.63</td>
<td>0.48</td>
</tr>
<tr>
<td>Turbulent-diffusion model</td>
<td>( \delta )</td>
<td>160.2</td>
<td>8.72</td>
<td>3.34</td>
<td>0.56</td>
</tr>
<tr>
<td>( \rho = 100 )</td>
<td>( \delta )</td>
<td>152.5</td>
<td>10.4</td>
<td>5.44</td>
<td>0.44</td>
</tr>
<tr>
<td>( \rho = 200 )</td>
<td>( \delta )</td>
<td>152.5</td>
<td>10.4</td>
<td>5.44</td>
<td>0.44</td>
</tr>
</tbody>
</table>

\( \log((\Delta_{\text{max}}/f_0)^2) \)

\[ -12 -10 -9 -8 -7 -6 -5 -4 -3 0 1 2 4 \log k \]

\[ 1.5 \times 10^{-8} \log(a, b) \left( \frac{1 \text{ light yr}}{R} \right)^{1/2} = \sqrt{\frac{\Delta_{\text{max}}}{f_0}} \]

The forced-oscillation equation (10) has been solved numerically by the Krylov--Bogolyubov--Mitropolski method for selected values of \( k \). Of special interest in these solutions is the maximum value of the oscillation amplitude, \( \Delta_{\text{max}} \), which need not occur precisely at the time of resonance (\( t = 0 \)). Figure 1 shows how \( \log((\Delta_{\text{max}}/f_0)^2) \) depends on \( \log k \). Notice that the behavior differs for \( k < \varepsilon \) and \( k > \varepsilon \), regions corresponding to different regimes whereby the frequency of the exciting force passes through resonance. If \( k < \varepsilon \) the frequency of the exciting force will change so slowly that at \( t = 0 \) the oscillation amplitude will reach its steady resonance value

\[ \Delta_{\text{max}} = \Delta_{\text{res}} = \sqrt{k f_0/2} \delta = k f_0/2, \]

just as would be true for an external force with an invariant resonance frequency. In the case \( k > \varepsilon \) the oscillations will be triggered in a regime of fast passage through resonance; the time scale for change in the frequency of the exciting force will be longer than the time required to establish a steady resonance amplitude.

Then the peak amplitude of the sun's natural oscillations will be

\[ \Delta_{\text{max}} \approx 2 \sqrt{\frac{k f_0}{\varepsilon}} \]

or

\[ \Delta_{\text{max}} \approx 0.7 \times 10^{-10} \log(a, b) \left( \frac{M}{M_\odot} \right)^{1/6} \left( \frac{\mu}{M_\odot} \right)^{1/6} \left( \frac{1 \text{ light yr}}{R} \right)^{1/2} \]

It is this second case that is germane to observational comparisons, because the amplitude of solar oscillations triggered by gravitational waves will increase with \( k \) and accordingly with the mass of the radiation source. One should recognize, however, that the parameter \( k \) will be bounded above by the lifetime of the binary system. In fact, according to the expressions (11) and (12), \( k \tau_0 = 5 \times 10^6 \text{ yr} \). Since Geminia has now been under observation for 8 yr and the 160-min pulsations for 10 yr, and since certainly \( \tau_0 > 10 \text{ yr} \), we do not have \( k < 5 \times 10^4 \) (note that if \( k = 5 \times 10^4 \) and \( m_1 \approx m_2 \), the total mass of the source would be \( M \approx 10^5 M_\odot \)), and Eq. (14) yields an upper limit on the oscillation amplitude:

\[ A_{\text{max}} < 4 \times 10^{-9} f_0 \approx 0.6 \times 10^{-9} \text{(1 light yr/}R) \text{).} \]

Hence if \( R \) is in the range indicated above (3-300 light yr), \( A_{\text{max}} \) should be less than \( 2 \times 10^{-10} \) to \( 2 \times 10^{-11} \). This limit is at least \( 10^6 \) times smaller than the amplitude of the 160-min pulsations observed on the sun: \( A_{\text{obs}} \approx \left( \frac{\delta R}{R} \right) \approx 10^{-4} \).

It is worth comparing the limit (15) with the oscillation amplitude estimated in the case of nonresonance excitation. To order of magnitude the relative oscillation amplitude will then be equal to the amplitude of the gravitational wave:

\[ A_{\text{nonres}} \approx k \approx 10^{-15} \left( \frac{M}{M_\odot} \right)^{1/6} \left( \frac{\mu}{M_\odot} \right)^{1/6} \left( \frac{1 \text{ light yr}}{R} \right) \]

\[ \times \left( \frac{1 \text{ light yr}}{R} \right)^{1/2} < 10^{-10} \left( \frac{1 \text{ light yr}}{R} \right), \]

or one order smaller than the limit (15).

Furthermore, if we neglect the energy loss by the binary system to gravitational radiation and assume that the gravitational-wave frequency is independent of time and equal to the sun's natural oscillation frequency, then for \( k = 5 \times 10^4 \) the expression (13) would give a grossly excessive estimate for the maximum amplitude; of order \( 10^{-2} \text{(1 light yr/}R) \). But actually, as shown above, Eq. (13) holds only for \( k < \varepsilon \).

In any event, then, our calculations show that gravitational waves arriving from the Geminia binary system cannot be responsible for the sun's 160-min pulsations.

I am deeply grateful to A. B. Severnyi for suggesting this approach to the problem and for valuable discussions.

Note added in proof. In a recent series of communications several authors arrive at the same conclusion: that the 160-min pulsations of the sun cannot be triggered by gravitational waves from the object Geminia.

15A. G. Kosovichev

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Earth rotation parameters from LAGEOS laser ranging in the 1980 MERIT campaign

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Laser tracking data acquired by 15 NASA and SAO stations on 841 passes of the LAGEOS satellite in August–October 1980 have been reduced by an analytic theory of satellite motion to yield improved values for the pole position and length of the day. The rms residuals for fitting 5° arcs of the orbit are ±50–80 cm.

Launched May 4, 1976, the Laser Geodynamics Satellite (LAGEOS) is a heavy (411 kg) sphere 60 cm in diameter whose surface carries 426 corner reflectors each of 3.8-cm diameter. Its orbit is practically circular (e = 0.004) and almost polar (i = 109.8°), and is effectively outside the earth's atmosphere (c = 12,285 km). Its ballistic properties (A/m = 0.0068 cm²/g) and orbit parameters make LAGEOS a near-perfect tool for studying the rotation of the earth.

A family of analytic theories, with the perturbations of the satellite orbital elements represented as truncated trigonometric series, was employed to calculate the topocentric distances. These calculations were performed in Coordinated Universal Time in a quasi-inertial reference frame specified by the instantaneous equator and the vernal equinox for epoch MJD 2,444,507. The conversion to the inertial system was based on Kozai and Kinoshita's expressions for the fictitious element perturbations. The satellite-motion model included:

1. An intermediate orbit, derived from Aksnes's theory, which gives the secular perturbations of a satellite to the third order in the flattening, and the periodic perturbations to the second order.

2. A 340-term series for the perturbations by the geopotential (the GEM 10 model complete to 20th order, the zonal harmonics to 28th order), evaluated using an algorithm described elsewhere by Romanova and the author.

3. Direct and tidal lunisolar perturbations, comprising series of 3004, 181, 122, and 46 terms for the moon, sun, lunar tides, and solar tides, respectively. This algorithm as well has been published separately.

4. Perturbations due to radiation pressure, according to Aksnes's theory.

5. Tidal shifts in the observing-station coordinates, calculated from Cartwright and Taylor's expansion of the tidal potential, with the Love numbers adopted from

A diagram showing the residual x, y coordinates of the pole derived from LAGEOS measurements.

A graph showing deviations in the length of the day from 86,400 sec.