EXCITATION OF NON-ADIABATIC QUASI-TOROIDAL OSCILLATIONS IN DOUBLE STARS

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1. Introduction

In a close binary system, the stars are subject to the time dependent tidal potential caused by their companion. Forced oscillations are induced in each component and constitute the dynamical tide. The tidal evolution of binaries with radiative envelopes was not understood until it was shown by Zahn (1975, 1977) that radiative damping of these forced oscillations could produce the required torque for synchronization and circularization by retarding the dynamical tide.

Recently it has been remarked by Chapellier (1984) that in some multiperiodic β CMa stars belonging to binary systems (σ Sco, 16 Lac, 12 Lac, ν Eri, θ Oph), the ratio between the beat period and the orbital period close to an integer. This remark leads to the question of the role of binarity for these stars and of the interest of studying extensively the forced oscillations in close binary systems.

Two methods have been used to study these forced oscillations: in the first one the perturbed motion is expressed in terms of series involving the eigenfunctions of the normal modes of the star. Taking into account the orthogonality properties of the eigenfunctions, resonance coefficients are derived, which inform about the importance of the excitation of a given mode (Zahn, 1970; Rocca, 1982, 1984). The other method consists in solving numerically the system of equations governing the forced oscillations. Non-adiabatic calculations, neglecting the star rotation, have been made for gravity modes firstly by Zahn (1975) and more recently by Savonije and Papaloizou (1981, 1984). In the present paper, we take into account the rotation of the star, assuming a small uniform rotation and we study numerically the excitation of non-adiabatic quasi-toroidal modes by the perturbing tidal potential.

2. Forced quasi-toroidal oscillations: basic equations

Forced oscillations are induced in each component of a binary system by the time varying gravitational potential U created by the other star. We consider a binary system with a small inclination θo between the orbital plane and the equatorial plane of the primary star, an orbit of small eccentricity e and a small rigid rotation Ω of the primary (in units of (GM/R^3)^1/2, Ω<<1). The gravitational potential U created at the point M (r, θ, φ) of the primary by the punctual secondary star can be expanded in spherical harmonics:

$$U = \sum_{n,k,m} U_{n,k,m} (r/R)^n P_n^m (\cos \theta) e^{-im\phi} e^{i\omega t}$$

In the frame corotating with the primary, each term of this potential is time varying with an apparent frequency $\omega_{k,m} = k\omega - m\Omega$, where $\omega$ is the orbital frequency. $U_{n,k,m}$ depends on three small parameters: the ratio R/d between the radius of the primary and the distance of the stars, the eccentricity e and the inclination $\theta_o$. The dominant terms relatively to R/d correspond to n=2. In the table I, the orders of magnitude of these terms are given.

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In this table, two terms are of order 1: the term $k=m=0$ induces an axisymmetric distortion of the star; the term $k=m=2$ corresponds to the equilibrium tide when $e = 0$ and $\omega = \Omega$. Note that no $m=1$ exists only if there is an inclination $\theta_0$, the principal terms being $k=0$ and $k=2$.

Each term of the potential $U$ will excite a forced oscillation in the primary star at the low frequency $\sigma_{k,m}$. This oscillation obeys to the following formal equation:

$$ -\frac{\partial^2}{\partial t^2} \xi_{k,m} + \mathcal{L}(\xi_{k,m}) = -\nabla \cdot \left( \hat{U}_{n}^{k,m}(r/R)^{n} P_{n}^{m} \cos \theta \right) e^{-i\omega t} $$

(2)

where $\mathcal{L}$ is the linear operator of the free stellar oscillations. If $\sigma_{k,m}$ is sufficiently close to a stellar eigenfrequency, resonance may occur with low frequency eigenmodes of the rotating star, i.e. gravity or quasi-toroidal modes.

Quasi-toroidal modes have an essentially toroidal horizontal displacement. Their radial displacement and pressure perturbation are by a factor $\Omega^2$ smaller:

$$ \begin{pmatrix} \xi_{r} \\ p' \end{pmatrix} = \Omega^2 \begin{pmatrix} \xi_{1-1} \\ p_{1-1} \end{pmatrix} P_{1-1}^{m} \cos \theta \begin{pmatrix} \xi_{1+1} \\ p_{1+1} \end{pmatrix} P_{1+1}^{m} \cos \theta \right) e^{i\omega t} $$

(3)

Their frequencies are proportional to the rotational frequency $\Omega$:

$$ \sigma = \frac{2 m \Omega}{l(l+1)} (1 + \Omega^2 \sigma_1) $$

(4)

The correction term $\sigma_1$ to the mode frequency is obtained by solving an eigenvalue problem (Papaloizou et al.1978; Provost et al.1981; Smeyers et al.1981; Saio 1982; Berthomieu et al.1983) and depends on the star distortion. It has been shown that if the star has both convective and radiative zones only $l=m$ modes exist. In this case, any physical quantity involves only one spherical harmonic. The geometric properties of the quasi-toroidal $l=m$ modes imply that they can be excited only by the $U_{l+1}^{0,m}$ of the potential, which exist for non-zero inclination. We will restrict this preliminary study to the dominant term: $l=m=1$. In this case the resonance condition is given by:

$$ \Omega \left(1 + \Omega^2 \sigma_1 \right) \sim \tau (k \omega - \Omega) \quad \text{with} \quad k = 0,1,2,3. $$

(5)

The forced non-adiabatic equations for quasi-toroidal modes are derived in the same way and with the same notations as in Berthomieu et al.(1983). The Cowling's approximation is made. The differences lie in the appearance of the source term $K$ and in the modification of the distortion term, due to the presence of the companion. The boundary conditions are: $x = 0$ and adiabatic behaviour of the perturbations at the center; $\delta p = 0$ at the surface and no radiation coming from the outside. For $l=m=1$, the equations governing the forced quasi-toroidal motions are:

$$ \ddot{r} \frac{dp}{dr} = (-U+3\alpha-V\delta\gamma)^{\alpha} (gp/\bar{U}+K/2) - \frac{dr}{dx} = \frac{\delta \theta + \kappa \nu_{p} + \nu_{P} + (6+\kappa \nu_{p}) x + (10 \sigma_{1} - 2\alpha(5+ra'/\alpha)(gp/\bar{U}+K/2))}{p} $$

(6a)

$$ \ddot{r} \frac{d\delta}{dr} = 4\nu_{p} \left[ \lambda/C_{3} - \kappa \nu_{p} + \kappa \nu_{p} \right] \frac{d\delta}{dx} = -6 C_{3} \left( j/4\nu_{p} + x \right) - i\Omega C_{2} \left( \theta - \nu_{p} \delta \right) $$

(6b)

$$ \ddot{r} \frac{d\lambda}{dr} = -6 C_{3} \left( j/4\nu_{p} + x \right) - i\Omega C_{2} \left( \theta - \nu_{p} \delta \right) $$

(6c)

with $j = 4\theta + i\Omega C_{2} (\theta - \nu_{p})$. The variable $\tilde{r}$ is constant on an equipotential surface, defined by:

$$ \tilde{r} = \text{constant} $$

(6d)
\[ \ddot{r} = r \left( 1 + \frac{2}{3} \Omega^2 a(r) P_2(\cos \theta) \right) \]

(7)

The distortion function \( a \) can be derived by a Clairaut-Legendre expansion (e.g., Tassoul 1980). It is equal to the coefficient given in Berthomieu et al., multiplied by the quantity \( (1 + 1.5 \frac{M_2}{M_1+M_2}) (\omega/\Omega)^2 \). The source term due to the tidal potential is given by \( k = 0.125 \frac{M_2}{M_1+M_2} (\omega/\Omega)^2 \sin 2\theta \). \( M_1 \) and \( M_2 \) are respectively the masses of the primary and secondary stars. Let us recall that \( \Omega^2 x \) is the displacement normal to the distorted equipotential surface. The rotational frequency \( \Omega \) appears explicitly only in the dissipative terms, \( \Omega C_2 \) and \( \Omega C_4 \), where \( C_2 \) and \( C_4 \) are characteristic thermal dissipative times.

The system has been integrated by the Henyey method, for some situations close to the resonance condition (5), using a 10 \( M_\odot \) evolved model \( (R/R_\odot = 6.9, \log T_{\text{eff}} = 4.31 \) \( \log L/L_\odot = 3.88 \)). This model has a convective core extending to 1/10 of the radius and a thin variable chemical composition zone. Preliminary results concerning the dominant terms of the tidal potential \( m=1, k=0 \) and \( k=2 \) (Table I) are given in the next section.

3. Results and discussion

![Figure 1](image1.png)  
![Figure 2](image2.png)

Figure 1: Modulus of the normal displacement at the surface in units of \( \xi_{\text{eq}} = 0.125 \frac{M_2}{M_1+M_2} (\omega/\Omega)^2 \), as a function of \( M_2/(M_1+M_2) (\omega/\Omega)^2 \), for \( \sin 2\theta = 0.1 \) and \( \Omega = 0.003 \). A modification of inclination \( \theta \) induces only a change of vertical scale.

Figure 2: Real part of the normal displacement in arbitrary units against radius, close to the dominant resonance of figure 1. The position of the variable chemical composition zone is indicated by the dashed area.

For \( k = 0 \), according to (5), resonance occurs for an orbital frequency such as \( \Omega_1 (\omega/\Omega) = 0 \). We have solved the system (6), with \( \omega_1 = 0 \) and by varying the orbital frequency both in the source term \( K \) and in the star distortion \( a \). The ratio between normal surface displacement and equilibrium tide displacement is plotted on figure 1, relatively to the amplitude of the source term. One essential resonant peak appears, corresponding to the eigenfunction given in figure 2. The other two resonances are much smaller and correspond to systems with an orbital frequency larger than the rotational frequency. If \( \Omega \) varies, the position of the resonant peaks in figure 1...
remains unchanged, but their amplitude differs as the magnitude of radiative dissipation changes. As a result of tidal and stellar evolution, the ration between orbital and rotational frequencies changes as well as the position of the resonant peaks in figure 1, which depends on the structure of stellar model. A more detailed analysis is required to determine if the binary system will pass through the resonance, during its evolution.

![Figure 3](image1)

![Figure 4](image2)

Figure 3: Modulus of the normal displacement at the surface, in units of $\xi_{eq}$ as a function of $\sigma_1$ for binary systems close to synchronism, $\sin \theta = 0.05$, $\Omega = 0.003$.

Figure 4: Real part of the normal displacement in arbitrary units against radius, corresponding to the two higher peaks of figure 3.

For $k = 2$, there are two possibilities of resonance: the first one, $2\omega = \Omega \sigma_1$, corresponds to a very small and unrealistic orbital frequency; the second one, $2 (\omega - \Omega) = \Omega \sigma_1$, corresponds to a system very close to synchronism. We have solved the system (6) around this resonance, by varying $\sigma_1$ and neglecting the departure from synchronism both in source and distortion terms. The ratio between normal surface displacement and equilibrium tide displacement is plotted on figure 3, relatively to $\sigma_1$. It shows a series of resonant peaks with decreasing amplitude when $|\sigma_1|$ increases. The corresponding eigenfunctions have an increasing number of radial nodes. Two of these eigenfunctions are plotted on figure 4.

Similar results are expected, when considering the resonances for $k = 1$ and $k = 3$, corresponding respectively to $\omega/\Omega = 2$ and $\omega/\Omega = 2/3$, but with amplitudes depending on the eccentricity.

Let us remark that in this work, we have considered independently the effect of each component of the tidal potential $U$. In fact, for values of orbital and rotational periods satisfying the relation (5), there exist terms of this potential, which vary with a period much longer than quasi-toroidal modes periods. Its effect will be to add a very slowly varying distortion term and thus to modify the properties of forced quasi-toroidal modes.

In conclusion, these preliminary results show that quasi-toroidal motions of
large amplitude may be excited in binary systems of stars with a small rigid rotation. Further investigation is needed to evaluate the consequence of this excitation on the evolution of the binary systems.

References.

Chapellier, E.: 1984, Thèse de 3ème cycle, Observatoire de Nice

DISCUSSION

G. DENIS: Using Curie's Principle of Symmetry, I do not quite understand how a tidal potential (which is a scalar potential) could cause toroidal modes, which correspond to the curl of a vector potential. Could you please comment on this.

J. PROVOST: I have improperly spoken of "toroidal" modes. In fact the modes I consider are quasitoroidal, which means that, due to the compressibility and stratification in the star, they have a small radial displacement of order $\Omega^2$. 

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