ON THE STABILITY OF SOLAR GRAVITY MODE OSCILLATIONS
AND THE STRUCTURE OF THE SUN

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Abstract: The g-mode oscillations stability for the interior of solar models with low heavy elements abundance and the models with turbulent diffusion mixing are investigated by quasi-adiabatic approximation. The models with low Z are found most unstable. It is pointed out that only the models with low Z are in agreement with the solar neutrino experiment and the observed solar gravity mode oscillations.

1. Introduction

Identification of the observed solar oscillations in the range $120^m - 200^m$ and related problem of their excitation are important in solar seismology. The most probable interpretation of the oscillations is to identify them with the non-radial g-modes. They are probably excited due to resonant nonlinear interactions with unstable modes as suggested by Dziembowski (1980) and Vandakurov (1981). The data recently obtained by Severny et al. (1984), provide a strong support for such point of view. The authors of cited work have detected 32 periods of the solar oscillations in the range $120^m - 200^m$ and interpreted them as oscillations for a model with low heavy elements abundance, so called the model C which is due to Christensen-Dalsgaard et al. (1979). Ten observed oscillations have been identified as nonradial gravity modes of degree $l = 4$ and orders $n = 11 - 20$, and the others as oscillations with combined frequencies of g-modes. It is very important that a great number of resonances have been found in observed power spectrum of the oscillations. So, as far as the $160^m$-oscillation is concerned there are the resonant couplings of the mode $l_1g_n = 2g_8 (P = 160^m.80)$ to following three couples of
oscillatory modes: \( 1\xi_1 - 2f \) \((160^{m.83})\), \(2\xi_1 - 2\xi_2 \) \((160^{m.24})\) and \(3\xi_4 - 4\xi_{14} \) \((160^{m.21})\). It follows the observed oscillations may be driven by nonlinear resonant interactions. A necessary condition of this mechanism is the instability of some modes in the resonant couplings.

This paper deals with the stability of gravity mode oscillations for solar models with low-Z interior as well as the models with turbulent diffusion mixing. It is known that these two kinds of solar models have the advantage due to the low flux of neutrinos (Bahcall and Ulrich, 1971; Schatzman et al., 1981). For a standard solar model the stability of \( g \)-modes has been previously investigated by several groups (Christensen-Dalsgaard et al., 1974; Boury et al., 1975; Shibahashi et al., 1975; Saio, 1980). The common conclusion of those papers is that all \( g \)-modes are stable, however there is a significant driving effect due to \( \varepsilon \)-mechanism for the \( g_1 \) and \( g_2 \) modes of \( l = 1 \).

2. Influence of chemical composition on the stability of \( g \)-modes

The stability have been investigated for the quasi-adiabatic approximation (Ledoux and Walraven, 1958; Ledoux, 1965; Boury et al., 1975). Six models of the Sun have been considered: a standard model (A), two models (B and C) with low-Z interior, and two models (1 and 2) with turbulent diffusion for the pseudo-Reynolds numbers \( \text{Re}^* = 100 \) and 200. Characteristic parameters of the solar models are given in Table 1.

<table>
<thead>
<tr>
<th>Model</th>
<th>( Z_0 )</th>
<th>( X_0 )</th>
<th>( X_c )</th>
<th>( \rho_c ) ( (g \text{ cm}^{-3}) )</th>
<th>( T_c ) ( (10^7 \text{K}) )</th>
<th>( D(\text{R}_\odot) )</th>
<th>( F_\nu ) ( (\text{SNU}) )</th>
<th>( P_0 ) ( (\text{min}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.02</td>
<td>0.739</td>
<td>0.369</td>
<td>153.4</td>
<td>1.51</td>
<td>0.272</td>
<td>7.9</td>
<td>36.8</td>
</tr>
<tr>
<td>B</td>
<td>0.004</td>
<td>0.832</td>
<td>0.482</td>
<td>129.0</td>
<td>1.41</td>
<td>0.203</td>
<td>2.3</td>
<td>37.9</td>
</tr>
<tr>
<td>C</td>
<td>0.001</td>
<td>0.861</td>
<td>0.512</td>
<td>123.4</td>
<td>1.39</td>
<td>0.135</td>
<td>1.7</td>
<td>37.6</td>
</tr>
<tr>
<td>1</td>
<td>0.02</td>
<td>0.733</td>
<td>0.572</td>
<td>108.0</td>
<td>1.46</td>
<td>0.260</td>
<td>2.7</td>
<td>56.1</td>
</tr>
<tr>
<td>2</td>
<td>0.02</td>
<td>0.718</td>
<td>0.619</td>
<td>99.6</td>
<td>1.43</td>
<td>0.242</td>
<td>1.8</td>
<td>61.3</td>
</tr>
</tbody>
</table>

The quantities listed are: the initial heavy elements and hydrogen abundances; the central hydrogen abundance, density and temperature; the depth of convection zone; the neutrino flux and the spacing period.
\[ P_0 = 2\pi^2 \left( \int_0^{R_c} \frac{N}{r} dr \right)^{-1} , \text{ in the asymptotic formula for the low degree and high order g-modes (} l \ll n\text{), (Vandakurov, 1967)}, \]

\[ P = P_0 \frac{n + 1/2 - 1/4}{\sqrt{1(1 + 1)}} \]

where \( R_c \) is the radius of the bottom of convection zone, \( N \) is the Brunt-Väisälä frequency.

The periods and damping times of the most unstable g-modes are listed in Table 2.

<table>
<thead>
<tr>
<th>Model</th>
<th>( \xi_1 )</th>
<th>( \tau_d )</th>
<th>( \xi_2 )</th>
<th>( \tau_d )</th>
<th>( \xi_3 )</th>
<th>( \tau_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>63.13</td>
<td>6.02 \times 10^5</td>
<td>85.46</td>
<td>-2.40 \times 10^8</td>
<td>109.90</td>
<td>1.10 \times 10^7</td>
</tr>
<tr>
<td>B</td>
<td>70.81</td>
<td>-1.68 \times 10^7</td>
<td>99.05</td>
<td>-1.35 \times 10^7</td>
<td>124.56</td>
<td>4.05 \times 10^7</td>
</tr>
<tr>
<td>C</td>
<td>72.87</td>
<td>-7.12 \times 10^6</td>
<td>102.25</td>
<td>-1.09 \times 10^7</td>
<td>128.12</td>
<td>-1.12 \times 10^8</td>
</tr>
<tr>
<td>1</td>
<td>88.61</td>
<td>-1.86 \times 10^7</td>
<td>122.59</td>
<td>-1.32 \times 10^7</td>
<td>155.12</td>
<td>1.35 \times 10^7</td>
</tr>
<tr>
<td>2</td>
<td>95.87</td>
<td>-2.94 \times 10^7</td>
<td>133.92</td>
<td>-2.37 \times 10^7</td>
<td>169.34</td>
<td>1.52 \times 10^7</td>
</tr>
</tbody>
</table>

As can be seen, both the decrease of the heavy elements abundance and the increase of the central hydrogen abundance due to turbulent diffusion intensify the instability of the g-modes. However, in the latter case the instability disappears when the ratio \( X_c/X_o \) is greater than 0.9. The important conclusion is that the instability of g-modes is the most strong in the model C. Therefore, for this model the mechanism of the resonant excitation of g-modes is of most importance.

3. Comparison of the solar models with the observational data

To compare the models with known observational data we have taken the following main characteristics of the Sun's interior: (1) the depth of convection zone, \( D \); (2) the neutrino flux, \( F_\nu \); and (3) the period spacing of the g-modes, \( P_0 \).

As it is known the models with low \( Z \) have the convection zone of a smaller depth as compared to the other models. In the model C, for example, the depth is equal to 0.13 \( R_\odot \). The shallow convection zone
does not seem to fit the known explanation of Lithium deficiency (Gabriel and Noels, 1977) and interpretation of the k-ω diagram for high-degree 5-min oscillations (Belvedere et al., 1983). On the other hand, there are observed phenomena such as sunspots and granulation which show that the depth of the convection zone does not exceed much the value of \(5 \times 10^4\) km (0.07 \(R_\odot\)). Therefore, it does not seem we can draw any definite conclusion on the depth of the convection zone, using the present data available.

Thus, we have proceeded from the two parameters: \(F_\nu = 2.1 \pm 0.3\) SNU (Bahcall et al., 1982) and \(P_0 = 37^m.4 \pm 2^m.7\) (Severny et al., 1984), \(38^m.6 \pm 0^m.5\) (Delache and Scherrer, 1983), \(41^m.18 \pm 0^m.14\) (Van der Raay et al., 1983). The \(F_\nu - P_0\) diagram for the computed models is shown in Fig. 1.

![Diagram showing solar models on the \(F_\nu - P_0\) diagram.](image)

**Fig. 1.** Location of the solar models on the \(F_\nu - P_0\) diagram (\(F_\nu\) is the neutrino flux, \(P_0\) - the period spacing of \(g\)-modes). A - the standard model, B and C - the models with low \(Z\), 1 and 2 - the models with turbulent diffusion mixing (\(Re = 100\) and \(200\)). The hatched area shows the observed values.

As seen only the low \(Z\) models, B and C are in agreement with the observed data. A model with turbulent diffusion mixing can fit only one of the parameters either \(F_\nu\), or \(P_0\) (see, also, Berthomieu et al., 1984).

Thus, the observations of the low-frequency solar oscillations
and the solar neutrinos and the calculations of the g-modes stability draw us to conclusion that a model of the Sun's interior with low heavy elements abundance is the most preferable.

References


Van der Raay, H.B. et al. 1983, Oscillations as a Probe of the Sun's Interior (European Study Conference, Catania ).