ON THE DETERMINATION OF THE HELIUM ABUNDANCE
OF THE SOLAR CONVECTION ZONE

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Abstract: With an appropriate analysis of data from solar oscillations, the
ground speed \( c \) in the solar interior becomes an observable quantity [1,2]. Since
\( c^2 = \gamma p / \rho \) the ionization zones are manifest in the sound speed, because \( \gamma \)
undergoes substantial variations (from 5/3 in fully recombined or ionized regions
to about 1.20 and 1.58 in the middle of the hydrogen and second helium ionization
zones, respectively). In the deeper layers of the solar convective envelope, the
nearly adiabatic stratification permits one to derive a direct relation between
thermodynamic quantities (\( \gamma \) and its first partial derivatives) and the sound
speed. The variation in \( \gamma \) due to helium ionization is evidently dependent on the
solar helium abundance. Therefore by measuring that variation it ought to be
possible to infer the helium abundance in the Sun.

INTRODUCTION

One of the issues one might expect helioseismology to address concerns the
helium abundance \( Y \) in the solar interior. Several attempts to determine \( Y \) have
already been made using low-degree oscillation data [3-7]. The methods that were
used rely to some degree on the theory of stellar structure and evolution, so the
results are subject to all the uncertainties in the assumptions and approximations
of that theory. Here, we discuss a new method using high-degree oscillations that
is based only on quite well established properties of the solar structure. The
principal hypothesis upon which the following analysis depends is that the bulk of
the solar convection zone is nearly adiabatically stratified. This is a well known
prediction of mixing-length theory, and appears to be a universal property of
convection at high Rayleigh numbers. To a lesser degree it has been confirmed
observationally by an analysis of the high-degree solar oscillations [1].

Using the constraint imposed upon the structure of the convection zone by
adiabatic stratification and the equation of hydrostatic support, the following
relation between thermodynamic quantities and the sound speed \( c = (\gamma p / \rho)^{1/2} \)
can be deduced [1]:

\[
\frac{1 - \gamma \rho - \gamma}{1 - \gamma_c^2} = \frac{r^2}{Gm} \frac{dc^2}{dr}.
\]

(1)

Here, \( m \) denotes the mass inside the spherical shell of radius \( r \),
\( \gamma = (\partial \ln p / \partial \ln \rho)_s \), the partial derivative being taken at constant specific
entropy \( s \), and \( \gamma_r, \gamma_c^2 \) are \( (\partial \ln \gamma / \partial \ln \rho)_c \), \( (\partial \ln \gamma / \partial \ln c^2)_\rho \) respectively. We define

\[
\Theta = \frac{1 - \gamma \rho - \gamma}{1 - \gamma_c^2}; \quad W = \frac{r^2}{Gm} \frac{dc^2}{dr},
\]

(2)
and note that \( \Theta \) and \( W \) are of order unity. These quantities are also well defined outside the quasiadiabatically stratified region, where, of course, \( \Theta \neq W \). Since in principle the depth dependence of the sound speed can be inferred from solar oscillation data [1,2] \( W \) is, again in principle, a measurable quantity. Having measured \( W \), the key idea is to find a solar envelope model with the right \( W \).

To see what kind of information one can extract by this method, one has to remember that \( \gamma = 5/3 \) in both fully ionized and fully recombined regions. There \( \Theta = -2/3 \). Partial ionization manifests itself in general by a lowering of \( \gamma \); in the Sun, \( \gamma \) drops to 1.20 in the middle of the H ionization zone and to 1.58 in the He II ionization zone. The influence of He I ionization is weaker, and produces only a shoulder in the function \( \gamma (r) \). Since \( \Theta \) contains derivatives of \( \gamma \), the ionization zones are expected to produce a more pronounced variation than in \( \gamma \) itself.

In the hydrogen ionization zone the stratification is too far from being adiabatic for \( W \) to provide an unambiguous measure of \( \Theta \). However that is not so in the helium ionization zones. There the features in \( \Theta \) that reflect helium ionization have the potential for measuring \( \gamma \). This is most readily seen by realizing that \( \gamma = 0 \) suppresses them altogether. We now demonstrate this diagnostic potential.

THE FUNCTIONS \( \Theta \) AND \( W \)

Figure 1 shows \( \Theta \) and \( W \), defined in eq. (2), computed from a model of the solar envelope. They are plotted against \( \zeta = \log(1-r/R) \), where \( R \) is the radius of the Sun. The envelope model was constructed by integrating inwards from a continuum optical depth of 0.63, using starting conditions obtained from the Harvard-Smithsonian Reference Atmosphere [8]. Opacities were obtained from a biquadratic interpolation in the tables of Cox and Stewart [9], and the convective heat flux was computed from the Böhm-Vitense mixing-length formalism as presented by Hofmeister et al. [10]. Reynolds stresses were ignored. The equation of state was derived from the simple Saha equation, assuming all bound species to be in their ground states; hydrogen and helium and the 10 most abundant heavier elements (whose relative abundances were taken from Ross and Aller [11]) were included. No attempt has been made to match the envelope onto the interior of an evolved model of the Sun.

The first point to notice in Figure 1 is the excellent agreement between \( W \) and \( \Theta \) at depths in the convection zone greater than \( \zeta = -2.2 \). Here, the stratification is within 0.2 per cent of being adiabatic. As expected, \( \Theta \) and \( W \) deviate in the radiative interior and in the superadiabatic boundary layer beneath the photosphere. The maximum in \( \Theta \) at \( \zeta = -4 \) and the broad shoulder between \( \zeta = -3 \) and \( \zeta = -2 \) are due to the combined influence of H and He I ionization, and the pronounced bump at \( \zeta = -1.8 \) is due to He II ionization. The general shape of
Figure 1: The functions $\Theta$ and $W$.

The curve is a signature of the path in the space of thermodynamic state variables, which is determined by the structure of the superadiabatic boundary layer at the top of the convection zone. The three small bumps that follow are due to oxygen, carbon and again oxygen (of a higher degree of ionization), respectively. For our purpose it is the He II bump that is the most important. There $\Theta$ is well represented by $W$. But we stress that Figure 1 suggests that there might be a much greater diagnostic potential, because fine details of the equation of state would become measurable if $W$ could be determined with sufficient accuracy.

DEPENDENCE OF $\Theta$ ON THE PARAMETERS OF THE ENVELOPE MODEL

Varying the mixing-length changes the entropy jump across the superadiabatic boundary layer. This shifts the positions and to a lesser extent alters the thicknesses of the ionization zones. Changes in the helium abundance, however, principally affect the magnitude of the helium-related structures in $\Theta$. A prognosis for the numerical matching experiment discussed in the next section, therefore, is that the different reactions to varying mixing length and helium abundance can be distinguished, and that $Y$ can be measured without assuming anything about the value of the mixing-length parameter. We note furthermore that since the structure of the convection zone beneath the superadiabatic boundary layer depends almost solely on the entropy jump across that layer, and is apparently rather insensitive to details of the variation within it [12], our determination of $Y$ is hardly dependent on the accuracy of the mixing-length formalism we have employed.
MEASURING THE HELIUM ABUNDANCE

In an initial assessment of the present method, the helium abundance of a test model of the solar envelope has been estimated by fitting the function \( W \) in a grid of functions \( W \) belonging to 9 other different models. The test model had a ratio of mixing-length to pressure scale height equal to 2.00, and \( Y = 0.235 \). The comparison grid of 9 models had \( \alpha = 1.40, 1.90, 2.40 \) and \( Y = 0.170, 0.220, 0.270 \) in all combinations. The heavy element abundance \( Z \) was held fixed at 0.02. We anticipate that small uncertainties in \( Z \) will introduce only small uncertainties in the determination of \( Y \).

A function \( W_0(\zeta;\alpha, Y) \) depending continuously on the parameters \( \alpha \) and \( Y \) was defined by biquadratic interpolation in the grid. Then the values of \( \alpha \) and \( Y \) that minimize by least squares the difference between \( W_0 \) and the function \( W \) of the test model were determined. The minimization was performed in the range of \( \zeta \) corresponding to that part of the He II bump in the test model satisfying \( W > -0.64 \). The result was \( \alpha = 1.96, Y = 0.231 \).

At the present level of refinement this excercise overestimates the power of the method, because the same physics was used for computing the model and the standard grid. We therefore performed a more realistic experiment by fitting three other test models constituted with more sophisticated equations of state. All three models had \( Y = 0.235 \), as before. One of the equations of state was that used by Berthomieu et al. [13] and the other was that described by Däppen [14]. Both take electron degeneracy into account and contain an excluded volume term in the free energy due to electrically neutral species (which has little significance in the convection zone). In the first case [13] the internal partition functions of the bound states of hydrogen and helium are computed with a static screened Coulomb potential, though the heavy elements are still treated with the simple Saha approximation; in the second case [14] the confined-atom model is used, and the gas is presumed to be a mixture of only hydrogen and helium. The third model uses the simple approximation of Eggleton et al. [15] for all species. The new calibrations differed from the correct answer by as much as 5 per cent. Nevertheless, unlike in our first calibration, the detailed shape of the best fitting interpolated functions \( W_0 \) differed markedly from the actual function of the test models. Therefore we have a direct indication that the microphysics used to construct the standard grid does not accurately mimic that in the test models.

DISCUSSION

In principle the technique presented here provides a method of measuring the helium abundance in the solar convection zone. In practice, of course, its success will depend on how well the sound speed and its derivative with respect to depth can be inferred from solar oscillations. Our next task, therefore, is to estimate the accuracy that will be required of the oscillation data by computing
from a theoretical model a large number of modes and inverting them to obtain $W$. It will be necessary to gauge the effect of uncertainties in the calibration that we have not yet considered, such as the heavy element abundance $Z$. The extent to which Reynolds stresses and sound-speed fluctuations induced by convection might influence the procedure, both via their effect on the mean stratification of the equilibrium model and by their direct interaction with the oscillations, must also be assessed.

In conclusion we stress that the helium abundance is merely one example of what can be deduced from the function $c^2(r)$. Comparison of $W$ computed with different equations of state has already revealed discrepancies of up to 10 per cent. There are still substantial uncertainties in the equation of state, especially in the treatment of bound states in an ionic environment, which could eventually set a limit to helioseismological interpretations. However, the method presented here might, with some good fortune, remove that limitation, rendering it possible to make direct observations of thermodynamical quantities in the Sun.

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REFERENCES

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