ALFVÉNIC RESONANCES ON SOLAR SPICULES
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Received 1983 December 12; accepted 1984 April 27

ABSTRACT
It is suggested that twisting and heating of solar spicules can be produced by Alfvén waves which enter the spicule from below. The spicule is treated as a region of constant Alfvén speed which is bounded above by a region of much higher Alfvén speed (the corona) and below by a region of exponentially increasing Alfvén speed (the photosphere and chromosphere). We show how the spicule can act as a resonant cavity. The transmission of the waves into the cavity is analytically determined to be enhanced at certain resonant frequencies. With reasonable spicule parameters, and assuming the spicule damping to be moderately large, we find that twisting velocities of \( \sim 20-30 \text{ km s}^{-1} \) can be induced on the spicule. We suggest that the Alfvén waves are dissipated via a turbulent cascade of their energy to higher wavenumbers. We show that the waves can thereby heat the spicules to the observed temperatures. It is further suggested that the continued input of energy can explain why Hz spicules fade, since the predicted heating rate is sufficient to heat the spicules to temperatures at which the hydrogen is fully ionized; thus Hz spicules may evolve into EUV spicules.

Subject headings: Sun: atmosphere — Sun: chromosphere

1. INTRODUCTION

Although ideas about mechanisms for producing solar spicules have been around for some time (see, e.g., Beckers 1968, 1972; Athay 1976; for discussions of some earlier suggestions), theoretical ideas based on nonlinear models have been advanced only recently.

Hollweg, Jackson, and Galloway (1982, hereafter HJG) considered spicules to be the chromospheric gas behind an uplifted transition region which has been moved along a selected set of magnetic field lines. Their model begins with an untwisted, vertical flux tube. Alfvén waves are launched at the base of the tube in the form of time-dependent, axisymmetric twists. These twists nonlinearly evolve into upward propagating fast shocks in the chromosphere. Upon interaction with the transition region, these shocks push the transition region upward. The upward velocities and density profiles of the chromospheric material behind the raised transition region are in reasonably good agreement with the observed properties of spicules. However, the temperatures yielded by the model are substantially lower than those observed in spicules. The low temperatures of the model are a direct result of the nearly adiabatic cooling of the parcel of chromospheric gas as it expands while being lifted.

Suematsu et al. (1982) consider the dynamics resulting from a sudden pressure enhancement at the base of a vertical flux tube with constant cross section in a model solar atmosphere. The disturbance evolves into a slow shock in the chromosphere, which pushes the transition region upward. The spicule is interpreted as the chromospheric material beneath the elevated transition region. As in HJG, the resultant model temperatures are significantly less than the observed temperatures in spicules. Hollweg (1982) presented a similar model where he considered the nonlinear evolution of acoustic gravity waves on a flux tube. In this case the transition region was pushed up by a train of rebound shocks which form as a result of nonlinear oscillations of the solar atmosphere at the acoustic cutoff frequency.

The object of this paper is not to present a spicule model, but rather to point out a new aspect of spicules which may account for their heating, and for the occasionally observed twisting motions (e.g., Beckers 1968; Livshits 1967; Paschoff, Noyes, and Beckers 1968). To this end, we show that spicules can be resonance cavities for Alfvénic twists. The resonance cavity forms because the spicule is bounded above by a region (the corona) where the Alfvén speed is much larger than in the spicule, while it is bounded below by a region (the photosphere and low chromosphere) where the Alfvén speed is a rapidly increasing function of height. Both of these regions can reflect Alfvén waves and trap wave energy in the spicule, forming a resonance cavity. We show that the spicules act much like antireflection coatings on camera lenses, allowing photospheric disturbances of certain resonance frequencies to propagate through the chromosphere and into the spicule without undergoing substantial reflection. Moreover, we show that this energy will heat the spicules, if a suitable wave dissipation mechanism is present in the spicule. We suggest that the dissipation rate may be determined by the rate at which a turbulent cascade feeds energy to higher wavenumbers.

Resonance cavities have been of recent interest in other contexts, particularly on connection with heating coronal active region loops (Hollweg 1981, 1984a, b; Ionson 1982, 1984; Zulga and Locans 1982). Hollweg and Sterling (1984) have recently shown that the theory of Hollweg (1984a) is consistent with coronal loop data, and we thus have some confidence that resonant heating may also apply to spicules. This paper is similar to Hollweg (1984a) in that a three-layer model is used to represent the behavior of the Alfvén speed. However, the behavior of the Alfvén speed assumed in this paper is fundamentally different from that assumed in Hollweg (1984a), and thus the mathematical details require reevaluation. However, our analysis will be restricted by the simplifying assumption that the spicule is not changing in time. Thus we will take \( L \) (the length of the cavity) to be constant, and we will ignore plasma flows within the spicule. This turns out to be a marginal assumption for the fundamental (i.e., longest period) resonance, since the period of the fundamental is not very much shorter than the time scale over which the length of an actual spicule changes.

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For example, the period of the fundamental is approximately $4L/v_{A2}$, where $v_{A2}$ is the Alfvén speed inside the spicule. Taking $L = 7000$ km and $v_{A2} = 357$ km s$^{-1}$ (appropriate for a spicular density of $10^{-11}$ g cm$^{-3}$ in a 40 gauss field) yields a fundamental period of 78 s, which is to be compared with spicule lifetimes of a few hundred seconds. Our approximation seems reasonable for the higher harmonics, however. For example, for the numerical values given above, the period of the first harmonic is only 26 s.

II. DISSIPATIONLESS ANALYSIS

Resonances on spicules will be analyzed subject to the following simplifying assumptions.

1. The wave fluctuations are small amplitude. Hence, linearized theory will be used. The time-independent background quantities will be denoted by the subscript 0, and the wave quantities by the prefix $\delta$.
2. The magnetic flux tube on which the spicule is to occur will be taken to have constant cross section.
3. All quantities are taken to be axisymmetric. The symmetry axis is vertical. Hence $\partial/\partial \theta = 0$, where $\theta$ is the azimuthal angle about the symmetry axis.
4. The background magnetic field, $B_0$, is untwisted, so $B_{0\theta} = 0$.
5. The only nonvanishing components of $\delta v$ and $\delta B$ are $\delta v_\theta$ and $\delta B_\phi$, where $v_\theta$ and $B_\phi$ denote velocity and magnetic field, respectively.
6. The effects of viscosity and electrical conductivity are not explicitly considered. However, some damping will be assumed at a later point.

Assumptions 3 and 5 together imply that the waves are noncompressive. Hence, gravity and thermal pressure play no role. Throughout the discussion, cgs units will be used.

Using basically the same assumptions, Hollweg (1984a) combined the MHD equations into the following equation for the magnetic field fluctuation:

$$\frac{\partial}{\partial t} \delta B_\theta = B_0 \cdot \nabla \delta v_\theta ;$$

and the following wave equation for the velocity fluctuations:

$$\frac{\partial^2}{\partial t^2} \delta v_\theta = v_{A2}^2 \frac{\partial^2}{\partial s^2} \delta v_\theta .$$

Equation (2) is to be solved for three different regions of the solar atmosphere, each with a different expression for the Alfvén speed, $v_A$. The geometry is given in Figure 1.

Region 1 corresponds to the photosphere and chromosphere, where $v_A$ is taken to be

$$v_A \propto \exp \left[-(s - L)/2h\right],$$

where $h$ is a positive constant. Region 2 represents the spicule, which will be taken to have constant Alfvén speed. Region 3 represents the corona, which will also be taken to have a constant, but much higher, Alfvén speed.

The source of the Alfvén waves is assumed to be in the photosphere, so there will be upward- and downward-propagating waves in regions 1 and 2, but only an upward-propagating wave in region 3. For harmonic waves varying as exp $(i\omega t)$, the solutions to equations (1) and (2) are easily verified to be:

Region 3 ($s < 0$):

$$\delta v_\theta = a \exp (i\omega t + ik_3 s),$$

$$\delta B_\theta = a \frac{k_3}{\omega} B_0 \exp (i\omega t + ik_3 s).$$
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Region 2 ($0 < s < L$):

\[
\delta v_g = [b \exp(-ik_2s) + c \exp(i k_2s)] \exp(i \omega t),
\]
\[
\delta B_g = -\frac{k_2 B_{ao}}{\omega} [b \exp(-ik_2s) - c \exp(i k_2s)] \exp(i \omega t),
\]

Region 1 ($s > L$):

\[
\delta v_g = [d H_0^{(1)}(\xi) + e H_0^{(2)}(\xi)] \exp(i \omega t),
\]
\[
\delta B_g = \frac{i B_{ao}}{v_A(s)} [d H_0^{(1)}(\xi) + e H_0^{(2)}(\xi)] \exp(i \omega t).
\]

Here, $\xi = \frac{2\hbar \omega}{v_A(s)}$; $\theta' = -\theta$, $k_2 = \omega/v_A$, $k_3 = \omega/v_{A3}$, and $a, b, c, d, e$ are complex constants.

The matching conditions at $s = 0$ and $s = L$ require that $\delta v_g$ and $\delta B_g$ be continuous at those locations. Therefore, we obtain

\[
b = \frac{k_2 - k_3}{2k_2} a,
\]
\[
c = \frac{k_2 + k_3}{2k_2} a,
\]
and

\[
e = \frac{\pi \beta}{4i} [(b' + c') H_1^{(1)}(\beta) - ib' - c' H_1^{(2)}(\beta)],
\]
\[
d = -\frac{\pi \beta}{4i} [(b' + c') H_1^{(2)}(\beta) - ib' - c' H_1^{(2)}(\beta)],
\]
where $\beta = 2\hbar \omega/v_{A2}$, and

\[
b' = b \exp(-ik_2L),
\]
\[
c' = c \exp(i k_2L).
\]

Equations (10)–(13) completely specify four of the constants in terms of the fifth. Here we will write four constants in terms of $a$, the amplitude of the wave leaving the spicule resonance cavity and escaping into the corona. Equations (10)–(14) can be combined to yield

\[
e = \frac{\pi \beta a}{4ik_2} [(k_2 H_1^{(1)} + ik_3 H_0^{(1)}) \cos(k_2 L) - (k_2 H_0^{(1)} - ik_3 H_1^{(1)}) \sin(k_2 L)],
\]
\[
d = -\frac{\pi \beta a}{4ik_2} [(k_2 H_1^{(2)} + ik_3 H_0^{(2)}) \cos(k_2 L) - (k_2 H_0^{(2)} - ik_3 H_1^{(2)}) \sin(k_2 L)],
\]
where the argument of the Hankel functions is $\beta$.

Hollweg (1984a) showed that the time-averaged Poynting flux in region 1, in the negative-$s$ direction, is proportional to $|d|^2 - |e|^2$. He therefore identified the $H_1^{(1)}$ part of equation (8) as the wave which propagates upward from the photosphere, while the $H_0^{(2)}$ part of equation (8) was identified as the downward-propagating wave. Thus an energy reflection coefficient can be defined as follows:

\[
R = \frac{|e|^2}{|d|^2}.
\]

The transmission coefficient at the region boundary between regions 1 and 2 is

\[
T = 1 - R.
\]

The quantities $T$ and $R$ can be calculated from equations (15) and (16). The analysis so far has only considered solutions to equation (2), which contains no dissipation. Therefore, the transmitted wave energy will flow through the spicule and out into the corona.

Figure 2 plots the transmission coefficient $T$ versus $\omega$. The frequencies correspond to periods ranging from 7 to 500 s. The coronal density was taken to be $\rho = 3.3 \times 10^{-16}$ g cm$^{-3}$, which is a typical value for coronal holes, and the spicule density used was $\rho_s = 1 \times 10^{-13}$ g cm$^{-3}$ (cf. Beckers 1972, Table 3). The chromospheric scale height, $h$, is 150 km, the spicule length $L = 7000$ km, and $B_0 = 40$ G. The figure displays the aforementioned transmission resonances. The period of the fundamental resonance is about 90 s, and the period of the first harmonic is about 30 s. The periods are closely approximated by equation (21), below.

Further insight into the properties of the resonances can be gained by simplifying the “full” solutions (eqs. [15]–[18]) using appropriate approximations. It turns out that $\beta$ is quite small for solar parameters, at least for the lower order resonances, so the small-argument approximations for Hankel functions, $H_1^{(1), (2)} \to \pm i Y_1$, $H_0^{(1), (2)} \to 1 \pm i Y_0$, and $|Y_1(\beta)| \gg |Y_0(\beta)|$, may be employed.

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Also, since $\rho_s \gg \rho_c$, we have $k_3 \ll k_2$. Using these one can reduce equations (15) and (16) to give the following approximate expression for $T$:

$$T \approx -\frac{4k_2 k_3 Y_1}{(k_2 - k_3 Y_1)^2 \sin^2 (k_2 L) + k_2^2 Y_1^2 \cos^2 (k_2 L)}$$

(19)

where the argument of the Bessel function is again $\beta$. Note that the coefficient of $\cos^2 (k_2 L)$ is large. Thus $T$ is a maximum when $\cos (k_2 L) = 0$. Thus the resonances occur when

$$k_2 L \approx (2n - 1) \frac{\pi}{2}$$

(20)

i.e.,

$$\omega_{\text{res}} \approx \frac{\pi}{2} \frac{v_{A_2}}{L} (2n - 1),$$

(21)

where $n = 1, 2, 3, \ldots$ denotes the order of the resonance.

The heights of the resonance peaks are thus

$$T_{\text{max}} \approx -\frac{4k_2 k_3 Y_1}{(k_2 - k_3 Y_1)^2}.$$  

(22)

Using the small-argument approximation $Y_1(\beta) \approx -2/(\pi \beta)$, and defining

$$L_n \equiv \frac{\pi^2 h}{2} \frac{k_2}{(2n - 1) k_3},$$

(23)

equation (22) may be rewritten

$$T_{\text{max}} \approx -\frac{4L/L_n}{(1 + L/L_n)^2}. $$

(24)

Note that $T_{\text{max}} = 1$ when $L = L_n$. In that case waves can propagate from the photosphere into the spicule without undergoing any reflections. However, for the spicule parameters used in Figure 2, we have $k_2/k_3 = v_{A_3}/v_{A_2} = (\rho_2/\rho_3)^{1/2} = 17.4$ and $L_n = 12900 \ (2n - 1) \text{ km}$ if $h = 150 \text{ km}$. Thus $L_n > L$ for all $n$, and $T_{\text{max}}$ is always less than unity for these parameters; this can be seen in Figure 2. The quantity $T_{\text{max}}$ could approach unity on longer spicules, however.

The approximate full width at half-maximum of the transmission resonance peak, $\Delta \omega$, can be found from equation (19) by noting that the coefficient of $\cos^2 (k_2 L)$ is large, and by expanding the denominator about $\cos^2 (k_2 L) = 0$. There results, in terms of $L_n$,

$$\Delta \omega \approx 2v_{A_2} k_3 \frac{L_n}{k_2^2 L} \left(1 + \frac{L}{L_n}\right).$$

(25)

The quality of the resonance is defined to be

$$Q \equiv \omega_{\text{res}} (\Delta \omega)^{-1}.$$  

Combining equations (21) and (25) yields

$$Q \approx \frac{L_n L}{2\pi h (L_n + L)}.$$  

Taking $L = 7000 \text{ km}$, $h = 150 \text{ km}$, and $L_n = 12900 \ (2n - 1) \text{ km}$ gives $Q = 4.8$ for $n = 1$ and $Q = 6.3$ for $n = 2$. Even in the absence of dissipation, the spicule resonances are not very high quality.
The energy flux density, $F$, entering the spicule is calculated by assuming that the convective motions launch a net upward-propagating energy flux density, $P$, in a frequency bandwidth $B_{\omega}$. After integrating over one resonance peak, we obtain (see also Hollweg 1984a)

$$F \approx PT_{\text{max}} \frac{\Delta \omega \pi}{B_{\omega} 2}.$$  

(26)

where it is assumed that the spectral density is flat for simplicity. Using equations (24) and (25), equation (26) can be written as

$$F \approx \frac{4\pi v_{\lambda}^2}{(1 + L/L_0) \omega^3 L B_{\omega}}.$$  

(27)

Take $B_{\omega} = 10^{-1}$ s$^{-1}$; this corresponds to assuming that the power is spread between periods of 60 and 1000 s. Setting the same numbers as in the previous examples, viz., $v_{\lambda} = 357$ km s$^{-1}$, $L = 7000$ km, $h = 150$ km, $v_{\lambda} = 6210$ km s$^{-1}$, $L_0 = 12900$ km ($n = 1$ resonance), we obtain $F/P = 0.24$. The flux $F$ is calculated by assuming that the photosphere twists the field lines with an rms velocity $\delta v_{\text{ph}}$. Then

$$P = \rho_{\text{ph}} \delta v_{\text{ph}}^2 B_0 (4\pi \rho_{\text{ph}})^{-1/2},$$  

(28)

where the subscript ph denotes photospheric values. Assume $\rho_{\text{ph}} = 3 \times 10^{-7}$ g cm$^{-3}$, $\delta v_{\text{ph}} = 1$ km s$^{-1}$, and $B_0 = 40$ G. Then for the first-order resonance, $P = 6.2 \times 10^7$ ergs cm$^{-2}$ s$^{-1}$. This implies a flux of $F = 1.5 \times 10^6$ ergs cm$^{-2}$ s$^{-1}$ entering the spicule. It is important to note that our estimate of the flux is only approximate and could easily be, say, increased by a factor of 2 or more by considering slightly larger photospheric motions, and/or slightly larger values for $B_0$, since $F$ varies as the square of both these quantities. Additional energy also becomes available when the higher order resonances are excited.

The velocity fluctuations in the spicule can be calculated from equations (6), (10), and (11):

$$\delta v_\theta = -\frac{a}{k_2} \left[ k_2 \cos (k_2 s) + i k_3 \sin (k_2 s) \right] \exp (i\omega t).$$  

(29)

Since $k_3 \ll k_2$ and since $\cos (k_2 s) \approx 0$ at resonance, we see that the amplitude of the fluctuations is greatest near the top of the spicule, where $s = 0$, at resonance. This is a reflection of the fact that the wave amplitudes are much larger in the corona than in the chromosphere because of the corona's much lower density. As a numerical example, suppose that a flux $F = 1.5 \times 10^7$ ergs cm$^{-2}$ s$^{-1}$ flows through the spicule and into the corona in one of the resonances. Thus $(4)\rho_{\lambda} a^2 = 1.5 \times 10^7$, and we deduce $a = 121$ km s$^{-1}$, if $B_0 = 40$ G and $\rho_{\lambda} = 3.3 \times 10^{-16}$; this is the velocity amplitude at the top of the spicule. The velocity amplitude at the bottom of the spicule ($s = L$) is only $7.0$ km s$^{-1}$ if $k_2/k_3 = 17.4$. Thus the resonances are capable of inducing large twisting motions on spicules. In fact, the predicted velocities near the top of the spicule are much larger than the typical observed values ($\sim 20$ km s$^{-1}$; e.g., Pasachoff, Noyes, and Beecers 1968; Beecers 1972). The contradiction can be resolved if we take wave damping into account. This is done in the next section.

III. WAVE DAMPING

Once energy has entered a spicule via a resonance, it must be dissipated in order for heating to occur. Without such dissipation, the energy simply flows through the spicule and into the corona.

In this section, the effects of damping on the transmission coefficient of the Alfvén waves entering the spicule from the chromosphere will be investigated. The damping mechanism is unspecified. Rather, some form of linear damping will be assumed. The effect of this is to change the spicule wavenumber, $k_2$, to a complex quantity. So we then write $k_2 = k_2 + ik_i$, where $k_i$ is negative for damping.

In redoing the analysis to determine the transmission coefficient of the Alfvén wave entering the spicule from the chromosphere, it must be noted that the condition $k_2 = \omega/v_{\lambda} \approx 0$ no longer holds.

Equations (15) and (16) are now

$$e = \frac{\pi \beta a}{4ik_2} \left[ k_2 H_{11}^{(1)} + ik_3 \frac{v_{\lambda} \sqrt{2}}{\omega} H_{01}^{(1)} \right] \cos (k_2 L) - \left( k_2 \frac{v_{\lambda} \sqrt{2}}{\omega} H_{11}^{(1)} - ik_3 H_{11}^{(1)} \right) \sin (k_2 L),$$  

(30)

$$d = -\frac{\pi \beta a}{4ik_2} \left[ k_2 H_{12}^{(1)} + ik_3 \frac{v_{\lambda} \sqrt{2}}{\omega} H_{02}^{(1)} \right] \cos (k_2 L) - \left( k_2 \frac{v_{\lambda} \sqrt{2}}{\omega} H_{21}^{(1)} - ik_3 H_{21}^{(1)} \right) \sin (k_2 L),$$  

(31)

with $k_2 = k_2 + ik_i$. The argument of the Hankel functions is still $\beta$.

The transmission coefficient is again given by equation (18). The form of $k_2$ is assumed to be

$$k_2^2 \approx \omega^2 \frac{v_{\lambda}^2}{v_{\lambda}^2} \left( 1 - \frac{k_2^2}{k_r^2} \right),$$  

(32)

which is a fairly general expression valid when the damping is not too strong (Hollweg 1984a, eq. [64]).

Using equations (30)-(32), the transmission coefficients for various values of $|k_i/k_r|$ were determined (see Fig. 3). The transmission peak heights are enhanced for the cases with a small amount of damping added. Note that the high-order resonances tend to blend together. This occurs in part because the damping increases $\Delta \omega$ (see below), and in part because the waves are so severely attenuated.
in propagating from \( s = L \) to \( s = 0 \) that effective contact with the transition region is lost. It is then no longer meaningful to talk about resonances, but it should be noted that \( T \) becomes very large in those cases.

The heights of the transmission peaks can be estimated from equations (30) and (31) by assuming that the peaks occur when \( \cos (k_s L) \approx 0 \). If also \( \beta \ll 1 \), \( Y_1(\beta) \approx -2/(\pi \beta) \), \( |Y_1| \gg |Y_2| \), \( k_s \ll k_2 \), \( k_2/k_s \ll 1 \), and \( |k_s/L| \ll 1 \), we deduce

\[
T_{\text{max}} \approx \frac{4\pi \hbar k_3 k_s^2}{(k_s + \pi \hbar k_3^2)^2} \left( 1 - \frac{k_s k_s}{k_3} \right) \left( 1 - \frac{k_s k_s}{k_3 + \pi \hbar k_3^2} \right)^2.
\]  

The width of the resonance peak can be found in the same manner as before (see also eq. [56] of Hollweg 1984b). We obtain

\[
\Delta \omega = \frac{2v_{A2}}{L} \frac{(\pi \hbar \omega_{\text{res}} k_3)}{v_{A2} k_s} - 2\omega_{\text{res}} \frac{k_s}{k_s}.
\]  

Note that dissipation \((k_s < 0)\) broadens the resonance.

The energy flux density, \( F \), entering the spicule can be calculated by combining equations (26), (33), and (34); the result will not be written down. This is the energy which flows out of the spicule into the corona, plus the energy which is deposited in the spicule as heat. Of this total, the fraction, \( f_H \), which is deposited as heat is

\[
f_H = \left[ 1 + \frac{k_s}{k_r} (k_s L)^{-1} \left( \frac{|k_s|}{k_r} \right)^{-1} \right]^{-1}.
\]  

See equation (58) of Hollweg (1984b). For example, consider the fundamental resonance \((k_s L = \pi/2)\) and take \( k_s/k_3 = 17.4 \); then \( f_H = 0.21, 0.50, 0.73, \) and 0.85 for \(|k_s|/k_r = 0.01, 0.037, 0.1, \) and 0.2, respectively. The average volumetric heating rate in the spicule can be obtained by dividing the energy flux density which goes into heat by \( L \). From equations (26), (33), (34), and (35) we obtain

\[
E_H = \frac{4n^2 \hbar v_{A3}(k_s L)^2 |k_s|}{L^2 B_0} k_s \left[ 1 + \frac{k_s}{k_3} \left( \frac{\pi \hbar}{k_3 L} + \frac{|k_s|}{k_r} \right)^{-1} \right]^{-1}.
\]  

For the spicule parameters used in this paper (\( h = 150 \text{ km s}^{-1}, L = 7000 \text{ km}, v_{A2} = 357 \text{ km s}^{-1}, v_{A3} = 6210 \text{ km s}^{-1}, B_0 = 0.1 \text{ s}^{-1}, \) and \( P = 6.2 \times 10^7 \text{ ergs cm}^{-3} \text{ s}^{-1} \)), and for the \( n = 1 \) resonance \((k_s L = \pi/2)\), we find \( E_H = 0.03, 0.04, \) and 0.045 ergs cm\(^{-3}\) s\(^{-1}\) for \(|k_s|/k_r = 0.1, 0.2, \) and 0.3, respectively. Let us take 0.04 ergs cm\(^{-3}\) s\(^{-1}\) as representative. This value of \( E_H \) may provide significant spicule heating. For example, take

\[
\frac{3}{2} n k \frac{\partial T}{\partial t} = E_H,
\]  

where \( T \) denotes spicule temperature, \( n \) denotes the particle concentration, and \( k \) is Boltzmann's constant. No allowance is made for losses due to ionization and radiation, and the adiabatic cooling due to the spicule expansion. For a fully ionized hydrogen spicule (molecular weight = 0.5) we have \( n = 1.2 \times 10^{14} \text{ cm}^{-3} \) and \( \partial T/\partial t = 1600 \text{ deg s}^{-1} \). Only 12\( s \) would be required to heat the spicule by 20,000 K. About 15 minutes (i.e., somewhat longer than a spicule lifetime) would be required to heat a spicule to coronal temperatures \((\sim 1.5 \times 10^6 \text{ K})\). And only 120\( s \) are required to heat a spicule by 200,000 K according to the above estimates. These estimates lead us to suggest that H\( \alpha \)-emitting spicules fade from view as the hydrogen becomes fully ionized as the gas is heated, and that the spicules eventually attain EUV-emitting temperatures and become EUV spicules. Observational evidence in support of this viewpoint can be found in a recent paper by Withbroe (1983).

However, it is important to note that radiation will be an important energy loss mechanism, which will severely limit the ability of waves (or any other process) to heat spicules to EUV-emitting or coronal temperatures. Unfortunately, the radiative energy budget of spicules is not known. If we were to treat an H\( \alpha \) spicule as typical of the upper chromosphere or lower transition region, then we would conclude that some \( 10^{-2} \text{ ergs cm}^{-3} \text{ s}^{-1} \) is needed to balance the radiative losses (Fig. 49 of Vernazza, Avrett, and Loeser 1981). This is comparable to our calculated value of \( E_H \). At EUV-emitting temperatures (few \( \times 10^{5} \text{ K} \)) we can assume that the
spicule is optically thin. The radiative losses are then \( \sim 6 \times 10^{-22} n_e^2 \) (in ergs cm\(^{-3}\) s\(^{-1}\) if \( n_e \) is in cm\(^{-3}\)) (see Fig. 10 of Rosner, Tucker, and Vaiana 1978). If the heating occurs at constant density, then \( n_e = 6 \times 10^{10} \), and the radiative losses are 2.2 ergs cm\(^{-3}\) s\(^{-1}\). Our predicted heating rate cannot sustain these losses, and we conclude that radiative losses prevent the resonances from heating spicules to EUV-emitting temperatures at constant density. However, if the heating occurs at constant pressure, then \( n_e \) would decrease by a factor of about 10 as the spicule is heated from Hz-emitting temperatures to EUV-emitting temperatures. The radiative losses are then 0.02 ergs cm\(^{-3}\) s\(^{-1}\), which is again comparable to our estimates of the resonant heating. We can only conclude that the resonances may be responsible for heating spicules to EUV-emitting temperatures if the heating occurs at constant pressure, but the radiative losses represent a major unknown.

A cautionary note: Equations (33)-(36) are valid only if there is a distinct resonance peak, unblended with neighboring resonances. Figure 3c shows an example where this condition is satisfied for the \( n = 1 \) resonance, but it fails for the higher order resonances. In such cases it is necessary to use the full result for the transmission coefficient, based on equations (30) and (31). Figure 3 indicates that the transmission coefficient becomes quite large at high frequencies, \( \omega \gtrsim 0.2 \ \text{s}^{-1} \). Thus most of whatever high-frequency power is generated by the convection zone can actually be available for spicule heating.

The velocities in the spicule can be computed in detail from equation (29), even for complex \( k_e \). However, we shall here content ourselves with a rough estimate. Since \( E_H \) refers to the heating averaged over the length of the spicule, we can write

\[
E_H = \frac{2\omega |k|}{k_e} \rho_e \langle \delta v_e^2 \rangle,
\]

where \( \langle \delta v_e^2 \rangle \) represents an average over time and over the length of the spicule; equation (38) assumes equipartition between magnetic and kinetic energies. Taking \( E_H = 0.04 \) ergs cm\(^{-3}\) s\(^{-1} \), \( \rho_e = 10^{-13} \) g cm\(^{-3} \), and \( \omega \approx 0.08 \) s\(^{-1} \) (the \( n = 1 \) resonance) yields

\[
\langle \delta v_e^2 \rangle^{1/2} = 15.8 (|k|/|k_e|)^{1/2} \ \text{km s}^{-1}.
\]

The predicted spicule velocities are compatible with the observed twisting velocities (Beckers 1972 quotes turbulent velocities of some 20 km s\(^{-1} \), while Pasachoff, Noyes, and Beckers 1968 give 30 km s\(^{-1} \) as an upper limit on the twisting velocities) if \( |k|/|k_e| \gtrsim 0.25 \).

IV. HEATING MECHANISMS

Hollweg (1984a) made speculations regarding the turbulent heating of coronal loops. Here we will apply similar arguments to the spicule heating problem.

Hollweg pointed out that Alfvén waves in a resonant cavity can be unstable to the Kelvin-Helmholtz instability, in virtue of the shearing motions intrinsic to the Alfvén wave. He further suggested that the Kelvin-Helmholtz instability could initiate and mediate a Kolmogorov turbulent cascade to higher transverse wavenumbers, where energy can be dissipated via microscopic processes, such as viscosity. The turbulent heating rate is, approximately,

\[
E_H \approx \rho_e k_{\perp o} \langle \delta v^2 \rangle^{3/2},
\]

where \( k_{\perp o} \) is the "outer" transverse wavenumber, i.e., the wavenumber at which energy is injected.

Unfortunately, the value of \( k_{\perp o} \) must be guessed. A conservative guess is \( k_{\perp o} = 2\pi/k_{\perp} \) (spicule diameter). Thus \( k_{\perp o} = 6.3 \times 10^{-5} \) cm\(^{-1} \) for a spicule diameter of 100 km. If we take Beckers's (1972) values for the turbulent velocities in spicules, viz., \( \langle \delta v^2 \rangle^{1/2} = 20 \) km s\(^{-1} \), we find \( E_H = 0.05 \) ergs cm\(^{-3}\) s\(^{-1} \). This value of \( E_H \) is sufficient to heat spicules to EUV-emitting temperatures, as discussed in the previous section. It is also comparable to the value of \( E_H \) deduced from equation (36). We conclude that Kolmogorov turbulence can dissipate the Alfvén wave fluxes which this paper predicts to enter spicules, and that the dissipation is consistent with the observed nonthermal motions on spicules. Note that equation (40) predicts more heating per particle near the top of the spicule, where the velocities are predicted to be largest. Our model may therefore account for observations indicating that spicules are hotter near their tops (e.g., Athay 1976, p. 440). However, other models also yield higher temperatures near the spicule tops (Hollweg 1982; Suematsu et al. 1982).

It should also be mentioned that ion-neutral frictional heating (Piddington 1956; Osterbrock 1961) may also play a role in heating spicules.

V. DISCUSSION

In this study we have suggested a possible explanation for the heating and rotational motions of spicules. The transition region with the corona forms the upper boundary of the spicule, while the chromosphere forms the lower boundary. In this way the spicule can be thought of as a cavity. We have shown that torsional Alfvén waves generated in the lower regions of the solar atmosphere can enter this cavity, and that the transmission amplitude of these waves into the spicule can be large at certain selected resonant frequencies. Large fluxes of energy enter the cavity and thereafter heat the spicule if dissipated. One possible dissipation mechanism is as follows: the torsional twists introduced at the base of the spicule induce shearing motions susceptible to Kelvin-Helmholtz instabilities, which in turn generate Kolmogorov turbulent rolls that cascade to ever-higher transverse wavenumbers and are subsequently dissipated via microscopic processes.

An interesting by-product of our work is a suggested explanation for the presence of EUV spicules and the fading of Hz spicules. We predict that spicules can be heated by Alfvénic twists at a rate of some 20,000 degrees in about 10 s. This rate is too high to conform to observed values, but we have assumed that all the energy goes into heat, when in fact some losses due to ionization and radiation would be expected. In any case, our scheme indicates that the spicule could continue to be heated even after its maximum height has been obtained. As the temperature continues to increase, the degree of ionization will increase. The intensity of the Hz
radiation will correspondingly decrease, and the spicule will fade from view if observed in Hz. If the spicule continues to be heated and attains a temperature of $1-2 \times 10^5$ degrees, emission in the EUV can be expected.

It is interesting to note that Dere, Bartoe, and Brueckner (1983) observed "chromospheric jets" which have properties similar to those of spicules, but with much shorter lifetimes ($\sim 40$ s). Our ideas could be consistent with these observations, since we suggest that the EUV emissions only occur when the spicule is near the end of its life. It is also interesting to note that Withbroe (1983) has called attention to some statistical similarities between EUV and Hz spicules. His results are consistent with our suggestion that Hz spicules can be Alfvenically heated to temperatures at which they eventually become EUV spicules.

Several aspects of the spicule phenomenon have not been addressed. We assume from the start that the spicule has been raised to a given height. The Introduction cites several recent ideas regarding spicule generation. For example, in HJG large-amplitude Alfvenic twists on flux tubes generate upward-propagating fast shocks in the chromosphere, which lead to structures resembling spicules, but which are not of spicular temperature. It is conceivable that such a process initiates spicule formation, while the smaller amplitude resonant processes discussed in this paper heat the spicule and superpose additional twisting motions on the spicule cavity. The present work is particularly reminiscent of HJG in the sense that the flux tube twisting is basically the same in both.

There is some question as to the origin of the twists on the flux tubes. Here we merely postulated their existence. However, some workers have suggested that twisted and untwisting flux tubes are fairly ubiquitous on the Sun (see, e.g., Piddington 1981; Parker 1982). A testable prediction of this paper is that the motions undergo torsional oscillations at the preferred resonant frequencies. The periods of the fundamental resonances turn out to be $\sim 100$ s, and so several complete oscillations would be expected in a spicule lifetime. Whether or not such motions have been observed is not clear. Pasachoff, Noyes, and Beckers (1968) did report some time dependence of the inclinations of some spicule spectral lines, but more detailed observations are needed before any conclusive statements can be made in this regard.

We also had to postulate the strength of the magnetic field on the spicule. We choose a value of $40$ G, corresponding roughly to average field strengths in the network. But this value is really a guess, since no data exist concerning the actual field strengths on spicules. However, it should be noted that a smaller value of $B_0$ would result in longer resonant periods. If this were the case, we would have to abandon our assumption that the spicule can be treated as quasi-steady, since that assumption requires that the resonant periods be small compared with the spicule lifetime.

We have benefited from discussions with Drs. J. Ionson, E. Priest, B. Roberts, and C. Smith. Part of this work was carried out while J. Hollweg was a guest of the Applied Mathematics Department of the University of St. Andrews, Scotland. He is grateful to the Science and Engineering Research Council of the UK for partial support, and to Drs. E. Priest and B. Roberts for their hospitality. This work has also been supported by the NASA Solar-Terrestrial Theory Program under grant NAGW-76, and by NASA grant NSF-7411.

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