A STUDY OF SOLAR PHOTOSPHERIC LIMB-DARKENING VARIATIONS

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ABSTRACT

We have obtained regular observations of photospheric limb darkening, using the McMath Solar Telescope, to study possible slow changes in the global temperature structure, in $T_{\text{eff}}$, and in the ultraviolet continuum flux from the quiet Sun. This paper reports on the analysis of data obtained on 15 days between 1980 September and 1982 December in a continuum window at $\lambda 4451$. We find no variations of global limb darkening exceeding 0.1% at the 99% confidence level. Variations reported by Rosen et al. (1982) were caused by inclusion of faculae near the limb. Interpreting these results with a model atmosphere, we find the relatively loose limits of $|\Delta T_{\text{eff}}| \leq 14$ K, or $|\Delta S/S| \leq 1\%$ during these 2 years. Our continuum observations easily rule out a change in temperature near $\tau_{0.5} = 1$ as large as inferred by Livingston and Holweger (1982) from equivalent width monitoring between 1976 and 1980.

The absence of significant change in continuum limb darkening at $\lambda 4451$ suggests that time variations of global convective efficiency at the top of the convection zone were less than 15%, but more realistic modeling of the transition between convective and radiative energy transport regimes near $\tau_{0.5} = 1$ is necessary to substantiate this constraint. Closer comparison of our continuum data with the line equivalent width measurements will be necessary to investigate possible slow changes in 0.2-0.3 $\mu$m ultraviolet flux from the quiet Sun, although the contribution of such global changes appears to be small compared with the changes caused by plage area variations over the sunspot cycle.

Subject headings: Sun: atmosphere — Sun: general — Sun: limb darkening

1. INTRODUCTION

The temperature distribution with optical depth in the photosphere, $T(\tau)$, determines both the intensity profile across the solar disk in visible continuum, $I(\mu)$, and the emergent radiative flux. This relationship suggests that slow changes in solar luminosity might be detectable by measuring changes in the photospheric limb-darkening function. Such global measurements would be insensitive to local changes of photospheric heat flow in magnetic spots and faculae (Wilkinson et al. 1981; Foukal and Vernazza 1979). But over time scales comparable to the solar cycle, precise relative photometry of limb darkening might reveal global changes of $T(\tau)$ and $T_{\text{eff}}$ that could be difficult to detect by radiometric measurements of the solar irradiance. For instance, Livingston and Holweger (1982) have suggested that systematic equivalent width changes observed in certain Fraunhofer lines may be caused by a global change of convective efficiency at constant $T_{\text{eff}}$. Their proposed variation of $T(\tau)$ would have easily observable consequences for $I(\mu)$.

Investigation of limb-darkening variations is also motivated by continuing uncertainty in the 11 year variation of solar ultraviolet fluxes between roughly 0.2 $\mu$m and 0.3 $\mu$m. The reproducibility of rocket-borne radiometry is no better than 10%-30% over this spectral range of direct importance to ozone chemistry (Donnelly 1977). Empirical models of the ultraviolet flux temporal behavior (e.g., Lean et al. 1982) assume that variations are caused only by plages and enhanced network, while the emission from the quiet photosphere remains constant. A series of limb-darkening measurements in the visible provides some test of this assumption since the contribution functions of UV emissions at $\lambda > 0.2$ $\mu$m have an appreciable component where the visible continuum is formed, i.e., $\tau_{0.5} > 0.2$ (Vernazza, Avrett, and Loeser 1981).

The first systematic investigation of limb-darkening variations was based on 13 years of regular observations between 1907 and 1920 at Mount Wilson (Abbot 1922). Although both Abbott's limb-darkening photometry and solar irradiance radiometry made pioneering contributions, their precision was insufficient to demonstrate any consistent relationship between changes in $I(\mu)$ and $T_{\text{eff}}$. Considerable advances have since been made in the techniques of limb-darkening photometry (e.g., Pierce and Waddell 1961; Pierce and Slaughter 1977; Wittmann 1980; Mitchell 1981) and in limb-darkening interpretation through photospheric modeling (Allen 1978). These considerations led us to initiate in 1980 a regular program at Kitt Peak National Observatory (KPNO) to study slow variations of $I(\mu)$ (Rosen et al. 1982). In § II of this paper we describe our instrumentation and observation procedures.

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Section III treats the reduction procedure and the data base analyzed here. In § IV we present the results and evaluate error sources. In § V we discuss the implications of our measurements for slow changes in solar luminosity, convection zone structure, and ultraviolet flux. Our conclusions are stated in § VI.

II. INSTRUMENTATION AND OBSERVING PROCEDURES

Our limb-darkening observations were obtained at KPNO using the McMath telescope and 13.7 m spectograph to make photometric drift scans across the solar disk. We used the drift-scan technique (in which the optical path through telescope and atmosphere remains constant during a scan) rather than two-dimensional imaging or telescope scanning to avoid air-mass or telescope vignetting corrections. The 2.5 minutes' duration drift scans were made along the celestial E-W diameter of the Sun by stopping the telescope drive and allowing the mass or telescope vignetting corrections. The 2.5 minutes' limb-darkening profile. Additional scans were made several 77 cm diameter solar image to drift across the spectrograph entrance slit. This drift path is coincident with a diameter of the solar semidiameter.

To begin, we produced daily mean scans for each of the 15 days by averaging individual scans, each corrected for zero point, changes in atmospheric transmission, image motion, seeing, and scattered light. The scans also contain noise due to solar granulation, faculae, and sunspots. The steps taken to remove or reduce these effects are described below. Throughout the remainder of this paper we use interchangeably the terms intensity and limb darkening, denoted by $I(\mu)$, to mean the intensity normalized by the disk center intensity, $I_0$. The disk center intensity is determined by least squares scaling of a polynomial limb-darkening function to the data near disk center ($\mu \geq 0.6$). Our aim in this study is to detect global limb-darkening changes, i.e., those which are symmetric about disk center and which are a smoothly varying function of $\mu = \cos \theta$.

The results reported are based upon 711 drift scans of the Sun and 46 drift scans of the solar aureole obtained on 15 days during 1980–1982 (Table I).

To begin, we produced daily mean scans for each of the 15 days by averaging individual scans, each corrected for zero point, scaled by the scan maximum intensity, and shifted to co-align the disk centers. The position of the disk center is the average position of the observed limb inflection points. Scans affected by clouds or detector malfunction were excluded from analysis. A typical raw drift scan is shown in Figure 1, and the 15 daily mean disk scans are shown in Figure 2.

TABLE I

<table>
<thead>
<tr>
<th>SCAN No.</th>
<th>RUN. No.</th>
<th>DATE (mo/day/yr)</th>
<th>NO. SCANS</th>
<th>INTENSITY STANDARD DEVIATION ($\times 10^4$)</th>
<th>BLUR FWHM (arcsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>9/27/80</td>
<td>48</td>
<td>101</td>
<td>4.1</td>
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<td>4.8</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>8/25/81</td>
<td>18</td>
<td>147</td>
<td>2.4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>8/26/81</td>
<td>29</td>
<td>124</td>
<td>2.7</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>2/19/82</td>
<td>62</td>
<td>135</td>
<td>5.3</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>2/20/82</td>
<td>83</td>
<td>141</td>
<td>4.8</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>2/21/82</td>
<td>42</td>
<td>139</td>
<td>4.0</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>8/09/82</td>
<td>81</td>
<td>69</td>
<td>3.9</td>
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<tr>
<td>9</td>
<td>9</td>
<td>8/10/82</td>
<td>19</td>
<td>116</td>
<td>3.5</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>8/15/82</td>
<td>18</td>
<td>119</td>
<td>3.1</td>
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<tr>
<td>11</td>
<td>11</td>
<td>8/16/82</td>
<td>21</td>
<td>131</td>
<td>2.7</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>8/17/82</td>
<td>18</td>
<td>104</td>
<td>4.1</td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td>12/14/82</td>
<td>51</td>
<td>93</td>
<td>5.2</td>
</tr>
<tr>
<td>14</td>
<td>14</td>
<td>12/16/82</td>
<td>73</td>
<td>116</td>
<td>4.0</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>12/17/82</td>
<td>56</td>
<td>112</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Average 47.4 118 3.8

A polynomial representation of the limb darkening, $I(\mu)$, was subtracted from the 15 daily mean disk scans. The poly-
Fig. 1.—A typical drift scan plotted against scan angle \( \phi/\phi_0 \). The intensities plotted here have been corrected for zero point and normalized. Representative values of \( \mu = \cos \theta \) are marked along the following half of the scan.

The heliocentric angle, \( \theta \), of a point on the solar surface, observed at a scan angle \( \phi \), was calculated in the approximation that the Sun is infinitely distant so that \( \theta = \sin^{-1} (\phi/\phi_\odot) \), where \( \phi_\odot \) is the scan angle of the solar limb. Faculae and sunspots were identified by inspecting plots of the daily mean intensity residuals. Figure 3 illustrates the removal of faculae and sunspots for both a typical day and an exceptional day when there were few faculae within the scan. The disk positions of our continuum faculae agree well with extended magnetic regions seen in daily KPNO magnetograms, although we did not rely on these magnetograms.

The daily scans were corrected for scattered light as illustrated in Figure 3 and described in Appendix A. The correction is calculated from a fit to observations of the solar aureole extending to a distance of \( 1.5 \phi_\odot \) beyond the limb. The size of the typical correction is illustrated by the dashed line in Figure 3. It varies smoothly from disk center to the limb and attains a maximum value of order \( 10^{-3} \) to \( 10^{-2} \) depending upon the observing run. The large-scale scattering correction is probably accurate to 10% of the peak aureole intensity (i.e., \( 10^{-3} \) for the days of strongest scattering to \( 10^{-4} \) for the days of least scattering), as suggested by an observed asymmetry between the preceding and following aureole (Table 1, col. [8]), and from examination of the aureole fit residuals. Seeing (instantaneous blur and image motion) has a negligible effect upon the limb-darkening photometry except near the extreme limb (Appendix A). We have not corrected the photometry for the effects of seeing, but have instead rejected all data closer to the limb than \( \mu = 0.15 \).

Plots of daily mean intensity residuals for the preceding and following semidisks were compared to test for detector hysteresis as noted by Rosen et al. (1982). As discussed in Appendix B and illustrated in Figure 10, data obtained on seven days (other than the 15 days reported here) with an ITT FW130 photomul-

Fig. 2.—A plot of the 15 daily average limb-darkening scans analyzed in this study. The scans are identified (sequentially in time) by their scan numbers assigned in Table 1. Sunspots and faculae near the limbs are especially apparent. The scans are offset, and each scan zero level is marked, but the linear flux scale is only labeled for scan number 1.
Fig. 3.—Removal of sunspots, faculae, and scattered light for two representative days. The data plotted represent residual intensity after subtraction of a mean polynomial, binned in 500 equal increments of $\mu = \cos \theta$. The dashed line shows the scattered-light correction to be subtracted. Panels (a) and (c) show the two days' data before rejection of spots and faculae and correction for scattered light; panels (b) and (d) show the final data.
Table 2

AVERAGE LIMB DARKENING

A. BEST FIT POLYNOMIAL COEFFICIENTS

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>1.18430</td>
</tr>
<tr>
<td>$u_2$</td>
<td>-0.60222</td>
</tr>
<tr>
<td>$u_3$</td>
<td>0.29796</td>
</tr>
<tr>
<td>$u_4$</td>
<td>-0.02886</td>
</tr>
<tr>
<td>$u_5$</td>
<td>-0.01958</td>
</tr>
</tbody>
</table>

B. OBSERVED LIMB DARKENING

<table>
<thead>
<tr>
<th>$\cos \theta$</th>
<th>Approximate Bin Width (arcsec)</th>
<th>Observed Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.975</td>
<td>300</td>
<td>0.9832</td>
</tr>
<tr>
<td>0.925</td>
<td>119</td>
<td>0.9501</td>
</tr>
<tr>
<td>0.875</td>
<td>87</td>
<td>0.9163</td>
</tr>
<tr>
<td>0.825</td>
<td>70</td>
<td>0.8821</td>
</tr>
<tr>
<td>0.775</td>
<td>59</td>
<td>0.8477</td>
</tr>
<tr>
<td>0.725</td>
<td>51</td>
<td>0.8123</td>
</tr>
<tr>
<td>0.675</td>
<td>44</td>
<td>0.7760</td>
</tr>
<tr>
<td>0.625</td>
<td>38</td>
<td>0.7398</td>
</tr>
<tr>
<td>0.575</td>
<td>34</td>
<td>0.7022</td>
</tr>
<tr>
<td>0.525</td>
<td>30</td>
<td>0.6645</td>
</tr>
<tr>
<td>0.475</td>
<td>26</td>
<td>0.6251</td>
</tr>
<tr>
<td>0.425</td>
<td>23</td>
<td>0.5852</td>
</tr>
<tr>
<td>0.375</td>
<td>19</td>
<td>0.5428</td>
</tr>
<tr>
<td>0.325</td>
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<tr>
<td>0.175</td>
<td>9</td>
<td>0.3577</td>
</tr>
</tbody>
</table>

tiplier tube were the only data to exhibit hysteresis in these plots. For this reason, all the data obtained with that detector were rejected from further analysis.

The daily average results for the 711 scans with no detectable hysteresis are summarized in Table 1. Columns (1) and (2) assign day and run numbers to each daily average scan. Columns (3) and (4) give the date and the number of scans averaged. Column (5) has the standard deviation of the observed intensity near disk center ($\mu \geq 0.95$). Column (6) gives the FWHM of the intensity derivative peak near the limb, which is a measure of the average seeing width. The last four columns provide information on the aureole. The intensity at 1.1 $\phi_0$ and the intensity difference between the preceding and following aureole at 1.07 $\phi_0$ are in columns (7) and (8). Either the power-law index or the exponential e-folding width of the scattering function (Appendix A) is listed in column (9) or column (10), respectively.

Table 2A gives the coefficients $u_i$ of the polynomial $I_p(\mu)$ defined in equation (1), as determined from fitting the intensities given in Table 2B. These intensities are the weighted average of the 15 daily mean scans described in Table 1. The intensity at a given value of $\mu$ is determined from the intensities measured over a range $\Delta \mu = 0.05$ centered at that value.

IV. VARIABILITY OF THE DRIFT SCANS

a) Global Solar Variations: 2 Year and 6 Year Limits

The variations seen between our six runs are shown in Figure 4 as residuals $\Delta I(\mu)$ from the polynomial $I_p(\mu)$ of Table 2A. The residuals of the weighted run average scan were binned over intervals $\Delta \mu = 0.05$. To test for low spatial frequency changes in global limb darkening that might be obscured by the bin-to-bin variations evident in Figure 4, we fitted the residuals $\Delta I(\mu)$ with a second-degree Legendre polynomial sum. A Legendre polynomial fit was chosen to reduce cross talk between the coefficients, while avoiding any specific assumptions regarding the shape of $\Delta I(\mu)$. The F-test (Bevington 1965) was used to determine whether the residuals from this fit have a significantly smaller variance than the run residuals $\Delta I(\mu)$ themselves. The F-values, degrees of freedom, and values of the Legendre polynomial coefficients $a_0$, $a_1$, and $a_2$ are given in Table 3. None of the six runs shows a significant second-degree polynomial limb-darkening perturbation at the 99% confidence level. Second-degree limb-darkening changes of amplitude below $1 \times 10^{-3}$ would be undetectable at the 99% confidence level given the noise in our data.

Pierce and Slaughter (1977) determined the coefficients of a fifth-degree polynomial representation of the limb darkening in the $\lambda 4451$ continuum window based upon a fit to a single scan obtained in 1974 using the McMath telescope and a technique.
TABLE 3

Legendre Polynomial Fits to Limb-Darkening Residuals

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Date (mo/yr)</th>
<th>No. Days</th>
<th>No. Scans</th>
<th>F*</th>
<th>a₀ (× 10⁴)</th>
<th>a₁ (× 10⁴)</th>
<th>a₂ (× 10⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9/80</td>
<td>1</td>
<td>48</td>
<td>0.51</td>
<td>13</td>
<td>-3.56</td>
<td>1.40</td>
</tr>
<tr>
<td>2</td>
<td>10/80</td>
<td>1</td>
<td>92</td>
<td>4.96</td>
<td>13</td>
<td>8.17</td>
<td>-1.30</td>
</tr>
<tr>
<td>3</td>
<td>8/81</td>
<td>2</td>
<td>47</td>
<td>0.79</td>
<td>14</td>
<td>-3.87</td>
<td>2.72</td>
</tr>
<tr>
<td>4</td>
<td>2/82</td>
<td>3</td>
<td>187</td>
<td>2.94</td>
<td>14</td>
<td>8.17</td>
<td>-1.30</td>
</tr>
<tr>
<td>5</td>
<td>8/81</td>
<td>2</td>
<td>157</td>
<td>3.14</td>
<td>14</td>
<td>-4.28</td>
<td>4.06</td>
</tr>
<tr>
<td>6</td>
<td>12/82</td>
<td>3</td>
<td>180</td>
<td>5.64</td>
<td>14</td>
<td>1.06</td>
<td>2.64</td>
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<tr>
<td>Total</td>
<td></td>
<td>15</td>
<td>711</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

* Residuals from polynomial of Table 2A. Fits are made to data between 0.15 < μ < 1.0 as binned in Table 2B.

The value of F must exceed 6.74 for ν = 13 and 6.55 for ν = 14 to achieve the 99% confidence level for the hypothesis that the coefficients a₀, a₁, and a₂ differ significantly from zero; i.e., that they represent an improvement in the fit over that achieved by the average polynomial I(μ) defined in eq. (1) and Table 2A.

V. INTERPRETATION

a) Implications for the Time Behavior of T_eff and Total Irradiance, S

To express the change in limb darkening ΔI(μ) at 4451 that would accompany a small change in T_eff, we define a sensitivity function, ζ(μ):

\[ ζ(μ) = \frac{Δ \ln T_{\text{eff}}}{ΔI(μ)} \]  

To evaluate ζ(μ), we use limb-darkening curves computed from radiative-convective photospheric models (Kurucz 1979) with T_{eff} = 5750 K, 5770 K, and 5800 K. These models indicate ζ(μ) ≈ 2.5 over 0.4 < μ < 0.8. Since the 3σ limit on the amplitude of run-to-run variations allowed by our F-test is 1 × 10^-3, our observations rule out variations |ΔT_{eff}/T_{eff}| ≥ 2.5 × 10^-3, or |ΔT_{eff}| ≥ 14 K. The actual run-to-run variation

...
of $\Delta T_{\text{eff}}$ determined by fitting the observed variations $\Delta I(\mu)$ with equation (2) is given in Figure 6. The 1980 October run shows an increase of $\Delta T_{\text{eff}}$ with a formal significance around 99%; this run also shows a high formal significance in the $F$-test of Table 3. But that run consists of only 1 day of final data, and also, we see in Figure 4 that the residuals $\Delta I$ show a large peak at $\mu \leq 0.5$ which we suspect to be of facular origin. Certainly, the peak seen in Figure 4 is not of the functional shape expected for a change in global limb darkening. This limit on $\Delta T_{\text{eff}}$ implies that the component of total solar irradiance variation over the period 1980-1982 specifically due to global changes in the quiet photosphere is below $|\Delta S/S| \sim 1\%$.

The sensitivity function $I(\mu)$ used here is based upon a single-component photospheric model which does not fit the observed shape of $I(\mu)$ given in Table 2 to better than 1%-2%. However, this sensitivity function does not depend strongly on $T_{\text{eff}}$; thus we do not expect that the use of a 2-stream model composed of a somewhat hotter (granule) and a cooler (intergranule) component would significantly affect our estimate. Figure 7a shows the difference of our 15 day grand-average $I(\mu)$ and the Pierce and Slaughter polynomial $I(\mu)$ of 1974 with the limb-darkening variation due to the Livingston and Holweger $T(\tau)$ variation between 1976 and 1980. The observed limb-darkening difference is in the same sense as that expected from the $I/H$ increase reported between 1976 and 1980, but the 0.7% peak amplitude and shape of the limb-darkening change expected from the $I/H$ increase do not agree well with the observed difference between 1974 and 1980-1982. However, given the uncertainty of the 1974 scan (Fig. 7a), we conclude that limb-darkening variation is not necessarily inconsistent with the variation expected if $I/H$ had increased as reported by Livingston and Holweger (1982).

The comparisons given above show that continuum limb-darkening observations at $\lambda 4451$ are relatively sensitive to changes in $T(\tau)$ which peak near $\tau = 1$, where radiative-convective models indicate that variations in $I/H$ might affect $T(\tau)$ most strongly. The absence of any detectable temperature variation near $\tau = 1$ in our data does not necessarily indicate inconsistency with the equivalent width data. For one, W.
VI. SUMMARY AND CONCLUSIONS

The main finding of this study is that we see no evidence for significant photospheric limb-darkening variations over a 2 year period near solar maximum in 1980–1982. We believe that our observations are sufficiently precise to detect global variations exceeding 0.1% in amplitude.

The relation between ΔI(μ) and ΔT_{eff} given in equation (2) is model-dependent. Relatively large changes in limb darkening can be produced by a few percent (of the total flux) variation of the convective flux around τ_{0.5} = 1 at constant effective temperature (see Fig. 7). Nevertheless, if an 0.1% upper limit on limb-darkening variation was to continue, it would seem to require ΔT_{eff} ~ 0 with roughly the tolerances implied by our sensitivity function, unless rather artificial assumptions are made on the redistribution of total energy flux through photospheric layers.

Our limits on ΔI(μ) during 1980–1982 rule out a change in photospheric temperature structure near τ = 1 as large as the 20 K suggested between 1976 and 1980 by Livingston and Holweger (1982). The difference in limb darkening between our data in 1980–1982 and the single scan of Pierce and Slaughter (1977) is large enough (and of the right sign) to accommodate the photospheric temperature gradient change suggested by Livingston and Holweger (1982), but the uncertainty of the single 1974 scan is sufficiently large that this difference can also be explained by granular and image motion noise. A closer comparison of our continuing limb-darkening observations with the equivalent width data being taken by Livingston will be necessary to determine whether the temperature changes reported in the lines formed near τ_{0.5} ~ 0.1 are associated with perceptible changes in the lower photospheric layers that we observed in the continuum.

Comparison of our limits on ΔI(μ) with the limb-darkening variations predicted when the mixing-length parameter l/H is changed in a standard convective atmosphere model argues against changes in global convective efficiency exceeding a few percent over the 1980–1982 time period. This appears to be comparable to the precision presently achieved in detecting l/H changes through interpretation of possible changes in the structure of high-degree 5 minute p-mode oscillations (E. J. Rhodes, private communication), although our data constrain only the very thin layer where convection is expected to be rapidly decreasing in efficiency. The theoretical treatment of the transition between convection and radiation requires more consideration to provide a better estimate of the temperature change near τ_{0.5} = 1 produced by changes in convective efficiency. The model developed by Nordlund (1982) seems to be suitable for such an improved estimate.

If changes in l/H exceeding a few percent are shown to cause variations in T(τ) at τ ~ 1 outside the limits placed by our photometry, it follows that our λ4451 photometry also constrains global changes in solar ultraviolet flux that might have occurred specifically through such variations of convective efficiency during 1980–1982. Holweger, Livingston, and Steenbock (1983) find variations of F_{λ} due to a 15% increase of l/H generally below 1% for 0.2 μm ≤ λ ≤ 0.3 μm, so we conclude that global changes during 1980–1982 were several times smaller.

The UV flux variation estimated from plage area changes during 1980–1982 considerably exceeds this level (J. L. Lean, private communication), so this conclusion supports the view that empirical models of UV flux variation in this spectral range can be constructed assuming the quiet photosphere to be invariant over the 11 year cycle. Nevertheless, even the quiet-Sun UV intensity could vary with little constraint from our photometry, if only layers above τ_{0.5} ≤ 0.2 (k ≥ 200 km) were involved. It is difficult to comment on such a variation given the present uncertainty regarding departures from radiative equilibrium in the high photosphere and the possible 11 year variability of wave heating in these layers.

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APPENDIX A

SCATTERED-LIGHT CORRECTIONS

Scans of the solar aureole shown in Figure 8 illustrate the range of peak aureole intensities (0.1%-1%) and the range of aureole forms encountered in our observations. To correct the disk scans for the scattered light implied by the aureole intensity distribution, we assume that the observed intensity distribution \( I' \) is the convolution of a point-spread scattering function \( \psi_s \) with the true intensity distribution \( I \) across the photospheric disk (David and Elste 1962; Zwaan 1965; Staveland 1972; Brahde 1974):

\[
I' = (1 - \epsilon)I + \epsilon \psi_s * I .
\]  

(A1)

Here, \( \epsilon \) is the scattered fraction of the true intensity, and the function \( \psi_s \) is assumed to be spatially symmetric and invariant. Column (8) of Table 1 demonstrates that the observed aureole asymmetry (preceding minus following intensity) is no greater than \( 2 \times 10^{-3} I_0 \) and is usually much less. Our observations show the aureole to be independent of hour angle, suggesting that it is caused principally by instrumental scattering.

The aureoles observed by us are of two basic forms, which may be represented, respectively, by the convolution of a limb-darkening function with the power-law scattering function

\[
\psi_s(r) = A_1/(r_0 + r) \]  

(A2)

or with the exponential scattering function

\[
\psi_s(r) = A_2 \exp(-r/w) .
\]  

(A3)

The power-law scattering function produces aureoles with distinct curvature in a semilog plot such as Figure 8, whereas the exponential scattering function produces linear aureoles in such a plot. Power-law scattering functions have been found by most previous investigators to represent the aureole intensity distribution on angular scales of order 10' (David and Elste 1962; Staveland 1972; Pierce and Slaughter 1977). An exponential scattering function has apparently not been utilized previously to represent the aureole on this angular scale, although Kinman (1953) has applied it to arcsec scale scattering at the limb.

The values of \( A_1, A_2 \) are set by the requirement that total flux be conserved by scattering; we have normalized \( \psi_s \) over 4\( \pi \) steradians. To determine the daily parameters of the scattering function, we convolved trial functions for a range of scattering parameters with our best estimate of the true limb-darkening function. For the power-law fits, we fixed \( r_0 = 20' \). The exact value of \( r_0 \) turns out to be unimportant in determining scattering over the scales of interest here. The convolution was performed numerically with 32 point Gauss-Legendre quadratures. To attain greater accuracy, the domain of integration was separated into a core and a wing scattering region with boundaries similar to those of Brahde (1972). The good fit to the aureole intensities achieved by this procedure is illustrated for five representative days in Figure 8.

---

Fig. 8.—Five representative aureole scans (daily averages) shown together with their respective power-law or exponential fits (open circles). The zero of scan angle is at disk center. Note that aureoles such as 1 and 4 show significant curvature on this semilog plot (and are well fitted using power-law scattering kernels), while other aureoles, such as 5, 6, 7, are straight and well fitted by exponential kernels.
Once the best fit $\psi_s$ function has been determined for a given day, we calculate the scattered-light correction $\Delta I_s(\mu)$ to be subtracted from a daily mean scan:

$$\Delta I_s(\mu) = I_s(\mu) - I_0(\mu).$$

The corrections $\Delta I_s(\mu)$ for two typical days are shown in Figure 3. The broad minimum of $\Delta I_s(\mu)$ near $\mu = 0.5$ can be understood (for the power-law kernel) by considering that for indices $\alpha < 2$ the long range of the scattering function scatters more energy into the beam from bright areas of the disk than is lost from the unscattered beam.

We have applied similar procedures to evaluate the effect of short-range Gaussian seeing upon the limb-darkening observations. The corrections for $2''$ and $6''$ FWHM Gaussians are plotted in Figure 9. The observed FWHMs presented in column (6), Table 1, are bracketed by this range of blurring. The effect of this redistribution is negligible for $\mu \gtrsim 0.15$, which we have taken, therefore, as the boundary of the bin closest to the limb to be analyzed.

**APPENDIX B**

**EVALUATION OF DETECTOR GAIN STABILITY**

Two photomultipliers and a photodiode were tried during this project in an effort to avoid detector fatigue that was noted in our first observations (Rosen et al. 1982). To check for fatigue in the data, we plotted the preceding and following semidisk residuals separately for each day. Figure 10a illustrates the behavior typically exhibited by the daily average curves on the 15 days utilized in this limb-darkening study. These residuals show a random bin-to-bin variability within roughly 0.3% of the zero level within both halves of the scan. In contrast, Figure 10b shows a systematic deviation of 1% between the preceding (west) and following halves of the scan. Anomalous deviations, such as shown in Figure 10b, occurred only when an ITT FW130 photomultiplier tube was used. This is consistent with the observation of Young (1974) that the slow movement of surface charges near the dynode support structure may more readily cause variable gain in box-type dynode chains (as employed in the ITT FW130) than in venetian blind–type dynode chains (such as employed in the EMI 9750). The contrast between the results obtained with the ITT FW130 and the EMI 9750 photomultiplier tubes is especially clear during the 4 consecutive days of a run in 1980 September. The first 3 days of data obtained with the ITT FW130 all show particularly bad hysteresis; the data from one of these days are illustrated in Figure 10b. The fourth day’s data, taken with the EMI 9750 (Fig. 10a), showed much more regular gain behavior.
APPENDIX C

CONTRIBUTION OF GRANULATION TO THE VARIANCE OF SPATIALLY BINNED, TIME-AVERAGED DRIFT SCANS

We wish to calculate the contribution of the partially coherent (in both space and time) solar granulation intensity pattern to the variance of a binned daily mean drift scan. We assume that a single drift scan is obtained rapidly (with respect to the granulation coherence time), and that \( N \) of these are obtained at regular time intervals, \( \Delta t \). We first compute the variance of the average intensity within a portion of a scan, and then we compute the variance of the mean of \( N \) such averages.

The observed binned intensity, \( I'(x, y) \), is the combined result of smearing the intrinsic intensity pattern, \( I(x, y) \), by Gaussian seeing, \( g(x, y) \), observing through a slit of height \( h \) (oriented perpendicular to the scan direction), and averaging over a distance \( l \) along the direction of scan:

\[
I'(x, y) = I(x, y) \ast g(x, y) \ast \left[ (h/t) \Pi(x/t) \Pi(y/h) \right],
\]

where \( \Pi(x) \) is the one-dimensional rectangle function (Bracewell 1965), and \( g(x, y) = b^{-2} \exp \left[ -\pi (x^2 + y^2)/b^2 \right] \). The variance of \( I'(x, y) \) may be expressed in terms of its power spectrum (Bracewell 1965):

\[
\text{var} \left[ I'(x, y) \right] = \int_{-\infty}^{\infty} \left| \tilde{I}(u, v) \right|^2 \text{d}u \text{d}v.
\]

Solar granulation is a randomly varying pattern, and, as such, the observed power spectrum may be expressed (Lee 1960)

\[
\left| \tilde{I}(u, v) \right|^2 = \left| \tilde{I}(u, v) \right|^2 |\tilde{g}(u, v) \text{sinc}(lu, hv)|^2 {h^2}.
\]

The power spectrum of the intrinsic disk brightness at \( \mu = \cos \theta \) is adequately represented by a Gaussian (Edmonds 1962), which may be written (taking into account projection)

\[
\left| \tilde{I}(u, v) \right|^2 = \sigma_0^2 \mu s_0^2 \exp \left[ -\pi (u^2 \mu^2 + v^2 s_0^2) \right].
\]

The intrinsic rms, \( \sigma_0 \), and spatial scale, \( s_0 \), are given in Table 4 as determined from data supplied by Edmonds (1962) and from the observation that the contrast of the granulation is zero \( \sim 15'' \) from the limb. We have not taken into account the effect of different observation wavelengths.

After some manipulation, the variance of the average intensity in a bin of length \( l \) and height \( h \) may be written

\[
\text{var} \left[ I' \right] = \sigma_0^2 \mu s_0^2 P(2\pi b^2 + \pi \mu^2 s_0^2, h) P(2\pi b^2 + \pi s_0^2, h),
\]

where

\[
P(p, x) = x^{-2} \left( \frac{\pi}{p} \right)^{1/2} \left[ \exp \left( -x^2 \right) + \pi^{1/2} x \text{erf} \left( x \right) - 1 \right].
\]

To compute the variance of the mean of the coarsely sampled (in time) bin averages, we use the definition of the variance in terms of the ensemble expectation of a function of a random variable:

\[
\text{var} \left[ f(I') \right] = \text{E}[f(I')^2] - \text{E}^2[f(I')],
\]

where

\[
f(I') = \text{time average of the bin-average intensities} = \frac{1}{N} \sum_{j=1}^{N} I_j = \langle I' \rangle,
\]

\[
I_j = \text{mean intensity in a given } \mu \text{-bin of scan } j.
\]

We assume that we are dealing with the residual intensities so that \( \text{E}[\langle I' \rangle] = 0 \). Therefore, the variance of the time-average intensity is

\[
\text{var} \left[ \langle I' \rangle \right] = \text{E}[\langle I'^2 \rangle] = \frac{1}{N^2} \left[ \sum_k E(I_k^2) + \sum_{k \neq j} E(I_k I_j) \right].
\]

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<tr>
<td>0.15............</td>
<td>0.0</td>
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</tr>
</tbody>
</table>
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The first term may be written in terms of the variance of the individual observations, and the second may be written in terms of the normalized autocorrelation function (ACF):

$$
\text{var}(\langle I' \rangle) = N^{-2} \left\{ N \text{ var}(I') + \sum_{k \neq j} \phi(k-j) \Delta t \right\}. \tag{C8}
$$

This may be further simplified to

$$
\frac{\text{var}(\langle I' \rangle)}{\text{var}(I')} = 1 + \frac{1}{N^2} \sum_{k = -(N-1)}^{N-1} (N - |k|) \phi(k \Delta t) \tag{C9}
$$

The normalized, time-lagged ACF of solar granulation may be represented by an exponential (Bahng and Schwarzschild 1961):

$$
\phi(k \Delta t) = \exp\left(-\frac{|k| \Delta t}{\tau}\right) \tag{C10}
$$

with $\tau = 6.26$ minutes. Therefore, the variance of the daily mean bin intensity is

$$
\text{var}(\langle I' \rangle) = \frac{\text{var}(I')}{N^2} \sum_{k = -(N-1)}^{N-1} (N - |k|) e^{-|k| \Delta t / \tau} = \frac{\text{var}(I')}{N} Q\left(N, \frac{\Delta t}{\tau}\right) . \tag{C11}
$$

The function $Q$ expresses the effective reduction of the number of independent scans.

Collecting results, the variance of the binned, time-average intensity $\langle I' \rangle$ is

$$
\text{var}(\langle I' \rangle) = \frac{\mu_0^2}{N} Q\left(N, \frac{\Delta t}{\tau}\right) \left(2 \pi \sigma_b^2 + \pi \mu_0^2 \sigma_h^2 \right) . \tag{C12}
$$

The spatial scale and the variance of the solar granulation are uncertain by approximately 30% judging from the range of measured values (see a summary in Edmonds and Hinkel 1977). Edmonds (1962) finds a maximum at $\mu = 0.6$ of the relative variance of the granulation, while others (Pravdjuk, Karpinsky, and Andreiko 1974; Kell 1977; Albregtsen and Hansen 1977) find only a moderate decline of the relative variance from disk center to the limb. The calculated observed variance must be regarded to be similarly uncertain. We believe that 30% is a reasonable estimate for the expected disagreement between calculation and observation.

APPENDIX D

CONTRIBUTION OF IMAGE MOTION TO THE VARIANCE OF LIMB-DARKENING MEASUREMENTS

We represent the limb darkening by a polynomial in $\mu = \cos \theta$,

$$
I(\mu) = 1 + \mu \sum_i u_i (\mu^i - 1) , \tag{D1}
$$

and consider the effect of a small error in $\mu$ upon the calculated intensity:

$$
\text{d}I = \sum_i u_i \mu^{i-1} \text{d}\mu . \tag{D2}
$$

We may express $\mu$ directly in terms of the observed quantities $t$, $t_p$, and $t_f$, via $x \equiv \sin \theta$:

$$
\mu = (1 - x^2)^{1/2} , \quad x = (t - t_p)/t_R , \quad t_c = (t_p + t_f)/2 , \quad t_R = (t_f - t_p)/2 . \tag{D3}
$$

The times of observation are: $t$, time of observation of intensity at $\mu$; $t_p$, time of preceding limb passage; and $t_f$, time of following limb passage. Because of image motion, the observed quantities $t$, $t_p$, and $t_f$ are in error. Image motion is caused by atmospheric seeing and fluctuations in the wind loading of the exposed McMath telescope heliostat. The errors in the three measured times are uncorrelated, except near the limbs where $t \approx t_p$ or $t \approx t_f$. We take the standard deviation of the measured times to be the same, namely $\sigma$, and denote the corresponding scan angle error as $\sigma$. Propagating these equal, independent random errors via the above formulae, we find

$$
\sigma(I) = \frac{\sigma}{R_0} \left\{ \frac{1}{\mu^2 - 1} \right\}^{1/2} \left\{ \frac{4 - \mu^2}{2} \right\}^{1/2} \sum_i u_i \mu^i . \tag{D4}
$$

For our 15 days of observation, we find from column (6) in Table 1 that the mean image motion FWHM is 3\'8, or $\sigma = 1\'62$. It should be noted that the error in limb position measurement causes a systematic error in $I(\mu)$ for any scan, but this error is uncorrelated from scan to scan. The final intensity error given in equation (D4) contains a randomly varying high spatial frequency component as well as a systematic component due to limb position measurement errors.

REFERENCES


