RESONANT HEATING: AN INTERPRETATION OF CORONAL LOOP DATA

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ABSTRACT

We show that the resonant heating theory of Hollweg can be used to organize the coronal loop data of Golub et al. When combined with a reasonable form for the input power spectrum, the resonant heating theory is fully compatible with the loop data.

Subject headings: Sun: corona — Sun: magnetic fields — Sun: X-rays

I. INTRODUCTION

It is widely believed that the solar corona is heated magnetically, and several magnetic heating mechanisms have been advanced in recent years (for recent reviews see Hollweg 1981a, 1983; Kuperus, Ionson, and Spicer 1981; Parker 1983; Spruit and Roberts 1983). Observational verification or refutation of any proposed mechanism obviously requires information about the coronal magnetic field. Unfortunately, there is very little information available in this regard.

A notable exception is a study by Golub et al. (1980) which provides estimates of the magnetic field strength inside a selected set of coronal loop structures. The point of this Letter is to reexamine the data of Golub et al. and to show how the data can be organized and interpreted in terms of the resonant heating theory of Hollweg (1984a).

II. DATA

Golub et al. (1980) have presented observations of seven coronal active region loops and four X-ray bright points (which are also interpreted as being magnetically controlled loop structures). Two of the loops and one of the bright points were observed on successive days. Thus a total of fourteen data points were provided. For each of the loops, Golub et al. provide the loop length, the average total (i.e., electron plus proton) pressure inside the loop, and an estimate of the average magnetic field strength in the loop derived from photospheric magnetograms. The availability of the magnetic field values makes this data set unique. The relevant data are summarized in Table 1 which is derived from Table 1 of Golub et al. (L is full loop length, p is total pressure, and B is magnetic field strength). We have also added representative values for coronal large-scale structures (LSS), as given in Table 2 of Golub et al.

To proceed further, we use the coronal loop scaling relations obtained by Rosner, Tucker, and Vaiana (1978). When these relations are combined with the data, we can derive values for the average volumetric heating rate, $E_H$, and for the peak temperature, $T$, within the loop:

$$E_H = 1.8 \times 10^5 \rho^{1.7} L^{-0.83},$$

$$T = 1.1 \times 10^3 (pL)^{1/3},$$

(all units are cgs). A representative value for the loop density, $\rho$, can be derived with the aid of equation (2):

$$\rho = 5.5 \times 10^{-12} (p^2/L)^{1/3}.$$

Once $\rho$ is derived, we can compute a representative value for the Alfvén speed inside the loop:

$$v_A = B/(4\pi \rho)^{1/2},$$

$$v_A = 1.2 \times 10^5 B^{1/6} L^{-1/3}.$$
limited justification for our use of equation (2), and, by implication, for our use of equation (1).

III. COMPARISON WITH THEORY

Hollweg (1979, 1981b) and Zugzda and Locans (1982) showed that coronal loops can act as resonant cavities for Alfvén waves. The resonances occur when an integral number of half-wavelengths fit into the coronal part of the loop. The resonant frequencies, \( f_{\text{res}} \), are thus

\[
f_{\text{res}} = m v_A / 2 L, \tag{5}
\]

where \( m = 1, 2, 3, \ldots \). The period of the fundamental resonance, \( \tau_1 \), is given by

\[
\tau_1 = 2 L / v_A. \tag{6}
\]

This quantity is tabulated in Table 1. It was shown that the transmission of wave energy into the loops is greatly enhanced in the vicinity of the resonant frequencies; the effect is closely analogous to the transmission resonances in an interference filter.

Hollweg (1984a) studied the resonances analytically by solving the Alfvén wave equation in a three-layer model of the solar atmosphere. The three layers consisted of the coronal part of the loop, in which \( v_A \) was taken to be a constant, and a chromosphere-transition region at each end of the loop, in which \( v_A \) was assumed to vary exponentially with height. Hollweg (1984a, eqs. [52] and [66]) derived the following expression for \( E_H \):

\[
E_H = \Lambda B v_A L^{-3} \sum_{m=1}^{\infty} m P(\omega_m), \tag{7}
\]

where

\[
\Lambda = 2 \pi^{5/2} \rho_{\text{ph}}^{1/2} \delta v_{\text{ph}}^2 h_A. \tag{8}
\]

Here \( \rho_{\text{ph}} \) is the density in the photosphere (where the Alfvén waves are assumed to be generated), \( \delta v_{\text{ph}} \) is the rms velocity associated with the waves in the photosphere, \( h_A \) is the scale height of the Alfvén speed in the chromosphere-transition region, and \( P(\omega_m) \) is a weighting factor related to the input power spectrum evaluated at the (angular) frequency corresponding to the \( m \)th resonance. The normalization of \( P \) is

\[
\int_0^{\infty} P(\omega) d\omega = 1. \tag{9}
\]

Equations (7) and (8) assume that the wave dissipation in the corona dominates leakage out of the cavity. In that case \( E_H \) is independent of the wave damping rate. The existence of such a regime was suggested by Ionson (1982), and the conditions for its existence were discussed by Hollweg (1984a, b) and Ionson (1984). It is also assumed that the resonant peaks are unblended. Independent estimates indicate that these are reasonable approximations for the coronal loops; see the forthcoming more detailed study by Sterling and Hollweg (1984).

Using equation (4b), equation (7) can be rewritten as

\[
E_H' = \Lambda' \Sigma m P(\omega_m), \tag{10}
\]

where

\[
E_H' = p^{1/3} L_{10}^{1/6} B^{-2} E_H, \tag{11}
\]

\[
\Lambda' = 5.6 \times 10^{-24} \Lambda, \tag{12}
\]

and \( L_{10} \) is \( L \) measured in units of \( 10^{10} \) cm.

The data points in Figure 1 display the derived values of \( E_H' \) versus the derived values of \( \tau_1 \) for the fifteen loop structures tabulated in Table 1. Note that the data become well organized when presented in terms of \( E_H' \) and \( \tau_1 \). The organization extends over some 3.5 orders of magnitude of \( E_H' \) and over some 1.2 orders of magnitude of \( \tau_1 \). Other possible schemes for organizing the data have been investigated, but the organi-

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Fig. 1.—The quantity $E_{H}$ (eq. [11]) plotted as a function of the fundamental resonance period (eq. [6]) for the fifteen coronal loops given in Table 1. The open circles, filled circles, and cross denote active region loops, X-ray bright points, and the LSS respectively. The solid curve is a model fit to the data for an input power spectrum which is flat for periods longer than 60 s, but which falls off as $\omega^{-4}$ at higher frequencies.

zation presented in Figure 1 seems to be the most satisfactory (Sterling and Hollweg 1984).

Figure 1 indicates that $E_{H}$ is an increasing function of $\tau_{1}$. According to equation (10), this implies that $\Sigma mP(\omega_{m})$ is an increasing function of $\tau_{1}$. We propose a simple explanation for this result. Suppose that $P(\omega)$ is reasonably flat at low frequencies, but falls off rapidly with frequency above some cutoff frequency. Now, if a given loop has a low-frequency fundamental resonance (i.e., $\tau_{1}$ large), then the flat part of the input power spectrum can be "sampled" by the fundamental resonance and by some of the higher harmonics. Only the highest harmonics will be above the cutoff. Thus $\Sigma mP(\omega_{m})$ will be large. As the frequency of the fundamental resonance is increased, then more of the higher harmonics will appear above the cutoff, and $\Sigma mP(\omega)$ will decrease. Finally, if the fundamental frequency exceeds the cutoff frequency, then $\Sigma mP(\omega_{m})$ will very rapidly decrease as $\tau_{1}$ decreases. This is precisely the behavior revealed by Figure 1.

We have numerically calculated the expected behavior of $\Sigma mP(\omega_{m})$, and thus of $E_{H}$, for a model input power spectrum which is flat for periods longer than 60 s, but which falls off as $\omega^{-4}$ at higher frequencies. The results of this calculation are displayed as the solid line in Figure 1. It is evident that a rather simple model for $P(\omega)$ succeeds in reproducing the observed behavior of $E_{H}$. Of course, in view of the scatter in the data and the paucity of data points, it is impossible to rigorously infer the input power spectrum. For example, we have found that cutoff periods between 60 s and 100 s and high-frequency power laws between $\omega^{-3}$ and $\omega^{-4}$ lead to fits which are essentially as good as the fit displayed in Table 1. The main point here is that the resonance theory of Hollweg (1984a), when combined with a reasonable form for the input power spectrum, successfully organizes the data of Golub et al. (1980).

The solid curve in Figure 1 implies $\Lambda \approx 9.3 \times 10^{15}$ (cgs). If we take $h_{A} = 300 \text{ km}$ and $\rho_{ph} = 3 \times 10^{-14} \text{ g cm}^{-2}$, equation (8) implies $\delta u_{pb} = 1.3 \text{ km s}^{-1}$. This value assumes that only one end of the loop acts as a source of waves. If both ends of the loop launch equal amounts of wave energy into the corona, then $\delta u_{pb} = 0.9 \text{ km s}^{-1}$. These are reasonable values for the photospheric motions.

IV. CONCLUSION

The resonant Alfvén wave heating theory of Hollweg (1984a) has been shown to be capable of organizing the coronal loop data of Golub et al. (1980). From model fits of the theory to the data, it is possible to infer reasonable forms for the input power spectrum and reasonable values for the wave velocity amplitudes in the photosphere. We conclude that the resonant heating theory is fully consistent with the available data.

Our analysis has made some assumptions which require further verification. We have assumed that the loop scaling relations of Rosner, Tucker, and Vaiana (1978) can be used to infer the average loop heating rates and temperatures. And we have assumed that the photospheric conditions and input power spectra are the same for all loops in the data set. Departures from these assumptions may account for some of the scatter in the data points in Figure 1.

Our analysis indicates that the harmonics play an important role in determining the total heating rate of a coronal loop. It is insufficient to consider only the fundamental resonance, as has been done by Ionson (1982, 1984).

In a subsequent paper (Sterling and Hollweg 1984), we will discuss further implications of the resonant heating theory and how it relates to the data given in Table 1. And we will also discuss to what extent other coronal heating theories succeed (or fail) in organizing the data.

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REFERENCES


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