OBSERVATIONS OF THE BRIGHTNESS PROFILE OF THE SUN IN THE 30–200 MICRON CONTINUUM

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ABSTRACT

We observed the brightness profile of the quiet Sun in broad continuum passbands centered at 30, 50, 100, and 200 μm with a resolution of 2′. Weak radial darkening was seen at all four wavelengths near disk center. This reverses to brightening toward the limb in the 100 and 200 μm continuum. Radial darkening at 100 and 200 μm is not expected from smooth model chromospheres consistent with absolute brightness measurements. These results do not support a homogeneous model of the low chromosphere, where the temperature reversal occurs.

Subject headings: infrared: general — Sun: chromosphere — Sun: limb darkening

I. INTRODUCTION

Continuum intensity profiles of the quiet Sun have long proved useful for studying the structure of the solar atmosphere. The far-infrared and submillimeter continua are of particular interest, since they emanate in local thermodynamic equilibrium (LTE) from the low chromosphere, where non-radiative heating becomes important. The brightness temperature of solar radiation undergoes a minimum in the 100–200 μm region (Rast, Kneubuehl, and Müller 1978). This implies the existence of a temperature minimum low in the chromosphere, where this radiation is emitted.

All smooth chromospheric models consistent with infrared brightness temperature measurements predict significant radial brightening for these and longer wavelengths, which come from the hotter overlying layers. However, the amount of radial brightening observed at submillimeter and radio wavelengths has generally been less than predicted by smooth models. These discrepancies are thought to be due to rough structure in the middle and upper chromosphere. If this is so, it should be possible to use infrared intensity profiles not only as a supplement for studying the vertical temperature dependence of the chromosphere, but also as a diagnostic for unresolved fine-structure inhomogeneities in the chromosphere (cf. Simon and Zirin 1969).

Lindsey and Hudson (1976) found considerably less radial brightening in the 680 pm continuum than predicted. Righini and Simon (1976) found no significant brightening in the 350 and 450 pm continuum. Lindsey et al. (1981) found a flat radial profile in the 350 μm continuum with significant brightening confined to within 90° of the limb. Horne et al. (1980), however, found significant radial brightening at 1.3 mm, possibly consistent with a smooth chromosphere at somewhat higher elevations.

In this paper, we report measurements of the radial brightness profile of the quiet Sun near disk center made in the 30, 50, 100, and 200 μm continuum. The observations were made with the 90 cm NASA Kuiper Airborne Observatory (KAO). We found evidence of radial darkening of the Sun near disk center at all wavelengths. An explanation of this result is probably to be found in unresolved rough structure extending far down into the chromospheric temperature minimum region.

II. PROCEDURE

a) Instrumentation

The KAO telescope is a 90 cm Cassegrain with f/17 secondary optics. For solar observations, a filter of 6 mm thick high-density polyethylene was installed over the primary mirror to reject visible light and prevent overheating of the telescope optics. This filter introduced optical aberrations that smeared the instrumental beam width to ~ 100″ at 200 μm, and to ~ 130″ at 30 μm. (This is considerably greater than the diffraction limit, which at 200 μm is 54″.)

The detector system was a dichroic cryogenic four-channel photometer (cf. Gatley et al. 1977). In this system incoming radiation enters a 4 mm (60″) common focal-plane aperture and is partitioned by dichroic beamsplitters into four well-defined passbands centered near 30, 50, 100, and 200 μm, with fractional width Δ/λ ~ 0.3. These bands are detected separately by four gallium-doped germanium bolometers. The infrared flux in the same region is thus monitored simultaneously in all four passbands.

b) Two-Beam Chopping

Two-beam chopping was used to compensate for variable sky and telescope emissivity and to eliminate low-frequency electronic noise in the detector system (cf. Lindsey and Hudson 1976; Lindsey et al. 1981). In this technique the secondary mirror of the telescope is rocked at 30 Hz in a square-wave mode, so that the focal plane aperture alternately samples two beams separated by a distance Δx ~ 6″ (= 0.38 solar radii). The detector outputs are fed into phase-switched amplifiers, which extract the difference signal and, thus,
continuously register the infrared flux difference between the two beams. We define the “positive beam” and the “negative beam” according to the sign of the signal voltage produced by an infrared source in the respective beam.

The infrared intensity profile of the Sun is determined by scanning the telescope across the Sun, in the direction of beam separation. The two-beam difference profiles that result can, in principle, be used to reconstruct the real intensity profile along the scan trajectory.

This procedure is illustrated conceptually in Figure 1. A trajectory (Fig. 1a) is chosen through disk center avoiding plage as much as possible. As the positive beam crosses the left limb (cf. intensity profiles Fig. 1b), a large positive signal excursion results (cf. profiles Fig. 1c). This is maintained until the negative beam crosses the left limb 6' behind. A similar signal excursion of opposite sign occurs as the beams cross the right limb. The region between the limb excursions will show a null signal if the infrared intensity profile is flat (solid curves). For relatively slow variations in intensity the two-beam-difference profile is approximately the derivative of the infrared intensity profile. The two-beam difference profiles that would result from a radially brightened profile show a positive slope, m, between the limb excursions, proportional to the amount of brightening (dot-dashed curves). Similarly, a radially darkened profile will result in a negative slope, m, between the limb excursions (dashed curves).

c) Observing Procedure

In practice, each scan across the Sun required about 1 minute to complete. All scans analyzed here passed through Sun center and were repeated several times to permit estimates of instrumental errors. Reproducibility to ±0.2% of the disk-center brightness was typical at all four wavelengths. We took care to scan several solar diameters outside of both limbs to account for artificial darkening of the infrared profile introduced by the far wings of the telescope beam.

Tracking was done automatically using the electronically encoded position of a video image provided by a visual reference telescope. Guiding stability was found to be better

![Figure 1](image-url)

**Fig. 1.**—Illustration of two-beam scan procedure across the solar disk. The beams are ~100' FWHM and are separated by a distance, Δx = 6'. The telescope scans across the solar disk in the direction of beam separation, in this case horizontally. As the positive beam crosses the left solar limb (cf. intensity profile b), a large positive excursion in the difference signal results. As the negative beam follows, the difference signal returns to nearly zero, maintaining a small positive or negative residual in accordance with the intensity gradient along the scan trajectory. The slope of this gradient profile is positive for a radially brightened intensity profile, and negative for a radially darkened profile.

The scan geometry shown in (a) against a map of Hα plage (from the Mees Solar Observatory at Mount Haleakala, Hawaii) applies to scan 3 in this data set, made on 23 July 1983. χ and ψ are the angles from the scan trajectory to the position vectors of the north celestial and north solar poles respectively.
than 5'' rms, and absolute positions of the geometric centers of the infrared beams were determined to 15''.

III. RESULTS

For convenience, we express our results in terms of the disk-center-normalized intensity, \( T(x) \), where \( x \) is the distance along the scan trajectory past Sun center in solar radii. Thus,

\[
T(0) = 1. 
\]

The two-beam difference signal \( \Delta T \), is then defined by

\[
\Delta T(x) = T[x + (\Delta x/2)] - T[x - (\Delta x/2)]. \tag{2}
\]

In Figure 2a, \( \Delta T \) for 200 \( \mu \)m radiation is plotted for a single scan through the Sun center, made on 1981 July 2. (A slight asymmetry seen in the limb signal excursions of \( \Delta T \) results from an irregularity in the core of the infrared beam profile caused by optical aberrations in the aperture filter.) The ordinate of this plot is expanded by a factor of 10 in Figure 2b (solid curve). The slope throughout the segment between the limb excursions is definitely negative—corresponding to extensive radial darkening.

\( \text{a) Correction for Wings of the Beam Profile} \)

Beam smearing, particularly that caused by the far wings of the telescope beam, will cause artificial darkening, which we must distinguish from real darkening. At this point, it becomes necessary to distinguish between the observed solar intensity profile, \( T \), and the real, unsmeared intensity profile, \( \phi \), which, like \( T \), is normalized to unity at disk center. The real intensity profile, \( \phi \), drops very suddenly at the limb from approximately unity to nearly zero. (The solar corona is so optically thin to submillimeter radiation [Newkirk 1961] that it is essentially invisible outside the sharp chromospheric limb.) The far wings of the telescope beam smear the real profile, \( \phi \), and, as we shall see, introduce artificial radial darkening inside the limb in the response profile, \( T \), and at the same time introduce corresponding artificial tails outside the limb. Evidence of the wings is seen in the tails of the limb excursions of the observed profile, \( \Delta T \), which extend well outside the limb. Using these tail profiles, we can estimate the two-beam profile, \( \Delta W \), that should result from a flat profile, \( \phi \). A straightforward but rough procedure, convolving a one-dimensional beam profile with an assumed one-dimensional solar intensity profile, for this is detailed in Appendix A. For the 200 \( \mu \)m profile, \( \Delta W \) is plotted as a dashed line in Figure 2b.

A comparison between \( \Delta T \) and \( \Delta W \) shows that the wings of the beam can be important a solar radius from beam center for both 100 and 200 \( \mu \)m radiation. For these two wavelengths, the beams have strong wings caused by diffraction around the secondary mirror. It is desirable to model this situation in two dimensions by a beam with radially symmetric wings (appropriately inverted and superposed to emulate two-beam differencing) convolved with an intensity over the solar disk for comparison with the observations. The procedure for this task is reasonably straightforward and is described in Appendix B. The result is shown by the dot-dashed curve in Figure 2b. The wings of the beam were tailored to fit the wings of the observed scan outside the limb excursions. The resulting profile inside the limb excursions shows a somewhat greater artificial-darkening slope than the one-dimensional approximation. From henceforth, \( \Delta W \) at 100 and 200 \( \mu \)m is computed by the procedure in Appendix B. For 30 and 50 \( \mu \)m, \( \Delta W \) is computed by the method in Appendix A.

\( \text{b) The Radial Brightening Index} \)

For a quantitative evaluation of the darkening near disk center, we define the “radial brightening index near disk center,” \( \beta \), by

\[
\phi = 1 + \frac{1}{2} \beta \rho^2, \tag{3}
\]

where \( \rho = |x| \) is the distance from Sun center in solar radii. [This expression is simply the lowest order expansion in terms of \( \rho \) of the more familiar expression]

\[
\phi = (1 + \beta) - \beta \mu, \tag{4}
\]

where \( \mu = \cos(\theta) \), and \( \theta \) is the angle of incidence of the optical path from the direction normal to the solar surface.] The index, \( \beta \), is related to the intervening slope, \( m \), of the two-beam difference signal profiles by

\[
\beta = m/\Delta x, \tag{5}
\]

where \( \Delta x \) is the beam separation.

To correct for smearing, we define the difference, \( \Delta S \), between the observed two-beam profile, \( \Delta T \), and the correction for the wings, \( \Delta W \). \( \Delta S(x) \) for 200 \( \mu \)m radiation is plotted for three scans made on three different days in Figure 3. Details of these scans are set out in Table 1. The angles \( \chi \) and \( \psi \), describing

<table>
<thead>
<tr>
<th>Date</th>
<th>( \chi ) (deg)</th>
<th>( \psi ) (deg)</th>
<th>30 ( \mu )m</th>
<th>50 ( \mu )m</th>
<th>100 ( \mu )m</th>
<th>200 ( \mu )m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981 Jul 23</td>
<td>41</td>
<td>34</td>
<td>-0.007</td>
<td>-0.08</td>
<td>-0.060</td>
<td>-0.048</td>
</tr>
</tbody>
</table>
the geometry of the scans are defined in Figure 1a. The $x < 0$ segment of scan 3 passes near weak Hα activity. Its geometry is that shown in Figure 1 on a map of Hα plage (from the Mees Solar Observatory at Mount Haleakala).

The slope of $\Delta S$ is consistently negative for $|x| < 0.4$ wherever the scan trajectory is clear of Hα plages. This indicates general radial darkening of the quiet Sun for $\rho < 0.6$ (recalling that the beam separation, $\Delta x$, is $\sim 0.4$).

The negative slope begins to turn around outside of this region. This nominal indication of a reversal toward limb brightening can mislead, since the procedures used to model the beam try to fit the scan profile toward the limb excursions. However, a preliminary analysis of 200 $\mu$m eclipse data from 1981 July 31, made later in this same observing sequence (cf. Lindsey et al. 1983), shows independent evidence of limb brightening at both 100 and 200 $\mu$m. At 30 and 50 $\mu$m, only gradual darkening is seen, with no significant indication of brightening toward the limb. A more detailed analysis of limb brightening at the longer wavelengths is proceeding. For the remainder of this paper, however, we will concentrate only on the radial profiles near disk center. The values of $\beta$ derived from scans 1–3 are tabulated for each wavelength in Table 1. Here $\chi$ and $\psi$ are the angles from the scan trajectory to the position vectors of the north celestial and north solar poles, respectively, as shown in Figure 1a.

In summary, we note that a transition from radial darkening to radial brightening is expected for wavelengths longward of 100 $\mu$m for smooth model chromospheres. Our observations show no evidence for such a transition. It appears instead that radial darkening in the region $\rho < 0.6$ persists out to 200 $\mu$m.

IV. DISCUSSION AND CONCLUSIONS

a) Perspective

Mankin (1969) seems to have found evidence for radial darkening at 115 $\mu$m in his 1968 balloon observations. Our 100 and 200 $\mu$m profiles show approximately the same degree of radial darkening as he found. Lindsey et al. (1981) observes a flat limb profile at 350 $\mu$m ($\beta = 0.00 \pm 0.02$) with substantial brightening appearing only at $\rho > 0.9$. At this wavelength, smooth models predict substantial radial brightening ($\beta \sim 0.03$).

b) Plage

Scans directly across bright plages show strong enhancements in 100 and 200 $\mu$m emission (Jefferies et al. 1981). This is consistent with recent results found by Cartier et al. (1982), who find brightness-temperature enhancements of order 200 K for active regions at 180 $\mu$m. Evidence of the significance of this is seen in the left segment of scan 3 in Figure 3, which passes near Hα plages in the northwest quadrant of the solar disk (cf. Fig. 1a). A reliable judgment of the quiet Sun intensity profile at the longer wavelengths must be based on scans that avoid plages as far as possible.

In actuality, calcium K line photos show ubiquitous plage enhancements of a small degree along latitudes passing near disk centers. We must consider the possibility that this brightens the Sun near the center of our scan trajectories and, thus, effects the observed radial darkening. The plage excess near disk center would need to be of order 60 K for both 100 and 200 $\mu$m radiation, which is typical of plage that is weak, but still clearly visible in Hα. Such plage does not appear near disk center in scans 1 and 2. A reliable determination of the true importance of plage may require further observations nearer to the solar activity minimum.

c) Chromospheric Fine Structure

A very important consideration, which has accounted for unexpected brightness profiles at radio wavelengths in the past, is unresolved rough chromospheric fine structure (cf. Simon and Zirin 1969). For smooth atmospheres, there is a close connection between the dependence of brightness temperature, $T_r$, on wavelength, $\lambda$, and on radial distance, $\rho$. The opacity of the chromosphere to infrared and submillimeter radiation increases approximately as the square of the wavelength. The opacity per unit depth is thus approximately proportional to $\lambda^2 \sec \theta$. As Simon and Zirin (1969) point out, the brightness temperature of solar radiation should therefore be a function of this single variable alone for a homogeneous chromosphere. This is conveniently expressed as follows:

$$T(\lambda, \theta) = T[(\lambda^2 \sec \theta)^{1/2}, 0)$$

(6)

For a smooth atmosphere, a knowledge of the dependence of the disk center brightness temperature on wavelength, $\lambda$, therefore, permits direct estimates of radial intensity profiles independent of the details of the atmosphere thus characterized. Table 2 compares our measurements of $\beta$ with estimates made this way based on the brightness-temperature measurements of Rast, Kneubuchl, and Müller (1978). On this basis, slight radial darkening ($\beta < 0$) at the shorter wavelengths is expected to give way to significant brightening ($\beta > 0$) at the longer wavelengths, for a smooth chromosphere.

<table>
<thead>
<tr>
<th>$\lambda$ ($\mu$m)</th>
<th>$\beta$ (observed)</th>
<th>$\beta$ (smooth chromosphere)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>$-0.07 \pm 0.02$</td>
<td>$-0.05$</td>
</tr>
<tr>
<td>50</td>
<td>$-0.08 \pm 0.02$</td>
<td>$-0.04$</td>
</tr>
<tr>
<td>100</td>
<td>$-0.06 \pm 0.03$</td>
<td>$-0.01$</td>
</tr>
<tr>
<td>200</td>
<td>$-0.05 \pm 0.03$</td>
<td>$+0.02$</td>
</tr>
</tbody>
</table>
Our observations show no evidence for such a transition at the longer wavelengths. Such a discrepancy cannot be resolved by a smooth chromospheric model, and in fact requires the existence of unresolved chromospheric fine structure extending far down into the temperature minimum region.

The situation need not come as a total surprise. Giovanelli (1980) argues convincingly for low-lying fine structure on the basis of extensive magnetic observations. It has been clear that nonradiative heating is strongly enhanced in magnetic flux tubes. We tentatively propose that expected radial brightening can be suppressed if heated material responsible for the brightness temperature reversal between 100 and 200 μm is recessed downward into magnetic flux tubes, so that it becomes obscured by surrounding cooler nonmagnetic material when viewed at increasing incidence. This downward recession may, for instance, amount to a simple Wilson depression consistent with magnetic force balance, also discussed in considerable detail by Giovanelli. We intend to explore this concept in the future by considering flux-tube models amenable to LTE analysis.

We wish to thank the entire KAO staff for their hard work and enthusiastic support in every area. We particularly thank Mr. K. Krisciunas for his special care and attention to the difficult data acquisition and analysis demands of this program. We are grateful to Mr. A. Meyer, who adapted the KAO visual reference telescopes for viewing the Sun and provided continual and indispensable technical support. We compliment Mr. J. Eilers for his care and patience in the design and construction of an infrared solar filter for the primary optics of the KAO. We thank Mr. J. Hollingsworth and Mr. D. Lesberg for photometer modifications and adaptations.

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APPENDIX A
CORRECTION FOR FAR WINGS OF THE BEAM PROFILE

A reliable estimate of the real radial darkening of the solar intensity profile requires some attention to the effect of the extended wings of the beam profile. To the extent that these wings are significant a solar radius or more from the core of the beam, they can cause artificial radial darkening near disk center even if the real intensity profile is flat. This is true with or without the complication introduced by two-beam chopping.

For simplicity we will therefore treat the problem of artificial radial darkening due to extended wings separately, assuming single-beam scan geometry, whereby it will be evident that one can then simply apply the two-beam difference analysis described in the text to the result.

For a rough estimate it is convenient to make the approximation that the intensity, $\phi$, is a function only of distance, $x$, along the scan trajectory past Sun center,

$$\phi = \phi(x), \quad \text{(A1)}$$

rather than a function of the two variables actually necessary to specify position on the celestial sphere. The response, $B$, of the telescope to a simple straight-line source lying perpendicularly across the scan trajectory at $x = 0$ will similarly be expressed as a function of this single variable:

$$B = B(x). \quad \text{(A2)}$$

The response $T(x)$ of the telescope scanning across the Sun will be expressed as a convolution in these two functions:

$$T = \langle B * \phi \rangle, \quad \text{(A3)}$$

where the convolution, $\langle \ldots \rangle$, is defined in terms of an integral,

$$\langle B * \phi \rangle(x) = \int_{-\infty}^{\infty} dx' B(x - x')\phi(x'), \quad \text{(A4)}$$

as a function with argument $x$.

Having thus artificially confined the problem to a single dimension, we now assume that $\phi$ is a function of the form

$$\phi(x) = U(x)[1 + \zeta(x)], \quad \text{(A5)}$$

where

$$U(x) = \begin{cases} 1 & \text{for } |x| < R \\ 0 & \text{for } |x| > R \end{cases}, \quad \text{(A6)}$$

$R$ is the solar radius, and $\zeta$ represents the variation of $\phi$ inside the limb, satisfying

$$|\zeta(x)| \ll 1. \quad \text{(A7)}$$
Similarly, we assume that the beam is composed of two components:

$$B(x) = \delta(x) + w(x) .$$  \hspace{1cm} \text{(A8)}

The first component, $\delta$, is a relatively narrow core ($\sim 100''$ in width) for which

$$\int_{-\infty}^{\infty} dx \frac{1}{2} \delta(x) = 1 ;$$  \hspace{1cm} \text{(A9)}

the second, $w(x)$, represents the extended wings, satisfying

$$\int_{-\infty}^{\infty} dx |w(x)| \ll 1 .$$  \hspace{1cm} \text{(A10)}

Without loss of generality, we can construct $w$ with a negative dip near $x = 0$ (appropriately compensated in $\delta$) so that

$$\int_{-\infty}^{\infty} dx \frac{1}{2} w(x) = 0 ,$$  \hspace{1cm} \text{(A11)}

whereby we retain the convenient relation

$$\int_{-\infty}^{\infty} dx \frac{1}{2} B(x) = 1 .$$  \hspace{1cm} \text{(A12)}

It is the convolution of the small component, $w$, with a large $U$ that may contribute artificial darkening that must be discriminated from the real variation, $\zeta$, also small. In these terms equation (A3) becomes

$$T(x) = \langle \delta + w \rangle \ast U(1 + \zeta)(x)$$

$$= \langle B \ast U \rangle(x) + \langle \delta \ast U \zeta \rangle(x) .$$  \hspace{1cm} \text{(A13)}

where the second-order cross-term, $\langle w \ast U \zeta \rangle$ has been ignored.

Let us examine the first term, $\langle B \ast U \rangle$, on the right-hand side of equation (A13). From equation (A6) one can see that

$$\sum_{n=\infty}^{\infty} U(x - 2nR) = 1$$

for all $x$. From this it follows that

$$\sum_{n=\infty}^{\infty} \langle B \ast U \rangle(x - 2nR) = \langle B \ast 1 \rangle(x) = 1$$

for all $x$. Therefore, in a more generalized sum, defined by

$$S(x) = \sum_{n=\infty}^{\infty} T(x - 2nR) ,$$  \hspace{1cm} \text{(A16)}

the first term on the right-hand side of equation (A13) sums to unity independent of $x$. Variations in $S(x)$ are consequently insensitive to the wings, $w$, of the beam, but rather are sensitive only to real variations, $\zeta$, in the brightness profile. In fact, near disk center ($|x| < R$),

$$S(x) = \langle \delta \ast \zeta \rangle(x) + 1$$

$$\sim \zeta(x) + 1$$

$$= \phi(x) ,$$  \hspace{1cm} \text{(A17)}

in as far as the narrow core of the beam, $\delta$, can be treated as a Dirac delta function.

The sum, $S$, then shows the intensity profile near disk center with the effect of the wings of the beam essentially subtracted out. Since the $n = 0$ term in sum (A16) is the telescope response to the untranslated solar profile, it follows that the telescope response to a flat solar profile, defined by $W = \langle B \ast U \rangle$, resides (in the negative) in the $n \neq 0$ terms of expression (A16). More rigorously, isolating the $n = 0$ term in equation (A15), we find

$$W = 1 - \sum_{n \neq 0} \langle B \ast U \rangle(x - 2nR) .$$  \hspace{1cm} \text{(A18)}

Now, when the argument, $(x - 2nR)$, is outside the range $\pm R$ (i.e., $|x| < R$ for $n \neq 0$ terms), we are sampling only the tails of the response, $\langle B \ast U \rangle$, which differ from the tails of $T(x - 2nR)$ only by the second order cross-term $\langle w \ast U \zeta \rangle$. Neglecting this difference, we can thus write

$$W(x) = 1 - \sum_{n \neq 0} T(x - 2nR) .$$  \hspace{1cm} \text{(A19)}

Note that

$$\zeta(x) = T(x) - W(x) .$$  \hspace{1cm} \text{(A20)}
The quantities $\Delta T$, $\Delta W$, and $\Delta S$, plotted in Figures 2 and 3, are seen to be simply the two-beam differences of $T$, $W$, and $S$, respectively. In particular, $\Delta W$ is the profile expected from a two-beam scan across a flat intensity source (dashed line in Fig. 2b); and $\Delta S$ is the profile expected from a two-beam scan for which the beams are free of the extended wings. Note that

$$\Delta \phi = \Delta S = \Delta T - \Delta W.$$  \hspace{1cm} (A21)

Note that for wings that extend less than a solar radius in either direction, the sum (A19) need be computed only for $n = \pm 1$. In this case the computation of $\Delta W$ is equivalent simply to switching the wing profiles, inverting them, and pointing them inward for comparison with $\Delta T(x)$. The wings of our beam were detectable further than a single solar radius in most of our scans, and we therefore always summed expression (A19) over $n = \pm 1, \pm 2$.

An important qualification to the preceding analysis is that in expressing $\phi$ and $B$ as functions of only a single variable (eqs. [A1] and [A2]), we introduce significant errors into the succeeding analysis. The actual effect of two-dimensional wings on a two-dimensional solar profile can leave $S$ contaminated with some residual artificial radial darkening. The effect of the wings of the beam is considerably reduced but not eliminated in $S(x)$. A more exact estimate of the effect of the far wings is possible along lines similar to the above treatment, accounting for the two-dimensional character of the beam and the solar disk.

**APPENDIX B**

**TWO-DIMENSIONAL TREATMENT OF RADially SYMMETRIC WINGS**

While the one-dimensional analysis of Appendix A is useful for determining the importance of the wings of the beam, it may be only a rough estimate for wings approaching the size of the source being observed. A more accurate two-dimensional treatment is desired when the extended wings are found to be important.

In this analysis, we model the wings by construction in terms of two-dimensional radially symmetric Gaussians. The Gaussians are differentiated in the direction of beam separation to realize the effect of two-beam chopping.

We proceed by representing the convolution of a Gaussian of width $a$ with a source of unit intensity over a disk of radius $b$ by an integral over a two-dimensional vector, $r = (x, y)$:

$$I(a, b; z) = \int_{|r| < b} d^2r \exp \left( -\frac{r - \hat{r}z}{a^2} \right),$$  \hspace{1cm} (B1)

where $z$ is the separation between the center of the Gaussian and the center of the disk, and $\hat{r}$ is the unit vector pointing from the center of the disk to the center of the Gaussian. Letting $\hat{r}$ lie along the $x$-direction, we find that

$$I(a, b; z) = \int_{-b}^{b} dy \exp \left( -\frac{y^2}{a^2} \right) \left[ E \left( \frac{(b^2 - y^2)^{1/2} - z}{a} \right) - E \left( \frac{(b^2 - y^2)^{1/2} + z}{a} \right) \right],$$  \hspace{1cm} (B2)

where

$$E(\eta) = \int_{0}^{\eta} e^{-t^2} dt.$$  \hspace{1cm} (B3)

Now let the direction of beam separation also lie in the $x$-direction. We are now interested in the convolution of the disk with the differentiated Gaussian

$$\frac{\partial}{\partial x} \exp \left( -\frac{y^2 + (x - z)^2}{a^2} \right) = \frac{\partial}{\partial z} \exp \left( -\frac{y^2 + (x - z)^2}{a^2} \right).$$  \hspace{1cm} (B4)

This convolution is, then, simply the partial derivative of $I$ with respect to $z$. The derivative of $E$ is a Gaussian, so

$$\frac{\partial}{\partial z} I(a, b; z) = \exp \left( -\frac{(b^2 + z^2)^{1/2}}{a} \right) \int_{-b}^{b} \exp \left( 2 \left( \frac{b^2 - y^2)^{1/2}}{a} \right) z \right) \sin \theta, \sin \theta d\theta d\theta.$$  \hspace{1cm} (B5)

Defining $y = b \cos \theta$, one finds that

$$I_z(a, b; z) = b \exp \left( -\frac{(b^2 + z^2)^{1/2}}{a} \right) \int_{0}^{2\pi} \exp \left( 2 \frac{b}{a^2} \sin \theta \right) \sin \theta d\theta d\theta.$$  \hspace{1cm} (B6)

If the argument of the exponential were pure imaginary, the integral would be an ordinary Bessel function of order 1 with a real argument. By analytic continuation to imaginary arguments, we extract the relation

$$I_z(a, b; z) = 2\pi b \exp \left( -\frac{b^2}{a^2} \right) \exp \left( -\frac{z^2}{a^2} \right) iJ_i \left( \frac{b}{a^2} \right) z.$$  \hspace{1cm} (B7)

For small arguments, the Bessel function is amenable to Taylor-series computation. For large arguments, the following asymptotic relation becomes useful:

$$iJ_i(z) \rightarrow \frac{(\pi z)^{1/2} e^z}{2}.$$  \hspace{1cm} (B8)
In this case \((2b/a > 1)\) the convolution takes the simple form

\[
I_z(a, b; z) = a \text{sgn}(z) \left(\frac{\pi b}{2|z|}\right)^{1/2} \exp \left[-\frac{(b - z)^2}{a^2}\right].
\] (B9)

Generally, a Gaussian that fits the near wings of the beam will vanish much too fast to extend to the far wings. Several superposed Gaussians may be required to duplicate diffraction wings over a reasonable area. The fit illustrated in Figure 2 was composed of two narrow Gaussians for a rough fit of the beam core \((a = 0.11\) and \(0.14\) solar radii) and two broad ones for the wings \((0.44\) and \(1.8\) solar radii). As one sees in Figure 2b, this can fit the wings snugly outside the cores of the limb excursions.

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