A WIND-TYPE MODEL FOR THE GENERATION OF ASTROPHYSICAL JETS

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ABSTRACT

We discuss wind-type solutions for the generation of astrophysical jets from active galactic nuclei and stellar sources such as those associated with SS 433 and protostellar objects. Acceleration, collimation, and morphology are consistently interpreted in terms of a flow starting from the galactic or stellar core inside the "throat" of a thick accretion disk.

Subject headings: galaxies: jets — galaxies: nuclei — hydrodynamics — radio sources: general — stars: winds

1. INTRODUCTION

Supersonic collimated outflows have recently been observed in a wide range of celestial objects. SS 433 (Beer 1981, and references therein), molecular jets from newborn stars (Bally and Lada 1983, and references therein), and jets from galaxies and quasars (Ferrari and Pacholczyk 1983, and references therein) all have remarkable structural similarities; these similarities suggest that similar hydrodynamic phenomena may underlie the dynamics of the flow over a wide range of spatial scales. In this Letter, however, we resist the temptation to generalize and restrict ourselves to steady, wind-type solutions for jets from active galactic nuclei. In particular, we consider how initiation of supersonic flow depends upon the flow geometry and possible sources of nonthermal momentum deposition. It will, however, be immediately evident that the same ideas—with appropriate scaling factors taken into account—may be applied to the above-mentioned, more general classes of astrophysical objects.

A rich collection of observational data on collimated supersonic fluid flows from active galactic nuclei is now available (Heeschen and Wade 1982; Ferrari and Pacholczyk 1983). These data indicate that jets originate deep within the nuclei, since their typical structures can be directly traced back to distances from the nucleus of the parent galaxy of less than approximately 0.1 pc (e.g., Readhead, Cohen, and Blandford 1978). In addition, short-period luminosity variability of parent nuclei (≤ hours) implies that the central power sources must have dimensions of the order of the size of our solar system.

Hence, because of these concentrations of large amounts of energy (and mass) in very small regions, it is commonly concluded that massive compact objects such as black holes must be involved.

In this context, several ideas have been proposed as to how jets may be accelerated in the vicinity of such black holes (Rees 1982). The general scheme is that matter with non-vanishing angular momentum accretes into the deep potential well of the compact massive central object and, in accreting, forms a hot "flat" disk (T > 10^6 K). However, particles accreting along paths at small angles to the spin axis have low angular momentum and therefore cannot reach stationary Keplerian orbits, hence forming a "throat" or channel symmetric about and perpendicular to the disk plane. This geometrical configuration (and its dynamical stability resulting from angular momentum conservation) may be suitable for generating directed flows if one can envisage an acceleration process for particles entering the throat. For example, electrodynamic acceleration (Lovelace 1976; Blandford 1976), radiation pressure acceleration (Abramowicz, Calvani, and Nobili 1980), and hydrodynamic acceleration in nozzles (Blandford and Rees 1974) all have been recently considered as possible candidates.

In this Letter, we discuss a somewhat different approach which is based on recent results of solar wind theory that indicate the possibility of multiple, steady, transonic solutions which can arise because of multiple critical points in the jet flow (Habbal and Tsinganos 1983, hereafter HT). We focus our discussion on the implications for the flow near the disk; the implications for collimated flow propagation far from the nuclear disk will be discussed elsewhere. An important outcome discussed in this Letter is the appearance of novel fluid-dynamical effects which arise because of the nonlinearity of the fluid equations and include the existence of degenerate stationary solutions. These results may have important consequences for current two-dimensional simulations of jet flows which have not been heretofore considered.

II. MODEL ASSUMPTIONS AND ELEMENTARY WIND THEORY

The basic scenario we adopt corresponds to the description given in Rees et al. (1982), which for illustrative purposes only is placed in the context of active galactic nuclei. The galactic core contains a compact mass, namely a black hole with mass \( M \geq 10^8 M_\odot \), surrounded by a rotating, (geometrically) thick disk with two narrow channels along its rotation axis. For our purposes, we assume that the channels have transverse radii (much) smaller than the disk thickness, as consistent with relatively rapid rotation. Such a configuration can be obtained both for large accretion rates, leading to dense, radiation-
supported bright disks (Calvani and Nobili 1983), and low accretion rates, leading to tenuous, cool, ion-pressure-supported disks (Rees et al. 1982).

The physical separation between disk and throat is dynamical; however, the disk’s walls are diffused by fluid and possibly magnetohydrodynamic effects (radiation, shears, magnetic stresses, etc.). In consequence, the disk’s throat is not empty; a tenuous, hot gas can be supported above a certain distance \(z_0\) (stagnation point) against the attractive gravitational force by pressure forces exerted by the disk radiation, as well as by possible plasma and MHD waves. In supercritical accretion models, such pressure forces are responsible for accelerating collimated flows to supersonic speeds along the throat. However, radio jets rarely correspond to bright nuclei, as would have been expected on the basis of the supercritical accretion rates required for fast radiative dissipation in dense disks (Rees et al. 1982). Here we indicate a way to generate supersonic flows even in the case of disks with low radiation pressure, using fluid effects in collimated flows.

Suppose that the plasma moves outward along the throat with a net nonzero velocity \(v(r) \leq v_s\), the sound speed), and that it is laterally confined by the disk’s “walls.” In the simplest case, we consider the steady flow along the channel’s symmetry axis. The dynamics of the flow is governed then by two independent hydrodynamic equations for the speed \(v(r)\) and density \(\rho(r)\) in the collimating channel (the energy balance relation is discussed separately below). If we adopt the stream line coordinates \((x_1, x_2, x_3)\) and consider flows along the symmetry axis (with \(z = x_1\), and \(x_2 = x_3 = \) constant), then the problem at hand can be regarded as quasi-two-dimensional because, although only \(z\) is allowed to vary, the change in transverse properties of a streamtube is taken into account by virtue of the metric elements (\(h_2, h_1\)), corresponding to \(x_2\) and \(x_1\) respectively. Thus, with the use of these coordinates, the two hydrodynamic equations are

\[
\frac{\partial(h_2 \rho v)}{\partial x_1} = 0, \quad \rho v^2 \frac{\partial v}{\partial x_1} = -\frac{\partial p}{\partial x_1} - \frac{GM}{x_1^2} + \rho D(x_1),
\]

(2.1)

where \(h_2 \rho v(z = x_1)\) is the streamtube cross sectional area [which may be approximated by the throat’s cross sectional area \(A(z = x_1)\)], and \(\rho D(z)\) is a nonthermal momentum deposition term (which represents the effects of flow acceleration by photons, relativistic electrons, plasma, MHD waves, etc., not explicitly included in eq. [2.1] Holzer 1977). For a polytropic gas, \(p = k \rho^\gamma\), and the speed of sound is \(v_s = (\gamma p/\rho)^{1/2}\). We represent the variation of a flow tube’s cross section by using the function \(f(z) = [A(z)/A(z_0)](z_0/z)^\gamma\); for a spherically symmetric flow, \(f(z) = 1\). Equation (2.1) can be combined to give the Mach number equation for the steady flow

\[
\frac{M_s^2 - 1}{2 M_s^2} \frac{dM_s^2}{dz} = \left(1 + \frac{\gamma - 1}{\gamma} \frac{M_s^2}{M_0^2} \right) \left[ \frac{d \ln A}{dz} + \frac{\gamma + 1}{2(\gamma - 1)} \frac{d}{dz} \ln \left[ \frac{E_0 + \frac{GM}{z} + \int_{z_0}^z D(\tilde{z}) d\tilde{z}}{v_s^2 \tilde{z}^2 \tilde{z}^2} \right] \right],
\]

(2.2)

where \(M_s = v/v_s\) is the Mach number and \(E_0\) is the conserved total energy per unit mass, \(E_0 = v^2/2 + v_s^2/(\gamma - 1) - GM/z - \int D(z) dz\). This equation can be studied quite easily in the isothermal case, \(v_s^2 = \) constant; in the following, we refer to this case, although the general polytropic equation (2.2) can be similarly studied [see, for example, Kopp and Holzer 1976 for the special case \(D = 0, f(z) \neq 1, \gamma = 1.1\) in eq. (2.2)]. We note that the disk radiation can maintain the flow at roughly constant temperature in its initial phases, when it is relatively close to the disk, so that our assumption may not be overly restrictive.

Consider first the effects of the geometrical terms, i.e., the second and third terms on the right-hand side of the Mach number equation,

\[
\frac{M_s^2 - 1}{2 M_s^2} \frac{dM_s^2}{dz} = -\frac{GM}{v_s^2 \tilde{z}^2} + \frac{2}{\tilde{z}} + \frac{d \ln f(z)}{dz} + D(\tilde{z}).
\]

(2.3)

The term \(2/\tilde{z}\) represents the radially decaying part of the thermal pressure term, which competes with the gravitational attraction in the standard spherically symmetric wind. The term \(d \ln f(z)/dz\), corresponding to flow in a nonspherically symmetric stream line geometry, can be regarded as a momentum deposition term, similar to \(D\). Therefore, \(d \ln f(z)/dz > 0\), i.e., rapidly diverging flow, corresponds to some positive momentum addition; and \(d \ln f(z)/dz < 0\), i.e., slowly diverging or even converging flow, corresponds to some effective momentum subtraction. These geometrical terms, as well as direct momentum deposition (i.e., \(D\)), have important physical effects because they substantially modify the classical wind solution: they change the solution topologies by introducing new critical points in the flow. Thus, in the spherically symmetric solution, the right-hand side of equation (2.2) contains the first two terms only; the classical Parker-type critical point corresponds then to the distance \(z^P = GM/2v_s^2\) at which the flow makes its transition from subsonic to supersonic values, i.e., \(M_s^2 = 1\), so that the Mach number equation becomes singular. On the other hand, when the last two terms are also included on the right-hand side of equation (2.2), the critical points correspond to values \(z_i\), \(i = 1, 2, 3, \ldots\), given by the solutions to the algebraic relation

\[
h(z_i) - g(z_i) = 0,
\]

where

\[
h(z) = \frac{d \ln f(z)}{dz} + \frac{D(z)}{v_s^2}, \quad g(z) = \frac{GM}{v_s^2 z^2} - \frac{2}{z}
\]

(2.4)

In HT it has been illustrated how a momentum addition term, with \(h(z) > 0\) and peaked at \(z < z_i^P\), progressively modifies the spherically symmetric solution as its amplitude is varied. One first obtains inessential \(X\)-type critical points, but as the momentum deposition term becomes sufficiently large (\(h \gg g\)), the continuous transonic solution goes through the innermost critical point and not the Parker-type critical point. This means that the flow becomes supersonic closer to its origin, an effect that can be produced not only by physical momentum addition (\(D > 0\)), but also by simply allowing for a rapid expansion of the flow \((d \ln f(z)/dz > 0\)), so that the
downstream pressure rapidly drops to low values, or both. This second situation may actually be encountered in jets at the exit of the "throat" of the accretion disk (see § III). Changes in the solution topologies of equation (2.3) for various amplitudes of the momentum deposition terms which are appropriate for jets from active galaxies have been presented by Tsinganos, Ferrari, and Rosner (1983).

A further important feature discussed in HT is that the wind solutions can become degenerate when several critical points are present; that is, multiple solutions for the same initial conditions may coexist. This phenomenon occurs because discontinuous shock transitions can connect two distinct transonic branches (see HT). The Rankine-Hugoniot relations for mass, momentum, and energy conservation across such a discontinuity yield (Landau and Lifshitz 1975)

\[
\begin{align*}
\rho_2 &= \frac{v_1}{v_2} = \frac{(\gamma + 1)M_1^2}{(\gamma - 1)M_1^2 + 2}, \\
T_2 &= \frac{\gamma M_1^2 - (\gamma - 1)}{\gamma - 1} \left[ \frac{(\gamma - 1)M_1^2 + 2}{(\gamma + 1)^2M_1^2} \right],
\end{align*}
\]

where the subscript 1(2) denotes quantities upstream (downstream) of the discontinuity. With \( v_2 < v_1 \), one obtains \( \rho_2 > \rho_1 \) and \( T_2 > T_1 \). Thus, the gas can be compressed and heated by the shock. We suggest that a galactic jet may develop discontinuities when its cross-section undergoes expansion or compression. Whether such discontinuities arise depends sensitively on the history of the flow (see Habbal, Rosner, and Tsinganos 1983, hereafter HRT; and Tsinganos, Habbal, and Rosner 1983, hereafter THR). More precisely (see HT), we see that suitable expansion \([h(z) > 0]\) before the Parker critical point, \( z_c^p \), or compression \([h(z) < 0]\) after this critical point are the necessary conditions for producing discontinuities. In fact, for \( z < z_c^p \), the flow must be accelerated to a new supersonic branch in order to be able to fall back onto the old Parker solution via a shock transition; conversely, for \( z > z_c^p \), the shock must slow the supersonic flow to a new transonic branch.

This scheme is quite general and can easily be extended to include rotational and magnetic stress effects. For example, if angular momentum conservation holds when the jet suddenly expands, rotational energy can be transferred to longitudinal kinetic energy, as in typical funnel flows; even in the absence of acceleration by the wind effect, the flow speed would then increase,

\[
\nu^2 = \omega_0^2 + A_0^2\omega_0^2(1 - A_0/A) > \omega_0^2, \quad \text{if } A > A_0,
\]

where \( \omega_0 \) is the spin angular velocity in the channel of \( z = z_0 \). This effect adds to the wind acceleration effect when the jet emerges from the disk.

III. Jets from Active Galactic Nuclei

We restrict ourselves to the nonrelativistic case and focus on the principal features of "wind stream" jets; a fuller treatment, including relativistic effects, will be presented elsewhere.

The first question then is of what relevance is a modified one-dimensional treatment to the jet problem, especially in light of recent fully two-dimensional numerical simulations (see Norman et al. 1982; Woodward 1983). The observational evidence suggests that astrophysical jets consist of magnetized plasma, and there is some evidence that such jets are pressure confined (see Rees 1982, and references therein). If this is indeed the case, then direct observational evidence for the closest astrophysical jet analog, namely high-speed solar wind streams (which have Mach numbers of \( \geq 3-4 \); see Zirker 1977, and references therein), suggests that a one-and-a-half-dimensional treatment is a fair description of the basic flow state near the Sun (Kopp and Holzer 1976); this is basically so because the flow is supersonic but sub-Alfvénic out to the solar Alfvén radius, and so flow occurs along the direction established by the magnetic field. Indeed, even beyond the Alfvén radius, observations show that stream lines coincide with magnetic field lines in the corotating frame, yielding the classical Parker Archimedean spiral form for the wind streams. This implies that, certainly within the domain \( v_0/v_A < 1 \) (\( v_A \) = the Alfvén speed), the flow—though two-dimensional—is channeled by the field, so that the complex stream morphology seen in the above-mentioned two-dimensional numerical simulation (such as the Mach disks) may not be present; this regime is precisely that expected to occur in the immediate vicinity of the disk, Figure 1, where our model is applied.

What then can our relatively simple "quasi-two-dimensional" treatment yield that would not be found in a more exacting treatment based on numerical (fully two-dimensional) simulations? The crucial new element introduced by our previous discussion is that the steady flow within the "throat" is not always uniquely defined by the inner boundary conditions; that is, for any given boundary conditions, the actual steady flow solution within the jet may be degenerate in the sense that there (always) exists one continuous solution and (at times) additional discontinuous solutions (satisfying the same boundary conditions; see HT). Hence, even numerical simulations can be extremely misleading if one is unaware of this solution degeneracy; which steady solution one obtains, in fact, turns out to depend sensitively (HRT, THR) on the detailed history of the flow, including the precise nature of imposed perturbations (as might occur as a result of unsteady accretion).

Let us then start with the classical wind theory and consider the (single) critical point in such wind solutions \( f(z, t) = 1, D(z, t) = 0 \) obtained from equation (2.3); for galactic core masses in units of \( 10^8 M_\odot \) and temperatures of the plasma in the disk's throat in units of \( 10^6 \) K, we obtain \( z_c^p = 4 \times 10^{19} M_\odot/T_\odot \) cm \( \sim 13 \) pc. This distance is much too large because we know that the jet is supersonic (relativistic in superluminal sources) much below the 0.1 pc scale. This difficulty was in fact encountered in the Blandford and Rees (1974) twin-exhaust nozzle model; these authors did not consider gravity in the jet propagation, but in their case the location of the nozzle point is fixed by the confining walls, whose shape in turn is determined by the hydrostatic equilibrium equation for the overall galactic mass distribution.

In our model, we instead consider a flow which is confined by rigid (dynamical) walls in the disk's throat and then suddenly expands in an underdense region as it emerges above.
the disk (see Fig. 1). This geometrical effect of rapid expansion of the flow, which is familiar from solar coronal hole studies, corresponds to a positive momentum deposition, as discussed in § II. The requisite additional critical points then arise if the expansion e-folding length is $l_{\text{exp}} \leq 6.24 \times 10^7 z_{\text{exit}} T_e / M_e$ cm. Since $l_{\text{exp}} / z_{\text{exit}} \ll 1$, we require a strong local departure from spherical expansion [in which case the approximation $h_1 h_2 \sim A(z)$ may fail]. Of course, the actual rate of expansion of the flow is bounded from above by the finite sound speed; hence, in a more realistic treatment, one must consider momentum sources in addition to sudden expansion to accomplish our goal. Indeed, we can invert our argument and ask—given that the jet flow must become supersonic near the throat exit—that the total requisite nonthermal momentum deposition must be; the presence of additional critical points near the throat exit then requires an amplitude $D_0$ (for a spatial $\delta$-function effective momentum deposition term) $D_0 \geq 2.6 \times 10^8 M_e / z_{14} - 2 n_{\text{ion}} z_{14} - 2.6 \times 10^8 M_e / z_{14}^2$ dyn cm$^2$. If one translates this momentum deposition rate to the corresponding energy deposition rate (in the form of electromagnetic, plasma, MHD waves, etc.), one obtains an energy requirement in the funnel of the order of the Eddington luminosity. On the other hand, large asymptotic flow velocities require a value of $D$ in excess of the Eddington limit, which can be provided by geometrical collimation of the energy input into funnels with small opening angles. We conclude that the fluid flow in the throat of a galactic disk may well become supersonic as soon as it expands above the disk; the geometrical effect of fast expansion, together with nonthermal momentum deposition, can bring the transonic point within the disk region. As already discussed, funnelling may further assist the acceleration process, while spinning of the flow may ensure collimation.

The subsequent confined jet propagation depends on the detailed interaction of the flow with the surrounding medium; in particular, modulation of its cross section leads to the possibility of further standing shocks, and associated emission enhancement, in the downstream flow. A detailed description of the possible association of bright knots along VLBI and VLA jets and shock discontinuities such as those discussed in the previous section will be presented elsewhere.

IV. SUMMARY

The process of generating supersonic flows largely depends on three factors: (1) the degree of deviation of the shape of the flow channel relative to a spherically symmetric flow tube [i.e., sign and magnitude of the term $d \ln f(z) / dz$ in eq. (2.3)]; (2) the amplitude and width of the nonthermal momentum deposition rate; and (3) the exact location of the region where most of the deviation of the flow tube from the spherically symmetric form or the nonthermal momentum deposition, or both, takes place. In this context, we can summarize our conclusions as follows:

1. The geometrical shape of the throat of an accretion disk offers the physical possibility for accelerating a subsonic fluid flow to supersonic velocities well inside the galactic core. We stress the fact that our solution differs substantially from the Blandford and Rees (1974) twin-exhaust nozzle because the gravitational attraction of the central mass plays a major and direct role in defining the location of the transonic transition.

2. In all stages of confined flow propagation, variation of the beam's cross section may result in shock discontinuities, which can result in local heating, compression, and particle acceleration.

3. The existence of degenerate solutions to the “quasi–two-dimensional” equations raises the question of the sensitivity of the nonlinear time-dependent equations to perturbations, especially in the far more complex, fully two-dimensional case; outstanding questions are thus: Do degenerate steady solutions exist in the two-dimensional case? How do current two-dimensional simulations (see Norman et al. 1982; Woodward 1983) relate to this degeneracy?

The present discussion summarizes the qualitative features of our model, which appears to be a more complete elaboration of the original nozzle model of Blandford and Rees (1974). We believe that an interesting aspect of our proposal is that we avoid (at least initially) the explicit treatment of complicated effects such as the interaction of the jet with the beam side-walls, etc. Obviously, a number of physical aspects, such as the presence of relativistic flows (see Ferrari et al. 1983), magnetic stresses, etc., have yet to be included; this remains for the future.
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REFERENCES

Zirker, J. B. 1977, Coronal Holes and High Speed Wind Streams (Boulder: Colorado Associated University Press).

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