THE PROTOSOLAR HELIUM ABUNDANCE

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Abstract

The theory of stellar evolution applied to the sun cannot alone determine the solar helium abundance. Only if the neutrino flux is taken into account can a formal calibration be carried out. But that procedure appears to give the wrong result. An astronomical argument for calibrating mixing-length theory, using a measure of chromospheric activity in lower main-sequence stars, suggests that the protosolar abundance by mass is about 0.25. More direct measurements, using five-minute high-degree oscillation frequencies to measure conditions in the upper regions of the convection zone, and five-minute low-degree oscillations to probe the radiative interior, corroborate this finding. A recent unpublished report of quadrupole g-mode frequencies is also consistent. It is difficult to assess the uncertainty in the estimated abundance, because there are systematic unexplained discrepancies between theory and observation. However, the spread in the various determinations, excluding that involving the neutrino flux, is less than 10 per cent of the mean value.

Introduction

My best estimate of the solar abundance by mass of $^4$He at the epoch when the sun was formed is $Y_{\odot} = 0.25$. It is difficult to assess the accuracy of this estimate; I judge that $0.23 \leq Y_{\odot} \leq 0.27$, but I cannot argue that these limits are reliable. Moreover, I must point out at the outset that my estimate is not universally accepted. In particular, it contradicts statements by two of the previous speakers to the effect that the solar helium abundance is anomalously low.

We can make no direct measurement of the protosolar helium abundance. At best we can try to measure the helium abundance now, and from that infer what the abundance must have been originally. Such inferences must necessarily be theoretical. They involve intricate calculations and they rely on many unconfirmed hypotheses. Thus even with a precise knowledge of the present solar helium abundance (which we do not have) it would be difficult to assess the accuracy of the extrapolation back to the time when the sun condensed from the interstellar medium. Nevertheless, as I shall discuss later, if one is prepared to confine consideration to the simplest theoretical models of the sun's interior, the variation in the amount of helium that has been produced during the sun's
lifetime is rather less than the uncertainty in our estimate of the amount of helium that is present in the sun today.

Estimates of the present helium abundance $Y$ in and outside the solar atmosphere vary quite widely. It is not possible to make a direct spectroscopic determination of $Y$ in the photosphere. However, the spectra of quiescent prominences are not inconsistent with $Y = 0.29 \pm 0.05$ (Heasley and Milkey, 1978), provided that non-LTE effects have been taken into account properly. Values ranging from 0.14 to 0.28 have been reported from solar cosmic rays, and particle counting yields a solar-wind abundance in the range $0.13 - 0.18$ (e.g. Hundhausen, 1972; Crawford et al. 1975). The diversity of the results no doubt arises in part from element-separation processes that occur in the wind and in dynamically active regions in the solar atmosphere. Moreover, it is not impossible that the sun has accreted substantial amounts of material from the interstellar medium during its main-sequence lifetime. Consequently none of these results can reliably be regarded as being representative of the helium abundance of the bulk of the solar material. Our only recourse is to indirect measurements that depend on the structure of the sun's interior. It is these that I discuss in this review. To put them into perspective, I must first summarize the essential features of the theory of the evolution of the sun.

Solar evolution

The theory of the sun's evolution, in its so-called standard form, considers a spherically symmetrical self-gravitating mass of gas in hydrostatic equilibrium. The microphysics leading to the constitutive relations, namely the equation of state, the opacity and the nuclear reaction rates, is treated with the utmost care. On the other hand, the macroscopic dynamics is treated quite crudely. In particular, it is normally assumed that there is no macroscopic motion other than small-scale turbulence in the convection zone which occupies the outer part of the envelope. It is usual to assume no mixing of the products of nuclear reactions by, for example, large-scale meridional flows or small-scale turbulence arising from shear instabilities induced by rotation (with or without magnetic fields). One assumes that there has been no material mixing beneath the convection zone, no substantial mass-loss or accretion, and no energy transport by mechanical waves. The sun is assumed always to be in hydrostatic and thermal balance.

These simplifying assumptions are made mainly because it is difficult to do otherwise. Their plausibility has been questioned by some, and defended by others. I shall not discuss that here. I shall merely accept that the present theory provides a simple zero-order structure from which to begin our investigation of the protosolar helium abundance, and recognise that it has deficiencies
which I cannot yet assess. I also draw your attention to Professor Schatmann's presentation, which follows mine, in which one of the assumptions of the standard theory is challenged. I shall remark on this work later.

For the purpose of my discussion I shall take as the origin of time \( t = 0 \) the epoch when the sun arrived on the main sequence. Although some minor nuclear reactions took place prior to that time, none of them had a significant direct influence on the abundance of \(^4\text{He}\). I shall assume, as is customary, that the initial composition was uniform, so that it is meaningful to consider \( Y_{\text{P0}} = Y(t = 0) \) to have a single value.

Under such circumstances the theoretical solar model at \( t = 0 \) can be specified by three parameters: \( Y, Z \) and \( \alpha \), where \( Z \) is the total abundance by mass of elements other than \(^4\text{He}\) and \( \text{H} \); the hydrogen abundance is \( X = 1 - Y - Z \). The parameter \( \alpha \) is, in the simplest form of the theory, a constant scalar quantity that is used to adjust a formula for convective heat transport. It can be regarded as a measure of the efficacy of convection. The heat-flux formula is almost always derived from mixing-length arguments, so for convenience I shall refer to \( \alpha \) as the mixing-length parameter.

**Calibration of solar models, and the solar neutrino problem**

The changing chemical composition brought about by the nuclear reactions in the solar core causes the entire structure of the sun to evolve. The parameters \( Y_{\text{P0}}, Z \) and \( \alpha \) must be adjusted to ensure that the sun has the correct radius \( R \) and luminosity \( L \) now. Such an adjustment is possible. Evidently there is a single infinity of solutions, which I shall label with \( Y_{\text{P0}} \). These solutions depend on the age of the sun, \( t_\odot \), which is assumed to be slightly greater than the age of the earth and the meteorites: \( t_\odot = 4.7 \times 10^9 \text{y} \).

Constraining \( R \) and \( L \) sets some limits on \( Y_{\text{P0}} \). However, it is important to realise that as it stands the theory does not determine \( Y_{\text{P0}} \). A common procedure is to adopt a value for \( Z/X \) at \( t = 0 \) that is typical of surface abundances of other Population I stars. Bahcall et al. (1982) discuss a model computed in such a way, and assess its uncertainties within the framework of the standard theory. They assume \( Z/X = 0.023 \) at \( t = 0 \), and so obtain \( Y = 0.25 \) at \( t = 0 \). Notice that this calibration relies on the assumption that the heavy-element abundance of the sun is the same as those typical Population I stars for which determinations of \( Z/X \) exist.

It is well known that a measure of the production rate of solar neutrinos is also available. In principle that could be used to provide a calibration of the solar models without resorting to assumptions about the heavy-element abundance. However, when such a procedure is carried out, agreement with
observation can be achieved only if $Z$ is extremely low (Abraham and Iben, 1971; Bahcall and Ulrich, 1971). The value required is so far from other estimates that it is normally considered to be out of the question. Thus the problem with which one is faced is regarded as being one of explaining the measured neutrino flux with a solar model of normal composition. According to Filippone and Schramm (1982) it is not out of the question that this problem will be resolved by small adjustments of the many uncertain parameters in the theory.

Some attention has been paid to the idea, discussed first by Joss (1974), that $Z$ is indeed very low in the solar interior, but that material rich in heavy elements has been accreted since the sun condensed, and has contaminated just its outer layers. If this were so, the convection zone would be quite shallow, permitting adequate contamination of the mixed layers without invoking implausibly large quantities of accreted matter. The neutrino flux could then be reproduced, but only if $Y_{p\theta} \leq 0.2$ (Christensen-Dalsgaard et al., 1979). This value is certainly in apparent conflict with the other estimates of the primordial helium abundance discussed at this workshop. Moreover, there are problems concerned with the stability of the solar model that results. However, I shall not discuss that here. The reason is that the weight of the evidence provided by the other observations with which I shall confront such a model appears to rule it out.

An astronomical determination of $Y_{p\theta}$

This is not a reliable determination because the observations, and their correlations with theory, have not yet been fully analysed. I discuss it first because it is an interesting consequence of assuming the physics of the sun to be similar to that of other main-sequence stars, and is thus the most closely related to classical astronomical methods.

The observations concern the chromospheric emission ratio $r_{HK} (FHK/vT^4)$ of those lower main-sequence stars in Wilson's (1978) long-term activity survey for which rotation periods have been inferred (Vaughan et al., 1980, Baliunas et al., 1983). Chromospheric activity is a measure of magnetic activity, and it is commonly (though not universally) believed that the latter is controlled by dynamo action. This in turn depends on such quantities as the Rossby number and the dynamo number, which influence important asymmetries such as helicity in the convective flow (e.g. Gilman, 1981; Stix, 1981). Subject to the assumption that the geometry of the flow varies along the main sequence more slowly than the spatial scale, the variation of these numbers depends primarily on the ratio $\tau$ of the convective turnover time to the period of rotation. That ratio is plotted by Gilman (1981) for three sequences of envelope models. According to R.W. Noyes (private communication), if one plots $r_{KH}$ against $\tau$, one can obtain

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a single-valued relation, but only for a particular value of the mixing-length parameter $\alpha$. It is tempting, therefore, to infer that that value is the one to adopt.

This procedure relies on many uncertain factors. In particular, we don't really know that chromospheric activity is directly related to dynamo action. However, if we accept that it is, the result is certainly suggestive. It is probably not important that we do not have an adequate theory of convective dynamos. The strength of the argument is that it does not rely on the details of the physics. It depends simply on scaling laws, and the hypothesis that a relation between important parameters that must at least influence dynamo action and a measure of chromospheric activity merely exists. We know that the mixing-length parameter $\alpha$ is hardly a universal constant, but our ability to fit the slope of the main sequence in the solar vicinity with theoretical models computed with mixing-length theory suggests at least that for solar-type stars the outcome of the formalism may be a good approximation to reality. Therefore in principle this calibration of $\alpha$ is probably more robust than assuming the initial solar value of $Z/X$ to accord with present spectroscopic determinations of surface abundances of other stars.

It is now necessary to confess that a careful calibration of $\alpha$ has not actually been undertaken. Nevertheless, I think it has been worthwhile mentioning this issue, because it may provide a useful constraint in the future. A preliminary comparison of the chromospheric data with the ratios $\tau$ quoted by Gilman (1981) yields $\alpha \approx 2$ (Noyes, private communication). This alone is inadequate information, since definitions of $\alpha$ vary, and Gilman does not state his; nor does he reveal the chemical compositions of his envelope models. However, recognising that the depth of the convection zone (which Gilman does give) depends quite sensitively on $\alpha$, one is tempted to use that to specify the model. Comparison with evolved models of the entire sun then yields a best fit with $Y_{\text{p0}} = 0.25$. It is not yet possible to assign a likely error to that result.

The dependence of solar models on $Y_{\text{p0}}$

All the other determinations of the protosolar helium abundance that I shall discuss depend on analyses of frequencies of free oscillation of the sun. The oscillations with diagnostic value that have been unambiguously observed and identified are all acoustic modes, whose frequencies depend principally on the sound speed $c$. Therefore it is instructive to consider how $c$ depends on $Y_{\text{p0}}$.

Models with low $Y_{\text{p0}}$ also have low $Z$. Consequently $X$ is high, and the luminosity $L$ can be generated by nuclear reactions in the core at a relatively low temperature. This leads to a lower neutrino flux. Because $Z$ is low, so is the opacity in the deep interior; $L$ can be transported by a lower temperature
gradient. Thus, as distance \( r \) from the centres of two solar models with different \( Y \) increases, the difference in temperature decreases until eventually there is a crossover: models low in \( Y \) and \( Z \) are cooler in the interior and hotter near the surface. Of course all calibrated solar models have the same photospheric temperature, implying that exterior to the crossover helium-deficient models must have steeper temperature gradients. This is accommodated by reducing \( \alpha \) to make convection less effective in transporting heat when \( Y \) is lower. That leads to a shallower convection zone.

The sound speed depends on both temperature and composition. In the convection zone the two influences reinforce. In the radiative interior they oppose. Christensen-Dalsgaard et al. (1979) found that except in the core (taken to be unmixed) the temperature dependence dominates. Thus, on the whole, lower \( Y_{\text{eq}} \) is associated with lower \( Z \), a lower neutrino flux, lower sound speed throughout most of the deep interior, higher sound speed immediately beneath the photosphere, a lower value of \( \alpha \) and a shallower convection zone.

**Description and classification of solar oscillations**

For the present purposes it is sufficient to regard the equilibrium state about which the sun oscillates to be spherically symmetrical. The evidence that we have at present suggests that deviations from spherical symmetry caused, for example, by rotation, magnetic fields or large-scale inhomogeneities in the convection zone, all influence the oscillation frequencies by less than the mean discrepancy between observation and the best-fit solar models. Therefore in a first approximation they can be ignored.

The oscillations observed have low amplitude. Therefore a linear analysis is no doubt adequate*. In that case the eigenfunctions separate into products of a function of \( r \) and \( t \) and a function of the spherical polar angles \( \theta \) and \( \phi \). The latter is a surface harmonic, whose degree \( \ell \) and order \( m \) are called the degree and the azimuthal order of the oscillation mode. The time dependence is sinusoidal, with cyclic frequency \( \nu \). For each value of \( \ell \) there is a discrete spectrum of eigenfunctions, which can be labelled with an integer \( n \). This is called the order of the mode. The eigenfrequencies of the acoustic modes, or \( p \) modes as they are usually called, increase with \( n \). It is customary for the gravest \( p \) mode to be labelled with \( n = 1 \). In addition there are \( g \) modes,

* Amplitudes may not be small in the upper atmosphere where the density is very low, and nonlinear distortions to the eigenfunctions might be substantial. Rosenwald and Hill (1980) have conjectured that the resulting modification to the eigenfrequencies is substantial too, though Belvedere et al. (1983) have subsequently argued that that is unlikely.
whose frequencies are always less than those of the p modes of the same degree. These are essentially internal gravity modes, and can be supported only in convectively stable regions. There remains the so-called f mode (fundamental mode) whose frequency lies between those of the p modes and the g modes. It is assigned the order \( n = 0 \). Frequencies are independent of \( m \); the order of a spherical harmonic depends on the orientation of the axis of the polar coordinates, which has no physical significance in a spherically symmetrical system. The degeneracy is split by symmetry-breaking agents, of course, and it is from reports of observations of such splitting that we judge these agents to produce only small perturbations. Thus we may denote the frequencies by \( \nu_{n,\ell} \), though I shall omit the subscripts where possible.

Given all the modes it is a straightforward matter to arrange a sequence with given \( \ell \) in ascending order of frequency. It is then possible to assign the order by labelling adjacent modes with consecutive values of \( n \), choosing the origin such that when \( |n| \) is large it is equal to the number of zeros in the vertical component of the displacement (for nonradial modes a node at \( r = 0 \) is not counted). We can think of \( |n| \) and \( \ell \) as measures of the vertical and horizontal components of a wave number that characterizes the mode. All the oscillations that have been observed and identified have either \( |n| \) or \( \ell \) large.

I conclude with a remark about f modes. When \( \ell \gg 1 \), they are the "compressible" generalization of deep ocean surface waves. I use the word "compressible" with reservation, for it signifies a physical property of the fluid without the implication that the fluid is actually compressed. In fact the motion is divergence-free, so it does not matter whether the fluid is compressible or not. As is the case in the ocean, therefore, the eigenfrequencies satisfy

\[
2\pi\nu = (g\ell/R)^{1/2},
\]

where \( g \) is the gravitational acceleration in the photosphere. This is independent of the stratification of the fluid; therefore identification of high-degree f modes does not depend on knowing the internal structure of the sun.

**Five-minute oscillations of high degree**

I discuss these first partly because they were the first oscillations to have their spatial and temporal structure resolved (Deubner, 1975), and partly because the observations are easier to interpret. Figure 1 is a two-dimensional power spectrum of some more recent observations by Deubner et al. (1979). Contours of \( \sqrt{\ell} \) times power are plotted with respect to \( \nu \) and \( \ell \). Several discrete ridges of power are evident, and are compared with the theoretical dispersion relations of two solar models. Each ridge corresponds to a sequence of modes with varying \( \ell \) and fixed \( n \).
Figure 1. Two-dimensional power spectrum of high-degree five-minute oscillations. The thin lines are contours of $\sqrt{\ell}$ times power (from Deubner et al., 1979). The thick continuous lines represent the theoretical dispersion relation for a model envelope with $\alpha = 2.5$ (from Berthomieu et al., 1980); the dashed lines represent the theoretical dispersion relation when $\alpha = 1.1$. The lowest ridge is produced by f modes ($n = 0$); higher-frequency ridges are produced by p modes. The orders of the modes are indicated on the diagram.

Since $\ell \gg n$, these modes can be regarded as resulting from the interference of acoustic waves propagating, on average, almost horizontally. Because sound speed increases downwards, the waves are refracted at depth. They cannot penetrate the atmosphere, where the scale height is shorter than the wavelength of the oscillation, and thus the waves are confined within a shallow wave guide. A typical path is illustrated in Figure 2. It stays well within the convection zone, and is predominantly in regions that, according to theory, are almost adiabatically stratified. Consequently buoyancy is unimportant to the dynamics of the waves, and locally the acoustic dispersion relation $2\pi v = kc$, where $k$ is the total wave number, is satisfied. Provided the wave guide is shallow, the horizontal component of the wave number, $k_h = \ell(\ell+1)^{1/2}r^{-1}$, is more or less constant. Therefore the vertical component must decrease with depth. The base of the wave guide is situated where it vanishes. There, propagation is exactly horizontal, and $c = 2\pi v_h^{-1}$.

One expects that in general the frequencies would depend on the sound speed. Indeed, if one approximates the upper envelope by a plane parallel polytrope,
Figure 2. Diagram of a ray path of a five-minute high-degree oscillation. A normal mode is a standing wave produced by the interference of many such waves displaced horizontally with respect to each other. The thin horizontal line represents the photosphere.

one finds it to be so. In that approximation the mean envelope provides no natural length scale, and we deduce immediately that the depth of the waveguide scales with \( k_h^{-1} \). Indeed, for a complete adiabatically stratified polytrope of index \( \nu \), the condition for horizontal propagation is satisfied at a depth \( (2n+\nu)k_h^{-1} \). Notice that at least when \( n \) is not small, the depth of the waveguide is only weakly dependent on stratification. Therefore the variation of frequency, as the model changes, must be controlled mainly by the temperature gradient. Recall that solar models with lower \( Y_{\odot} \) have higher subphotospheric temperature gradients (and shallower convection zones). Therefore their high-degree \( p \)-mode frequencies are higher.

Studies by Lubow et al. (1980) and Berthomieu et al. (1980) have revealed that the theoretical high-degree five-minute oscillation frequencies depend predominantly on the mixing-length parameter \( \alpha \). As \( \alpha \) increases, frequencies decrease, but the variation saturates at large \( \alpha \). In Figure 1 are plotted dispersion relations for two model envelopes. Evidently the data favour the model with \( \alpha \geq 2 \). This agrees with the calibration I discussed earlier based on chromospheric activity of lower main-sequence stars, and again suggests \( Y_{\odot} = 0.25 \).

This determination of the protosolar helium abundance appears to rely quite heavily on the theory of stellar structure and evolution. It is therefore perhaps not out of place to digress slightly at this point to report some recent remarks that add credence to the models. Lower main-sequence stars, including the sun, have convective envelopes. According to the mixing-length prescription, which, as is widely acknowledged, is suspect, the convection zones are nearly adiabatically stratified everywhere except in a thin boundary layer near the photosphere. It is interesting, therefore, that we now have some evidence in support of that aspect of the configuration. Recently Duvall (1982) has pointed out that the observations of high-degree \( p \) modes indicate a dispersion relation
of the form \( (n+\tilde{\delta})/\nu = f(\nu/\lambda) \), where \( f \) is a function and \( \tilde{\delta} = 1.5 \). That is a property of acoustic waves trapped in an adiabatically stratified envelope. In the limit \( n \gg 1 \) the dispersion relation can easily be obtained either by ray theory or from the JWKB approximation to the governing wave equation. In the former case one integrates the travel time along a ray path of Figure 2, and applies the condition that many similar such waves interfere constructively. As reported by Christensen-Dalsgaard and Gough (1981a), one finds that the constant \( \tilde{\delta} \) is roughly \( \bar{\nu}/2 \), where \( \bar{\nu} \) is a mean polytropic index characterizing the upper layers of the wave guide. Such a relation is destroyed by buoyancy, as is evident from the dispersion relation for nonadiabatically stratified polytropic envelopes (e.g. Gough, 1978; Christensen-Dalsgaard, 1980); consequently Duvall's result is direct observational confirmation that most of the solar convection zone is approximately adiabatically stratified.

**Five-minute oscillations of low degree**

Low-degree five-minute modes provide a more direct measure of conditions in the solar interior. For these waves \( n \gg \ell \), propagation is nearly vertical, and the waves penetrate almost to the centre of the sun. An example of the motion, in the case \( \ell = 2 \), is illustrated in Figure 3. The apparent zone of avoidance around the centre results from destructive interference amongst the convergent waves. The oscillation period is twice the sound travel time, or acoustical distance, \( \tau_{AB} \) from A to B, divided by the number \( n \) of wavelengths in the path A→B→A. Thus \( n \) is also the number of nodes between A and the centre of the star (excluding the central node), and thus is the order of the mode. To estimate the position of B we note that when \( \ell \) is even, as in Figure 3, oscillations at A and D (and oscillations at B and C) are in phase, so that a plane-wave component of the mode crossing the centre (vertically downwards in the figure) must have the same phase at B and C. Hence \( \tau_{BC} \) must be an integral number of periods (which is an even number of half-periods). Similarly, when \( \ell \) is odd, the oscillations at A and D are \( \pi \) out of phase, whence \( \tau_{BC} \) must be an odd-integral number of half-periods. It can be shown that the number of half-periods is simply \( \ell \).

Thus \( \nu^{-1} = \nu_{0}^{-1} - (\ell/2)\nu_{o}^{-1} \), yielding \( \nu = (n + \ell/2)\nu_{o} \), where \( \nu_{o}^{-1} \) is the acoustical distance between A and D. When \( \ell = 0 \) there is no zone of avoidance, and the waves converge at the centre.

A formal expansion for \( n \gg \ell \), ignoring perturbations in the gravitational potential, has been carried out by Tassoul (1980), yielding

\[
\nu = (2n + \bar{\nu} + \ell + \frac{1}{2})\nu_{o}/2 - \Lambda[\ell(\ell+1) + \delta]\nu_{o}^{2} - \nu_{o}^{-1} + \ldots
\]  

(1)
Figure 3. Motion of the axisymmetric quadrupole $p$ mode of order 13. The radial lines represent the magnitude of the displacement, where it is positive, in a meridional plane. Negative displacements have been omitted. The radial coordinate has been stretched in such a way that equal distances in the diagram represent equal acoustical distances in the sun. The magnitude of the displacements has been exaggerated, and the variation of the amplitude with radius has been reduced. The circle indicates the position of the undisturbed photosphere.

with

$$\nu_o = \left(2\int_0^R c^{-1} \, dr\right)^{-1}.$$  \hspace{1cm} (2)

Once again $\bar{\mu}$ is a polytropic index characterizing the upper layers of the envelope where the waves are reflected, and $\Lambda$ and $\delta$ are more complicated constants that depend on the equilibrium model. The term $\bar{\mu} \nu_o$ in the leading term of (1) was not found by the argument from ray theory outlined above simply because the effects of stratification were ignored. The reason for presenting the argument in so simple a form was to highlight the fact that the term $n + \ell/2$ is basically a geometrical property, and does not depend on details of how $c$ varies with $r$. The second term in (1) does depend on $c(r)$, and is therefore a useful diagnostic of the sun's structure; I shall return to this point later. First I shall consider the structure of just the leading term.

If $\ell$ is increased by two and $n$ decreased by unity the leading term in (1) is unchanged. This fortunate property permitted the first resolution of discrete frequencies in a power spectrum of spatially unresolved Doppler shifts measured by Claverie et al. (1979). Subsequent observations by Grec et al. (1980) over a longer time interval enabled the frequencies of modes with different $\ell$ but the
same $n + \ell/2$ to be distinguished. By comparing their separations, which are crudely estimated from the second term in (1), with theory (and taking into account the sensitivity of the detector to modes with different $\ell$) it was then possible to identify the degrees of the modes unambiguously (Christensen-Dalsgaard and Gough, 1980).

There are now available several measurements of frequencies of modes with $\ell \leq 3$ (Claverie et al., 1981a; Grec et al., 1983; Woodard and Hudson, 1983) obtained in light integrated over the entire solar disk. In addition there are some frequencies of modes with $\ell = 3, 4$ and 5, obtained by Scherrer et al. (1982) using a technique with limited spatial resolution. These higher values of $\ell$ were also inferred by comparing the frequency spectrum of the observations with the theoretical distribution of the frequencies of those modes to which the detector is most sensitive, though in that case it was also necessary to register the results with frequencies identified in the spatially unresolved observations.

By comparing with observation the low-degree eigenfrequencies of a sequence of solar models with different initial helium abundances, one can attempt to determine $Y_{\text{p}0}$. However, because the order $n$ of a mode is not measured, the result is uncertain. Indeed, though Christensen-Dalsgaard and Gough (1981b) found that the best least-squares fit to the data available to them at the time chose a model with $Y_{\text{p}0} > 0.25$, the second best solution (which has $Y_{\text{p}0} \approx 0.18$ and required assigning larger values of $n$ to the observed frequencies) is not so much worse as to leave no room for doubt. However, there is now a new direct observational determination of $n$. I shall therefore discuss that next, and proceed to the determination of $Y_{\text{p}0}$ afterwards.

**Five-minute oscillations of intermediate degree**

Recently Duvall and Harvey (1983) have obtained frequencies of five-minute oscillations of low and intermediate degree. By projecting longitudinally averaged, spatially resolved Doppler measurements onto zonal harmonics, it was possible to infer the degrees of modes from 0 to 140. Thus the new data connect the modes of low degree to those of high degree; the ridges evident in Figure 1 are thus extrapolated to the axis $\ell = 0$.

When $\ell >> 1$, the $f$ modes can be identified unambiguously, because their frequencies are independent of the subphotospheric stratification. Indeed, even when $\ell$ is only 140, the frequency is within 2 per cent of that given by the asymptotic formula for deep ocean waves. Hence there is no doubt which of the oscillations in Figure 1 are $f$ modes. The $p$ modes have frequencies lying immediately above, and by counting from unity one can assign the order $n$. Of course one might wonder whether it is possible that entire sequences of modes corresponding to certain values of $n$ are missing, so that the counting yields
the wrong identification. This is quite unlikely for two reasons. First, the 

modes are probably excited by broad-band noise in the convection zone, and are 

likely to be coupled to each other to a sufficient degree for substantial amounts 

of energy to be exchanged amongst them. This renders it difficult to imagine 

how all the modes with particular values of n can be selectively suppressed. 

Second, changing the assignment of n would require implausibly severe changes 

to the solar model to reproduce the power spectrum in Figure 1. Varying the 
mixing-length parameter a from 1 to infinity in an envelope model, for example, 

changes the frequencies by typically 5 per cent, which is insufficient to move 

from one ridge to the next. The values of n quoted in Figure 1, and those 

inferred by Duvall and Harvey for their higher values of \( \ell \), can therefore be 

trusted.

By following the ridges down to \( \ell = 0 \), Duvall and Harvey were able to 

assign values of n to the low-degree modes. In the discussion that follows, 

these values will be adopted.

**Calibrating solar models using low-degree p-mode frequencies**

The most straightforward calibration is to perform a least-squares fit of 

the theoretical eigenfrequencies to the data. This has been done independently 

by Christensen-Dalsgaard and Gough (1981b) and by Shibahashi *et al.* (1983), 

yielding \( Y_{P0} = 0.27 \) and \( 0.23 \) respectively. However, in both cases the best-

fitting model deviates in a systematic way from the observations by more than 

the observational error. Therefore the determination of \( Y_{P0} \) is uncertain.

Part of the difference between the two determinations comes from the fact 

that Shibahashi *et al.* had available more accurate observations. But part comes 

from numerical error. In particular, the analysis of Christensen-Dalsgaard and 

Gough (1981b) used a sequence of solar models computed with an ordinary stellar 

evolution programme that was not written with high accuracy in mind. Recently 

Christensen-Dalsgaard (unpublished) has repeated the calculation using the more 

carefully written programme described by Christensen-Dalsgaard (1982). Com-

parison was made with the data reported by Claverie *et al.* (1981a), Grec *et al.* 

(1983), Woodard and Hudson (1983) and Scherrer *et al.* (1983). The result is 

\( Y_{P0} = 0.23 \), in agreement with Shibahashi *et al.* (1983).

One should not infer from this coincidence that the determination of \( Y_{P0} \) 

is complete. There are details of the solar models that differ in an important 

way. These appear to arise, at least in part, from differences in the equations 

of state that were used. Christensen-Dalsgaard *et al.* used an equation of 

state based on the simple formalism proposed by Eggleton *et al.* (1973) which, 

in particular, ignores the influence of bound states on the thermodynamic
partition function. Shibahashi et al. (1983), and also Ulrich and Rhodes (1983), have taken some explicit account of the effect of electron screening on the energy levels of bound atomic and ion states, and have approximated the partition function using the Planck-Larkin formalism. Christensen-Dalsgaard and Gough (unpublished) have also used variants of the procedure described by Fontaine et al. (1977), one of which was used in the computations of high-degree oscillations by Berthomieu et al. (1980). The divergence between the results, particularly in quantities such as the small separations between frequencies of modes with the same value of $n + \frac{1}{2}$, is substantial. That reflects differences between the models in the deep interior.

The high-frequency cutoff, and the photospheric $\nu$

It is well known that there is a high-frequency cutoff to the amplitudes of solar oscillations. This was observed in the high-degree modes before their spatial structure was resolved (Noyes, 1967), and is evident in the power spectrum in Figure 1. A drop at 5.5 mHz was reported by Brown et al. (1978) in the power spectrum of their observations of oscillations of presumably low and intermediate degree, and an abrupt termination of power at about 5.7 mHz has been reported recently by Claverie et al. (1981b) for low-degree oscillations.

The reason for this behaviour is often considered to be related to Lamb's (1919) acoustical cutoff. As I mentioned earlier, it is only when the frequency of acoustical oscillation of a stratified medium under gravity is so high that the wavelength of the wave is less than about the scale height of the medium that the wave can propagate. For five-minute modes this condition is not satisfied in the sun's atmosphere, and so the wave energy cannot leak out into the chromosphere and corona. But as the frequency rises energy does leak, and the oscillations are damped. The criterion for propagation is quite complicated (Christensen-Dalsgaard et al., 1983), but for an isothermal atmosphere it simplifies to $\nu > \nu_c = \gamma g / (4\pi c)$, where $\gamma$ is the adiabatic exponent ($\partial \ln p / \partial \ln \rho$)ad. At a given temperature the critical frequency $\nu_c$ depends on composition; and Isaak (1983) has suggested that the sharp break he finds in his power spectrum occurs at $\nu_c$ and so provides a measure of the atmospheric helium abundance.

Unfortunately the situation is probably not that simple. For example, the reflecting barrier presented by the atmosphere to subcritical oscillations is finite in extent, and some penetration takes place. Also the reflecting layers are not isothermal, which invalidates the simple criterion. Christensen-Dalsgaard and Frandsen (1983) have analysed the behaviour of the atmospheric waves and find no sudden increase in damping near $\nu_c$. They suggest that the abrupt change in the spectrum reported by Isaak is related to a chromospheric resonance. If that is so, it will provide a useful diagnostic in the future, though one somewhat different from that originally envisaged.
Gravity modes

P. Delache and P.H. Scherrer (in preparation) have found evidence for a sequence of gravity modes with periods between 3.5 and 5 hours. As \(|n|\) increases at fixed \(\ell\), the period difference between adjacent modes tends to a constant; it was seeing this property that led to the tentative identification. The mean period difference is 15.3 min, which is what one would expect from high-order quadrupole modes. Fitting this value to the mean period separations computed by interpolating between and extrapolating beyond the eigenfrequencies of appropriate \(g\) modes of the models of Christensen-Dalsgaard \textit{et al}. (1979) yields \(Y_{p0} = 0.22\). This is somewhat less than the result obtained from the low-degree \(p\) modes, and is probably more uncertain.

A remark on mixed solar models

One is tempted to wonder whether the systematic discrepancy I mentioned earlier between the observed and the theoretical low-degree \(p\)-mode frequencies results from an error in one of the assumptions of the so-called standard solar model. One possibility is that the products of the nuclear reactions do not remain \textit{in situ}, but are mixed, perhaps only partially, with the outer envelope. I have selected just this possibility for discussion because it has recently received some attention, and because Professor Schatzmann will discuss the matter at this workshop. I mention it also because it is one of the few apparently plausible suggestions for reducing the theoretical neutrino flux to the level observed.

An important aspect of mixing is that it causes the molecular weight to vary more gradually in the core. More gradual variation is the case also in unmixed models with lower \(Y\) and \(Z\), because in these the reacting core is more extensive. Core conditions are reflected in the \(\ell\)-dependent part of the second term of equation (1); in the outer parts of the star acoustical communication across a horizontal wavelength is not possible in an oscillation period, so the motion cannot be directly dependent on the spherical harmonic. Therefore a determination of the value of \(\Lambda\) is likely to be an indicator of core inhomogeneity. Unmixed models have the greatest variation of \(Y\) in the core, which maximally opposes the influence of temperature on the sound speed, and yields the smallest values of \(\Lambda\).

It is actually more useful to consider the frequency separations
\[
\delta \nu_{n,\ell} \equiv \nu_{n,\ell} - \nu_{n-1,\ell+2}
\]
because the neglect of the gravitational potential renders equation (1) too inaccurate (Dziembowski and Gough, unpublished). This quantity is listed in Table 1 for various theoretical models, together with the observational determinations: a general rise in the values of the separations as \(Y\) and \(Z\) decrease is evident. Included are two mixed models, which also exhibit
<table>
<thead>
<tr>
<th>Observation</th>
<th>$\delta v_o$</th>
<th>$\delta v_l$</th>
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</thead>
<tbody>
<tr>
<td>Claverie et al., 1981</td>
<td>8.3</td>
<td>-</td>
</tr>
<tr>
<td>Grec et al., 1983</td>
<td>9.4</td>
<td>15.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Theory</th>
<th>$Y_{p0}$</th>
<th>$Z$</th>
<th>$\delta v_o$</th>
<th>$\delta v_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Models</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Christensen-Dalsgaard and Gough, 1980</td>
<td>0.25</td>
<td>0.020</td>
<td>10.1</td>
<td>16.0</td>
</tr>
<tr>
<td>Shibahashi et al., 1983</td>
<td>0.19</td>
<td>0.004</td>
<td>13.2</td>
<td>17.2</td>
</tr>
<tr>
<td>Ulrich and Rhodes, 1983</td>
<td>0.23</td>
<td>0.020</td>
<td>10.7</td>
<td>11.9</td>
</tr>
<tr>
<td>Ulrich and Rhodes, 1983</td>
<td>0.23</td>
<td>0.018</td>
<td>6.7</td>
<td>16.7</td>
</tr>
<tr>
<td>Ulrich and Rhodes, 1983</td>
<td>-</td>
<td>0.021*</td>
<td>8.3</td>
<td>16.3</td>
</tr>
<tr>
<td>Ulrich and Rhodes, 1983</td>
<td>0.27</td>
<td>0.018*</td>
<td>9.2</td>
<td>16.0</td>
</tr>
<tr>
<td>Ulrich and Rhodes, 1983</td>
<td>-</td>
<td>0.005*</td>
<td>9.7</td>
<td>17.5</td>
</tr>
</tbody>
</table>

Table 1. The observed splittings $\delta v_4$ are averages over n of $\delta v_{n,4}$ over the range of observation. In the case of Grec et al., the frequencies are weighted with the power. Aside from those of Ulrich and Rhodes, for whom $\delta v_4 \equiv \delta v_{22,4}$ with n = 22, the theoretical values are averages from n = 16 to n = 28 weighted with the approximation to the envelope of the observed power given by Christensen-Dalsgaard and Gough (1982); these averages never differ from $\delta v_{22,4}$ by more than 3 per cent. The mixed model of Christensen-Dalsgaard is chemically homogeneous; the mixed model of Ulrich and Rhodes has had the inner 5 per cent of the mass homogenized.

* Only the ratios of the heavy-element abundances of the unmixed models of Ulrich and Rhodes are given. I have presumed that the "standard" has $Z = 0.018$, in agreement with that of Bahcall et al. (1982). The composition of the mixed model is probably similar.

The larger separations. Though the corresponding data for the models of Schatzmann et al. (1981) are not available, one would expect them to share the trend. Indeed, Ulrich and Rhodes (1983) report that the eigenfrequencies of a mixed model (with a helium distribution corresponding apparently to that of the model of Schatzmann et al. with a 'pseudo-Reynolds number' of about 200) are similar to those of their low-Z model listed in Table 1.

The outcome of this comparison is that if $Y = 0.25$, then the high frequency-separations of the mixed models are in conflict with observation. Evidently the gap could be reduced by changing the initial chemical composition, but that is likely to demand a rather large value for $Y_{p0}$. Alternatively one could increase

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t_\theta$. In either case the theoretical neutrino flux would appear to exceed the value that Schatzmann et al. maintain. Of course there may be other factors, which have not yet been taken into account, that could alter the situation.

**Future prospects**

The oscillations I have discussed are essentially adiabatic. Their dynamics is determined by pressure gradients causing acceleration, the latter being opposed by inertia. Fundamentally, this is true of both acoustic and gravity modes. In linearized theory, the perturbed pressure is determined uniquely in terms of the equilibrium value of the adiabatic exponent $\gamma$ and density perturbation; the equilibrium density and pressure are related through the Poisson equation that determines the gravitational potential and the equation of hydrostatic support.

Were it not for the dependence of $\gamma$ on the composition of the gas, oscillation eigenfrequencies would depend solely on the mass distribution in the sun. There would certainly be no direct method of determining the helium abundance from seismology alone. It is necessary to know the temperature too.

In fact, $\gamma$ varies, particularly in the ionization zones of hydrogen and helium. Therefore in principle a helioseismological determination of the present helium abundance of the convection zone might be possible.

In the sun's core ionization is almost complete, and the dependence of $\gamma$ on $Y$ is extremely weak. Therefore it seems unlikely that a direct unambiguous determination of the composition will actually be possible. It will no doubt be necessary to consider the nuclear reactions and the transport of radiation, which will demand invoking an assumption about the energy balance. It may be that the sensitivity of the eigenfrequencies is weak, permitting tight constraints on $Y$ to be imposed. If the converse were true, it might be possible to test the thermal balance. In any case, it seems likely that our deductions from helioseismology will lean on theoretical arguments beyond those concerned with the dynamics of the oscillations.

Solar observations can at best give us the sun's helium content now. To ascertain the protosolar abundance $Y_{p0}$ entails extrapolating backwards to $t = 0$. How reliable is that procedure?

Barring massive accretion or substantial mass loss, the evolution of the sun's energy output $L(t)$ appears to have been quite well determined. According to theory, the luminosity on the main sequence can be well approximated by

$$L(t) = \left[ 1 + \beta E(1 - t/t_\theta) \right]^{-1} L_\theta$$

(3)
where $E = L_\odot t_\odot /c = 0.046$ is the ratio of the energy that would have been radiated in the sun's lifetime had $L_\odot$ been fixed at $L_\odot$ to the energy $c$ (= 0.0070 Me$^{-2}$, where $M$ is the solar mass and here $c$ is the velocity of light) that would be liberated by converting a solar mass of hydrogen into helium. The parameter $\beta$ is about 6.5 if the core is unmixed (the precise value varies with $Y_{\odot}$, and is also dependent on opacity and nuclear reaction cross-sections), and about 9.5 if the sun were completely mixed. Provided the sun is in or near thermal balance, which it almost certainly is, these values probably bracket the plausible possibilities.

It is now possible to estimate the total amount of helium the sun must have produced in its lifetime. If the total mass of helium at any time is $\tilde{Y}$, then

$$ \frac{d\tilde{Y}}{dt} = \frac{L}{c}. $$

(4)

With the help of (3) this equation can be integrated to yield

$$ \Delta \tilde{Y} = \tilde{Y}(t_\odot) - \tilde{Y}(0) = \beta^{-1} \ln(1 + \beta E). $$

(5)

When $\beta = 9.5$, the production is $\Delta \tilde{Y} = 0.038$, and $\Delta \tilde{Y} = 0.040$ when $\beta = 6.5$. The difference between these values is rather smaller than the error in any estimate of the current helium content that we can anticipate making in the immediate future.

**Conclusion**

$Y_{\odot} = 0.25 \pm 0.02$ ?

I am grateful to W. Düppen, W.A. Dziembowski, R.W. Noyes and J. Perdang for useful conversations.

**References**


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**DISCUSSION**

D. Schramm: I believe your uncertainties will be even larger if you include the uncertainties in the input parameters which affect the solar neutrino flux. In particular, possible opacity and nuclear physics input could enable somewhat different neutrino fluxes than your model yielded, thus further complicating the issue.

D. Gough: At the present state of the art I agree with you. I indicated in my talk the kind of variation one finds when one changes the opacity or the equation of state; uncertainties in nuclear reaction cross-sections or the age of the sun, for example, aggravate the problem further. Our hope is that many more frequencies of free oscillation will be measured: in particular, those of both p and g modes of low order and low degree. Because the oscillations are essentially adiabatic, and therefore depend only on mechanics and adiabatic thermodynamics, we might then be in a position to measure at least the distribution of pressure and density in the sun irrespective of opacity, nuclear reaction rates, and the simplifying assumptions of stellar structure theory. Of course we shall need to rely somewhat on these uncertain issues to infer the helium abundance.