Granulation and Supergranulation as a Diagnostic Test of Solar Structure

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One of the most difficult tasks facing theoretical astrophysics today is to find a satisfactory model for the convective transport of energy in regions where the temperature gradient is superadiabatic. A number of such models have been proposed, such as the mixing-length theory and its extensions (Vitense 1953, Böhm-Vitense 1958, Spiegel 1963, Travis and Matsuhashi 1973 a, b, Parson 1969), to take into account the combined effects of convection and turbulence but it is generally agreed that, to-date, no satisfactory theory has been put forward.

In most stars, certain layers are unstable against convection and it would be a great help if we were possible to decide on the validity of competing theories (Zahn 1981). The Sun could prove to be most useful in this regard since it has an extensive convective region in its outer layers and since it is near enough for us to investigate in some detail its surface structures. In particular large-scale convective phenomena such as granulation and supergranulation could improve our understanding of the outer layers of the Sun, and provide a test for the accuracy of the models that have been proposed.

In the case of granulation we know (Beckers 1979, Bray and Loughhead 1967, Böhm 1977) that the granules have an average horizontal extent of about 2000 km, that horizontal and vertical velocities of the order of 1 km/s (Dravins 1975, Mattig 1980, Canfield 1976, Durrant et al. 1979, Keil and Canfield 1978) have been observed, that the granules show an intensity fluctuation of about 15% (Gibson 1973, Deubner and Mattig 1975, Beckers 1979, Keil and Canfield 1978) that a temperature difference of about 900 K (Keil and Canfield 1978, Keil 1980) exists between the ascending and descending currents and that the average life-time of a granule is about 20 minutes (Bray and Loughhead 1967).

On the other hand, supergranules have an average horizontal extent of 30,000 km (Böhm 1977, Bray and Loughhead 1967), horizontal and vertical velocities of 0.3 to 0.5 km/s (Keil and Canfield 1978, Gibson 1973, Bray and Loughhead 1967) have been observed but no observable intensity fluctuation (Böhm 1977, Gibson 1973, Bray and Loughhead 1967) is present. This last observation seems to indicate that supergranulation is essentially a very weak type of convection which is unlikely to contribute much to the overall transport of energy.

For hexagonal cells, the horizontal wave number $a$ is given by

$$a = \frac{8\pi d}{3\ell} \quad (1)$$

where $d$ is the depth of the convection layer and $\ell$ is the diameter of a convection cell.

It is known from the linear theory of convective motions that the critical horizontal wave number is of the order of $\pi/\sqrt{2}$. This corresponds to the highest degree of instability and approximately to maximum efficiency in convective transport of energy.

With this value of $a$ it follows from equation (1) that the average depth of the granules and supergranules is of the order of 530 km and 7954 km respectively.

The medium in which granulation and supergranulation occur is highly turbulent. This can be seen in a number of ways:

(i) Over a depth $d$ of 530 km, with a characteristic velocity $U$ of 1 km/s and adopting a molecular kinematic viscosity $\nu_0$ of $5 \times 10^9$ the Reynolds number $R_e$ is

$$R_e = U d / \nu_0 = 10^9$$

This number is several orders of magnitude larger than the value of $5 \times 10^8$ for which, according to theory and laboratory experiments, turbulence is about to start.

(ii) In the Boussinesq approximation, the Rayleigh number $R$ is defined as follows

$$R = \frac{g \beta}{\kappa} \left( \frac{\partial T}{\partial z} \right)^2 - \frac{\partial T}{\partial z} \left( \frac{\partial T}{\partial z} \right) \frac{d_T}{d_T + d_X} \kappa \nu \quad (2)$$

where $x$ and $\nu$ are the thermal diffusivity and kinematic viscosity respectively.

Assuming that the Prandtl number is equal to unity, i.e. that $\kappa = \nu = 5 \times 10^8$ and using the Böhm-Vitense model it follows that, for a layer with a depth of 530 km the Rayleigh number $R$, as given by (2), equals $2 \times 10^{11}$. This is again much larger than the linear value $R_L = 1.3 \times 10^4$ for which convection is about to start. Steady convection at such high Rayleigh numbers would be impossible since it would lead to enormous velocities, which are not observed, and extremely thin boundary layers which could not establish themselves in such a medium with free boundaries. In other words, the medium will become turbulent.

No fully satisfactory theory of turbulence exists but most studies of large-scale convective motions in a turbulent...
medium which have been developed so far use the concept of global-convection and assume that the effects of turbulence can be adequately represented by a turbulent kinematic viscosity $\nu$, and a turbulent diffusivity $\kappa$, both of which may of course be depth dependent.

Ideally both these quantities should be given by the model under investigation, whereas the value of $\kappa$ can be inferred from the data contained in the model, the value of $\nu$ must be assumed unless given independently by the theory of turbulence.

The radiative flux $F_r$ is given by

$$ F_r = 4 \pi r^2 \frac{dT}{dz} / (3k) $$

where $k$ is the opacity of the stellar material.

According to the Böhm-Vitense model the ratio of radiative to observed flux $F_r/F_{\text{tot}} = 0.00615$ at a depth of 500 km and only reaches the value of 1 at a depth of 200,000 km. The transport of energy in the layer under consideration, i.e. down to a depth of 8,000 km, is mainly turbulent and at each level the turbulent thermal diffusivity $\kappa$, is given, in first approximation, by

$$ \kappa = \kappa_0 \frac{\rho_0 q_0}{(1-\tau)} \frac{dT}{dz} $$

and it can easily be seen, from the Böhm-Vitense model, that $\kappa$ is of the order of $10^{27}$ and therefore very much larger than either the molecular diffusivity $\kappa_m = 5 \times 10^{10}$ or radiative diffusivity $\kappa_r = 5.5 \times 10^{9}$.

If $\kappa_0$ is defined as the turbulent diffusivity at the top of the convective layer, then

$$ \kappa = \kappa_0 \xi(z) $$

where $\xi(z)$ is a function of the depth $z$ given by the model to be tested and $\xi(0)=1$ at the top of the layer.

The turbulent kinematic viscosity $\nu$ can be written as follows

$$ \nu = \mu / \rho = \nu_0 \xi(z) / \rho_0 q_0 \xi(z) / \nu_0 \xi(z) $$

where $\mu_0$ and $q_0$ are the turbulent viscosity and the density at the top of the layer, $\xi_0$ is a function of depth given in the model under consideration and $\xi(z)$ is a function of depth given either independently by the theory of turbulence or by making suitable assumptions as to its form.

If we assume that the viscosity $\mu$ is not depth dependent it follows that $\xi = 1$ and we have model A as defined by Graham and Moore (1978). If, on the other hand we assume that the kinematic viscosity $\nu$ is a constant it follows that $\xi = \xi_0(z)$ and we have case B of the above paper.

The Prandtl number is defined as follows

$$ \sigma = \nu_0 / \kappa_0 $$

and its value has to be determined in such a way as to reproduce one of the observed characteristics of granulation or supergranulation.

Once convective motions, such as granulation and supergranulation, set in they will carry a certain amount of energy, especially in the case of granular motions. The theoretical flux $F_\alpha$ obtained from non-linear convective theory, will be larger than $F_m$ used to find the value of $\kappa$, as given in equation (3) which is only approximate or, in other words, the value of $\kappa$ has been underestimated.

It follows that the value of $\kappa_0$ used in the numerical integrations, must be obtained from

$$ \kappa_0 = \kappa(1-\tau) \frac{dT}{dz} / \tau $$

where $F_m$ is the observed flux and $\tau$ is the fraction of energy transported by the convective motions, in fact by granulation since supergranulation is unlikely to contribute much to the energy flux as indicated by its insignificant flux modulation.

Of course, $\tau$ is not known in advance and several numerical integrations have to be carried out until the total flux due to turbulence and granulation equals the observed flux.

In order to take time-dependence into account the numerical integrations could be carried out for say 20 minutes with values of $\tau$ and $\sigma$ chosen in such a way as to obtain the correct flux and a horizontal velocity of 1 km/s for granulation at the top of the convective layer.

The other observed characteristics, such as flux modulation in granules and velocity measurements in supergranules can then be used to test the accuracy of the various models that have been proposed. For instance, the ordinary mixing-length theory and its non-local extensions (Norlund 1974) vary in one important respect. The non-local theories have a smaller temperature gradient in the upper layers. This is likely to have a marked effect on the numerical integrations and probably on the observed flux modulations of the granules.


