1.5 Lie-Integration of Planetary Motions

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The aim of this work is the construction of a fast inte-
gration method for differential equations, especially
the equations of the motion of celestial bodies
taking into account non-conservative forces. Although
a number of integration schemes are available none of
them seems to be adequate for treating n-body systems
with variable masses, which arise in some cosmogonic
problems of the early solar system. As a first step
we are now able to present a high-speed numerical
integration scheme of the classical n-body system.
The basic ideas of solving differential equations with
Lie-series is due to W. Gröbner (1967) but, unfortu-
nately, he did not elaborate on this method
and stopped after some numerically unsatisfactory
results. We could simplify the calculation of the
Lie-terms and derived finally a recurrence formula
for the Lie-terms. Whereas W. Gröbner tried to solve
the two-body and three (n-body) problem by two
different approaches we solved, at first, in an
optimal way the 2-body-problem. Then we were able
to derive in a quite similar way the solutions of
the 3-body and n-body system.

Our integration method for planetary motions has two
major advantages:
First, it is a relatively fast method (about the factor
3-10 faster in comparison with the n-body program by
Schubart-Stumpf, which is commonly used by Astronomers).
Second, because larger step lengths can be used, round-
off errors are smaller (e.g. step length 135 days for
Jupiter).

1.6 A First Order Planetary Theory with Elliptic
Functions. C. W. WILLIAMS, U. of South Florida. U. VAN
FLANDERN, U. of South Florida -- Using a variation of elements approach the
differential equations representing the perturbations on
one planet by another are solved to first order complete-
liness in terms of elliptic integrals of the first and second
kind and Jacobian elliptic functions. The reference or-
bit for both planets is a circle. The solutions may be
extended to the elliptic case. The first order solution
for n and a (where the mean longitude, \lambda = nt + \epsilon )
can be written in closed form and is equivalent to the so-
lution presented by Richardson (B.A.S., 13: 2, 571,
1981). The rest of the solution is written as an in-
finite series whose terms can be easily generated by re-
currence relations. The contribution to the secular
terms is isolated to all orders. This form of the
solution may have more rapid convergence in powers of \epsilon
than more traditional methods as a approaches unity,
where a is the ratio of the semi-major axes.

1.7 Diffusion of Cometary Orbits into the Plane-
tary Region. P. R. WEISSMAN, Jet Propulsion Laboratory
-- New studies of the dynamical evolution of cometary orbits
in the Oort cloud are made using Monte Carlo simulation
techniques. The program uses an improved computer code
which more accurately models the perturbation of Oort
cloud comets by the major planets. It is shown that
perturbations by Saturn provide a substantial barrier to
the diffusion of cometary perihelia into the inner solar
system. Jupiter perturbations do this also, though to a
lesser extent. A substantial fraction of incoming comets
are ejected to hyperbolic orbits or evolved to shorter
period orbits, \rho < 10^5 yrs, before their perihelia can
diffuse to values in the observable region, \rho < 5 AU.
Perturbations by Uranus and Neptune do not substantially
stop comets from diffusing to smaller perihelia but do
result in greater dispersion in the orbital energy of the
dynamically new comets. Thus, the inner solar system is
undersupplied in dynamically new comets. Conversely, the
observed flux of Oort cloud comets for \rho < 4 AU implies a
significantly greater total cloud population than previ-
ously thought. The new estimates of the Oort cloud pop-
ulation range from 1.8 to 2.2 x 10^{12} comets. The flux of
new comets at the orbits of Jupiter, Saturn, Uranus, and
Neptune are, respectively, 1.3, 3.1, 3.6, and 5.0 times
that at the earth's orbit. For Saturn, Uranus, and Nep-
tune the relative flux of all long-period comets is even
greater because these planets are not as efficient as
Jupiter in dynamically removing comets from the solar
system. The flux of new comets from the Oort cloud con-
tinues to increase with increasing perihelion distance
until \rho ~ 80 AU where it levels out with a flux 8.5 times
that at 1 AU. This work was supported by the NASA
Planetary Geophysics and Geochemistry Program.

1.8 The Critically Spinning Earth. R. S.
HARRINGTON & T. C. VAN FLANDERN, USNO. The critically
spinning Earth has been numerically simulated by a
spinning disk with an outer ring broken into pieces
irregularly spaced around the circumference and free
to escape and gravitationally interact. The subsequent
Earth spin was computed from conservation of angular
momentum. Often, a significant number of pieces would
coalesce into a body of sizable mass in an orbit
completely outside the inner synchronous orbit (which
is at 2.3 Earth radii). The implications for lunar
origin are evident.

1.9 Poynting-Robertson Force Allowing for
Wavelength-Dependent Reflection Coefficients and Non-
Spherical Shapes. V. J. SLADINOSKI, Communications
Satellite Corp. - The Poynting-Robertson force appears
to be the largest drag force on synchronous-altitude
satellites. This paper derives the force for a body
large compared to a wavelength of light by computing
the solar radiation force in the reference system in
which the body is momentarily at rest. The approach
allows easy inclusion of non-spherical shapes and the
wavelength-dependent reflection of incident sunlight.
This leads to the actual demonstration that the Poynting-
Robertson force does not depend greatly on body shape,
or on the reflection coefficients with a normal star
like the sun.

Reflection can drastically change the force in the
artificial case of a star that emits radiation in a
single spectral line. When the line falls at the edge
of an absorption band for a spherical body orbiting
the star, the force can change from a drag to an
accelerating force. The orbital semi-major axis then
increases rather than decreases (as occurs for
perfectly absorbing bodies).