1.5 Lie-Integration of Planetary Motions

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The aim of this work is the construction of a fast integration method for differential equations, especially the equations of the motion of celestial bodies taking into account non-conservative forces. Although a number of integration schemes are available none of them seems to be adequate for treating n-body systems with variable masses, which arise in some cosmogonic problems of the early solar system. As a first step we are now able to present a high-speed numerical integration scheme of the classical n-body system. The basic idea is the solution of the Lie-series is due to W. Grünber (1967) but, unfortunately, we did not succeed in this method and stopped after some unsatisfactory results. We could simplify the calculation of the Lie-terms and derived finally a recurrence formula for the Lie-terms. Whereas W. Grünber tried to solve the two-body and three (n-body) problem by two different approaches we solved, at first, in an optimal way the 2-body-problem. Then we were able to derive a quite similar way the solutions of the 3-body and n-body system.

Our integration method for planetary motions has two major advantages:
First, it is a relatively fast method (about the factor 3-10 faster in comparison with the n-body program by Subbarat-Stumpf, which is commonly used by Astronomers). Second, because larger step lengths can be used, round-off errors are smaller (e.g. step length 135 days for Jupiter).

1.6 A First Order Planetary Theory with Elliptic Functions

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Using a variation of elements approach the differential equations representing the perturbations on one planet by another are solved to first order completely in terms of elliptic integrals of the first and second kind and Jacobian elliptic functions. The reference orbit for both planets is a circle. The solutions may be extended to the elliptic case. The first order solution for \( n \) and \( \alpha^2 \) (where the mean longitude, \( \lambda = n t + \alpha \)) can be written in closed form and is equivalent to the solution presented by Richardson (J.A.S., 13, 2, 571, 1961). The rest of the solution is written as an infinite series whose terms can be easily generated by recurrence relations. The contribution to the secular terms is isolated to all orders. This form of the solution may have more rapid convergence in powers of \( \alpha^2 \) than more traditional methods as a approaches unity, where \( \alpha \) is the ratio of the semi-major axes.

1.7 Diffusion of Cometary Orbits into the Planetary Region

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New studies of the dynamical evolution of cometary orbits in the Oort cloud are made using Monte Carlo simulation techniques. The program uses an improved computer code which more accurately models the perturbation of Oort cloud comets by the major planets. It is shown that perturbations by Saturn provide a substantial barrier to the diffusion of cometary perihelia into the inner solar system. Jupiter perturbations do this also, though to a lesser extent. A substantial fraction of incoming comets are ejected to hyperbolic orbits or evolve to shorter period orbits, \( p < 10^7 \) yrs, before their perihelia can diffuse to values in the observable region, \( q < 5 \) AU. Perturbations by Uranus and Neptune do not substantially stop comets from diffusing to smaller perihelia but do result in greater dispersion in the orbital energy of the dynamically new comets. Thus, the inner solar system is undersupplied in dynamically new comets. Conversely, the observed flux of Oort cloud comets for \( q < 4 \) AU implies a significantly greater total cloud population than previously thought. The new estimates of the Oort cloud population range from \( 1.8 \) to \( 2.2 \times 10^{12} \) comets. The flux of new comets at the orbits of Jupiter, Saturn, Uranus, and Neptune are, respectively, \( 1.3, 2.1, 3.6, \) and \( 5.0 \) times that at the earth's orbit. For Saturn, Uranus, and Neptune the relative flux of all long-period comets is even greater because these planets are not as efficient as Jupiter in dynamically removing comets from the solar system. The flux of new comets from the Oort cloud continues to increase with increasing perihelion distance until \( q 

1.8 The Critically Spinning Earth

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The critically spinning Earth has been numerically simulated by a spinning disk with an outer ring broken into pieces irregularly spaced around the circumference and free to escape and gravitationally interact. Subsequent Earth spin was computed from conservation of angular momentum. Often, a significant number of pieces would coalesce into a body of sizable mass in an orbit completely outside the inner synchronous orbit (which is at 2.3 Earth radii). The implications for lunar origin are evident.

1.9 Pointing-Robertson Force Allowing for Wavelength-Dependent Reflection Coefficients and Non-Spherical Shapes

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The Pointing-Robertson force appears to be the largest drag force on synchronous-altitude satellites. This paper derives the force for a body large compared to a wavelength of light by computing the solar radiation force in the reference system in which the body is momentarily at rest. The approach allows easy inclusion of non-spherical shapes and the wavelength-dependent reflection of incident sunlight. This leads to the actual demonstration that the Pointing-Robertson force does not depend greatly on body shape, or on the reflection coefficients with a normal star like the sun.

Reflection can drastically change the force in the artificial case of a star that emits radiation in a single spectral line. When the line falls at the edge of an absorption band for a spherical body orbiting the star, the force can change from a drag to an accelerating force. The orbital semi-major axis then increases rather than decreases (as occurs for perfectly absorbing bodies).