MAGNETOHYDROSTATIC MODEL OF SOLAR FACULAE

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ABSTRACT

A self-similar magnetohydrostatic model of solar faculae is presented. The model is based on the Schlüter-Temesvary equation, originally derived for sunspots. Magnetic tension and twisted magnetic field are taken into account. The exact magnetohydrostatic solution obtained from an observationally derived pressure deficit basically confirms Chapman's semiempirical facular model without tension.

The difference between a facula and a sunspot and the thermodynamic consequences of a twisted magnetic field are discussed.

Pressure and temperature profiles at different heights are presented.

Subject headings: hydromagnetics — Sun: faculae

I. INTRODUCTION

During the last decade several attempts to construct facular models (or solar network models) have taken place (Livshits 1963; Chapman 1970, 1979; Shine and Linsky 1979; Koutchmy and Stellmacher 1978; Stenflo 1973; Unno and Ribes 1979; Spruit 1976). From these models the facula appears as a magnetic tube with a cool bottom, resembling a sunspot, and a hot top.

The attempts mentioned above demonstrated two difficulties in modeling faculae. From the line profiles it is possible to construct a facular atmosphere (P and T vertical distributions) and to compare it with the quiet atmosphere outside the facula. What is really difficult is to make a geometric shift of these two atmospheres. Combining intensity measurements with magnetohydrostatic theory, we can determine the proper shift between the atmospheres. Physically this shift depends on the magnetic field, and as a first guess the Wilson depression can be chosen on the basis that the deficit of pressure is equal to the magnetic pressure (Chapman 1979). This means neglecting magnetic tension forces. This choice is possible if one has some measurements of B for the facula. The second difficulty appears in the analysis of Stokes's parameters. In order to do the analysis one has to assume some field-strength profile (Stenflo 1973). For example, for this purpose Rees and Semel (1979) took a self-similar distribution of magnetic flux without solving any force balance. Both of the difficulties mentioned demand a self-consistent magnetohydrostatic model, which can provide a correct photometric picture as well as opportunities for Stokes's parameters analysis. The aim of this paper is to construct that magnetohydrostatic model of solar faculae.

Because of a lack of information on the facula energy transport mechanism, the basic equation will be solved for the observationally derived pressure deficit AP (z). As a final result, a photometric picture in the continuum will be obtained to demonstrate the appearance of the presented model. The particular questions of the influence of tension forces on the solution will be considered in detail.

II. BASIC THEORY

The plasma equilibrium inhomogeneous gravitational field can be described by the set of equations:

$$\frac{1}{4\pi} (\nabla \times B) \times B = \nabla P - \rho g,$$

and

$$\nabla \cdot B = 0.$$

Schlüter and Temesvary (1958; hereafter ST), assuming that the relative distribution of the magnetic flux through

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the horizontal cross section of a tube is everywhere geometrically similar, reduced the system (eqs. [1]-[2]) to a nonlinear ordinary differential equation for a sunspot, neglecting the azimuthal magnetic field.

The first attempt to include the azimuthal magnetic field in ST sunspot theory was made by Yun (1971) under a strong restriction that the azimuthal component $B_\phi$ has to be proportional to the radial component $B_r$. Then Low (1975) demonstrated that the system (1)-(2) for axisymmetric configurations may be reduced to the scalar equation

$$\frac{\partial^2 \psi}{\partial z^2} - \frac{r}{\partial r} \left( \frac{1}{r} \frac{\partial \psi}{\partial r} \right) + I_A \frac{dI_A}{d\psi} = -4\pi r^2 \frac{\partial}{\partial \psi} P(\psi, z),$$

where

$$B_r = -\frac{\partial \psi}{\partial z}, \quad B_\phi = \frac{\partial \psi}{\partial r}, \quad B_\psi = \frac{I_A(\psi)}{r},$$

and

$$P = P_0(\psi) \int_0^\infty \exp \left[ -\frac{\mu}{kT\psi(z')} \right] dz',$$

where $\mu$ is molecular weight, $k$ is the Boltzman constant, $T$ is temperature, and $P$ is gas pressure.

On the basis of equation (3) it is possible to obtain an ST-type equation with a twisted field and having no special geometrical restrictions on $B_\phi$ (Osherovich 1979). The ST similarity assumption corresponds to one parametric representation of a flux function:

$$\psi = \psi(\alpha), \quad \alpha = r\xi(z).$$

Using equations (6) and (7), we write equation (3) in this form:

$$\frac{1}{2} \frac{d}{dx} \left( \frac{d\psi}{dx} \right)^2 + \frac{1}{\alpha} \frac{d}{dx} \left( \frac{d\psi}{dx} \right)^2 \xi'' + \frac{1}{2} \frac{d}{dx} \left( \frac{d\psi}{dx} \right)^2 \xi^4 + \frac{1}{2\alpha^2} \frac{d}{dx} I_A^2 \xi^2 = -4\pi \frac{\partial P}{\partial \alpha}.$$

Integration over $\alpha$ gives us the basic equation:

$$(\xi')^2 J_1(\alpha) + \xi\xi'' J_2(\alpha) + \xi^4 J_3(\alpha) + \xi^6 J_4(\alpha) = -4\pi [P(\infty, z) - P(0, z)],$$

where

$$J_1(\alpha) = \frac{1}{2} \int_\alpha^\infty \frac{d}{d\alpha} \left( \frac{d\psi}{d\alpha} \right)^2 d\alpha,$$

$$J_2(\alpha) = \frac{1}{\alpha} \int_\alpha^\infty \frac{d}{d\alpha} \left( \frac{d\psi}{d\alpha} \right)^2 d\alpha,$$

$$J_3(\alpha) = \frac{1}{2} \int_\alpha^\infty \frac{d}{d\alpha} \left( \frac{d\psi}{d\alpha} \right)^2 d\alpha,$$

$$J_4(\alpha) = \frac{1}{\alpha^2} \int_\alpha^\infty \frac{d}{d\alpha} I_A^2 \xi^2 d\alpha.$$

For the Gaussian distribution,

$$\psi = \psi_0(0)e^{-z^2}.$$ 

Integration from 0 to $\infty$ gives us the basic equation

$$\xi' \xi'' - 2\xi^4 + \xi^6 J_4(0) = -\frac{4\pi \Delta P(z)}{\psi_0^2(0)},$$

where

$$\Delta P(z) = P(\infty, z) - P(0, z).$$

The only requirement for the azimuthal field is that the integral (13), i.e., $J_4(0)$, be finite. The first term in the equation (15) represents the tension force, which exists even without twisting. In our paper we will solve the basic equation (15) taking the pressure deficit from Chapman's (1979) facular model.

Studying twisted field models we will consider cases a and b.

Case a: $I_A = I_A(0)z^2e^{-z^2},$
which leads to

$$J_4(\alpha) = \frac{I_4(0)}{4} (1 - 2\alpha^2)e^{-2\alpha^2},$$

and to the equation

$$\xi^4 - 2\xi^4 + S^2 \xi^2(0) \xi^2 \frac{1}{4} = -\frac{4\pi \Delta P(z)}{\psi_0(0)},$$

where the nondimensional parameter

$$S = \frac{I_4(0)}{\xi(0)\psi_0(0)}$$

characterizes $B_\phi$.

Case b: \( I_A = I_A(0) e^{-2\alpha^2} \),

which leads to

$$J_4(\alpha) = \frac{I_4(0)}{8} (1 + 2\alpha^2 - 4\alpha^4)e^{-2\alpha^2}$$

and to the equation

$$\xi^4 - 2\xi^4 + S^2 \xi^2(0) \xi^2 \frac{1}{8} = -\frac{4\pi \Delta P(z)}{\psi_0(0)}.$$

Case a corresponds to Yun's model when the angle between $B_\phi$ and $B_r$ at the same geometrical level does not change with radius. Case b describes a model in which the tangent of the angle mentioned increases linearly with radius.

III. NUMERICAL SOLUTIONS OF THE BASIC EQUATION

First we construct a model without $B_\phi$. We assume the Wilson depression to be

$$\Delta Z_W = 175 \text{ km}.$$

The shift between two atmospheres defines $\Delta P(z)$ (Chapman 1979). We took the zero level ($z = 0$) to be the point where $B_z(0, 0) = 2050$, assuming that at this level

$$\frac{B_z^2(0, 0)}{8\pi} = \Delta P(0).$$

We defined the radius of our facula $R_f$ at the zero level as follows:

$$R_f = 1.63 \left[ \frac{\phi}{\pi B_z(0, 0)} \right]^{1/2},$$

where $\phi$ is the total magnetic flux through the horizontal surface. This definition differs from Chapman's by a factor of 1.63, which comes from the empirical spot formula for the magnetic flux, restricted by the boundary radius $R_f$:

$$\phi_f = 0.35 B_{\text{max}} R_f^2 \pi.$$

The difference in definitions has little influence on the final results if we have $R_f = 200-400 \text{ km}$. (Actually we took $R_f = 200 \text{ km}$.) However, for larger tubes the difference is important.

It will be shown that choice (25) is valid for a small $R_f$ (200-400 km). For that $R_f$, a tension-free solution has special significance in the sense that even if one starts out with a boundary magnetic pressure on the axis not equal to $\Delta P(z)$, it will rapidly relax to the tension-free curve.

Then starting from $z = 0$ we solve the basic equation (15) for $J_4(0) = 0$. Among all possible solutions for the structural function $\xi(z)$ we consider only $\xi(z) \to 0$ as $z \to \infty$ to be a physical solution. From formulae (14) and (4) we can derive components of $B$. In Figure 1 the continuous line represents the vertical distribution of the magnetic field for the $B_\phi = 0$ model. Pressure distribution is derived from formula (9) (see Fig. 3), and temperature profiles at different geometrical heights are obtained from the formula

$$\frac{-1}{T} = \left[ \frac{k}{\mu \partial \xi} \left|_{\xi = \text{const}} \right. \right] \frac{1}{P},$$

which follows from equation (5). Temperature distributions (Fig. 4) demonstrate that a facular bottom really resembles
Fig. 1.—The vertical distribution of the magnetic field at the axis of symmetry $B = B_0(0, z)$. Dashed line corresponds to the model with twisted magnetic field (case b) with $S = -0.5$; dot-dashed line to that with $S = -1$.

a sunspot, but the top is hotter than the surrounding plasma. With azimuthal field having the same $\Delta P(z)$, the initial magnetic field $B(0)$ will be a little bit stronger. For case a we have

$$\Delta P(0) = \frac{B_0^2(0, 0)}{8\pi} \left( 1 - \frac{S^2}{4} \right), \quad (29)$$

and for case b, instead of $S^2/4$ we have $S^2/8$. The vertical distribution $B_0(0, z)$ for a facula with twisted magnetic field (case b) is represented by a dashed line ($S = -0.5$) and dot-dashed line ($S = -1$). Horizontal distribution for $B_0(0, r)$, $B_0(0, r)$, $B_0(0, r)$ are presented in Figure 2 (case b, $S = -1$). In the same figure, for comparison, we draw $B_0(0, r)$ and $B_0(0, r)$ for the untwisted model (dashed lines). We found that the azimuthal field ($S = -1$) changes our curves for $T$ and $P$ only a little (less than 5%). For example, in the presence of $B_0$ the pressure is a small percentage lower than the pressure for the untwisted model. For our $R_f$, if we go up not more than a few hundred kilometers, then $B_0 \ll B_0$, and we can neglect the first term in equation (15); i.e., we can neglect tension.

Then for the case $B_0 = 0$ we have a simple representation of the structural function:

$$\xi(z) = \left[ \frac{2\pi \Delta P(z)}{\psi_0^2(0)} \right]^{1/4}, \quad (30)$$

IV. THE CALCULATION OF INTENSITY

Using the presentation (30) from equations (9) and (28) we can derive $P-T$ distributions (given in Figs. 3 and 4) and then calculate the intensity as a final check of our choice of the Wilson depression $\Delta Z_w$. It becomes clear that variations of $\Delta Z$ effectively change $\Delta P(z)$ and $P-T$ profiles along with the magnetic field. Adjusting the theoretically derived photometric picture to an observational photograph, we could finally find $\Delta Z_w$, which would give us a self-consistent magnetohydrostatic model for a particular facula.

In this paper we present an example of the calculation of intensity. Using the $P-T$ profiles derived from Equation (30), we have constructed profiles of electron pressure and absorption coefficient assuming LTE. Calculations of intensity in the vertical direction were calculated for every 5 km from the axis to a distance of over 100 km, by which position the contrast was less than 0.5% above the quiet Sun. Contrast profiles are shown in Figure 5 for four values of the Wilson depression, $\Delta Z_w$. The light and solid curves are for $\Delta Z_w = 10$ and 162 km, respectively, the dashed curve is for $\Delta Z_w = 206$ km, and the dot-dashed curve is for $\Delta Z_w = 220$ km. All values of contrast are for a wavelength of 0.53 $\mu$m. As one can see, the mean contrast and the contrast profile depend sensitively on the Wilson depression. The inner three contrast values for $\Delta Z_w = 220$ km are not accurate because the largest value of optical depth for these ray paths was less than unity (at the greatest geometrical depth $t = 0.69$ and 0.94 at $r = 0$ and 5 km, respectively). The trend is clear in the four models, however, that the contrast profile should have a “hole” in the
Fig. 2.—Horizontal distribution for the components $B_z(0, r)$, $B_r(0, r)$, and $B_\phi(0, r)$ (case b, $S = -1$). Dashed lines represent $B_z(0, r)$ and $B_r(0, r)$ for the untwisted model.

Fig. 3.—Pressure distribution at different geometrical heights ($B_\phi = 0$)
Fig. 5.—Horizontal distribution of intensity contrast in the vertical direction for four values of the Wilson depression, $\Delta Z_w$. The Wilson depressions of 101, 162, 206, and 220 km are represented by light solid, heavy solid, dashed, and dot-dashed lines, respectively. The central contrast is not known for the two larger values of $\Delta Z_w$ because the facular model had insufficient opacity.
Fig. 6.—Magnetic field on the axis of the facula as a function of height for different radius of faculae, $R_f$.

Fig. 7.—Role of tension for solar faculae
center. This hole will have a FWHM of ~35 km and thus is below the resolution in the mid-visible of a 1.25 m aperture space-borne telescope. Such a telescope, if diffraction-limited in the near UV, might detect this hole. The effects of two-dimensional radiative transfer could reduce the contrast and detectability of the calculated contrast profile.

V. DISCUSSION

Here we will discuss the influence of tension on the facular model, where $B_o = 0$. Comparing the first term in equation (15) with the second one, which represents magnetic pressure $B^2/8\pi$, we note that two factors are important: the radius of the magnetic tube and the height. Figure 6 shows five curves for the magnetic field on the axis of the facula. All five models are constructed for the same $\Delta P(z)$; the only difference is the choice of the radius $R_f$ at $z = 0$. From Figure 6 one can see that for $200 \leq R_f \leq 600$ km, Chapman’s model works well. For larger tubes, tension begins to play a more important role. But even for small faculae when we go up to 500 km, we also cannot neglect tension. All these conclusions are summarized in Figure 7. The curve presented separates the region where tension does not play a significant role from the region where the tension term is more than 10% of the magnetic energy $B^2/8\pi$ (all points above the curve). The dashed vertical line indicates the usual facular region. The vertical scale is the geometrical height. All calculations were made for $\Delta P(z)$ from Chapman’s semiempirical model. As long as we model small faculae (200–400 km), tension is small, and ST theory adequately describes the physical situation. For large plages some magnetic lines can return to the horizontal plane, increasing tension. Probably in such a case we have to employ the recently developed return flux model to get a correct description of facular phenomena (Osherovich 1982).

VI. CONCLUSIONS

We have constructed a self-consistent magnetohydrostatic model for solar faculae. As a result of our belief that there is no deep physical gap between the magnetic field of a facula and a sunspot, we have described faculae using ST sunspot theory. There are exact magnetohydrostatic solutions with small tension close to Chapman’s facula model. At least for small tubes, $R_f = 200–400$ km, photometric pictures can be calculated neglecting tension. We found that in terms of magnetic structure the main difference between facular and sunspot magnetic structures comes from the vertical gradient of the magnetic field. For a facula, tension is not so important, $dB/dz \sim 3–4$ gauss km$^{-1}$, and the magnetic field does not penetrate to the chromosphere. For a sunspot, the tension is important, $dB/dz \sim 0.3–0.5$ gauss km$^{-1}$, and the magnetic field does penetrate to the chromosphere.

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REFERENCES


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