OBSERVATIONS OF GLOBAL-SCALE PHOTOSPHERIC FRAUNHOFER LINE SHIFTS

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ABSTRACT

Transform techniques have been applied to a 1 yr sequence of solar photospheric Fraunhofer line shift data. In agreement with earlier studies, we find no evidence of large-scale structure in mean spatial power spectra near the level of 12 m s$^{-1}$. It is argued that such spectra are easily dominated by random supergranule noise, and that these data may even be used to estimate supergranule characteristics. By considering a rotation signal in the temporal transformed data, we find statistical evidence (at the 3σ level) of a large-scale, long-lived, photospheric line shift field. A simple model suggests spatial scales of $10^5-10^6$ km, with lifetimes of at least 3 days and line shifts corresponding to velocity amplitudes near 2 m s$^{-1}$. While the interpretation of the residual line shift is not unambiguous, we suggest that this is the Doppler velocity signature of large-scale convective cells.

Subject headings: convection — Sun: atmospheric motions — Sun: granulation

I. INTRODUCTION

It is likely that the interior properties of the Sun will be inferred only from measurements of large-scale photospheric parameters. For example, global-scale photospheric velocity measurements have already provided data concerning a radial gradient in the solar rotation rate (Deubner, Ulrich, and Rhodes 1979). In particular, the search for conclusive evidence of giant convective cells is important for understanding the physics of the convection zone. Several models of giant-cell convection (cf. Durney and Spruit 1979; or Roxburgh and Tavakol 1979) have been proposed. An important common feature of these models is that transverse convective-cell scales comparable to the convection zone depth should be present near the photosphere. Considering a standard solar model (Sears 1964), this implies transverse scales of at least $10^5$ km. Several authors have reported qualitative measurements of large-scale features. Howard and Yoshimura (1976) reported observations of 30° velocity regions that persist for at least one rotation period. Bumba (1976) has seen qualitative evidence of an angular scale near 35° in his magnetograph data.

After this paper was submitted, the work of Labonte, Howard, and Gilman (1981) appeared, reporting an upper limit to the giant-cell velocity amplitude of between 5 and 12 m s$^{-1}$. This bound was derived from the mean spatial power spectra of their velocity maps. Because the spatial form, rotation rate, lifetime, and solar latitude distribution of such convective regions are unknown, they found that a systematic search for these features is difficult. The 5 minute oscillation, supergranulation, and other velocity "noise" at shorter wavelengths and presumably larger amplitudes were shown to be a limiting factor in the Mount Wilson data. We report here on an alternative method for looking for large-scale velocity features that assumes very little about their wavenumber or rotation rate.

These data are obtained from only one observation per day; thus we also expect the velocity measurements to be dominated by granule, supergranule, and 5 minute oscillation noise. It is straightforward to show how these, generally, small-scale structures contribute power to low-spatial frequency velocity measurements. This is most evident in mean spatial power spectra. If there is a global coherence to the low-frequency components of these velocity structures, with a lifetime longer than the interval between observations, then we may hope to see a rotation signature in the two-dimensional $k-\omega$ power spectra at velocity amplitudes much smaller than the incoherent low-spatial frequency noise. This paper reports on the quantitative detection of large-scale velocity features by using a spatial and temporal transform approach. The next section describes the data, § III outlines the analysis procedure, and the last sections discuss the results and conclusions.

II. DATA COLLECTION

Line-of-sight velocity information is obtained from the spatial location of the cores of solar Fraunhofer lines recorded on film. Terrestrial absorption lines serve as a
wavelength calibration on the film. The observed velocities, determined from the line core displacement, are seen to be closely related to the actual photospheric velocity near the temperature minimum along the line of sight.

Photographic data were obtained with the Sacramento Peak Observatory Littrow Spectrograph during the period between 1978 July and 1979 August. One hundred and forty sets of observations were collected during this time. A 1.1 cm image was oriented on a 50 μm slit with a dove prism and a short–focal length objective mounted on a translation stage. Approximately 10 spectra were taken during each observation period with the slit oriented perpendicular to the solar rotation axis. The heliographic latitude of each exposure was changed by translating the objective a fixed distance. The spectrograph was operated at a dispersion of 11 mm Å−1 with resolution of 3.6 × 10^5. Spectra were recorded on high-contrast (gamma = 4.5) Eastman Kodak KPAN type film. Film response in the 630 nm range of interest was near a maximum, requiring exposure times of less than 4 s. Negatives were digitized with a 25 μm circular aperture in steps of 25 μm along the dispersion and 50 μm in the spatial dimension.

Two Fraunhofer lines near 630 nm were chosen to measure velocities. Two terrestrial oxygen absorption lines are within 0.1 nm in wavelength and provided wavelength calibration. Table 1 describes the Fe i lines as identified by Moore, Minnaert, and Houtgast (1966) and Corliss and Tech (1968). Although the Lande factors for these lines are nonzero, we have estimated that the contribution to the velocity measurements from magnetic fields is less than 0.8 m s−1 and is negligible.

The observed line shift at any point on the photosphere is a function of the point velocity, weighted by a line contribution function and integrated along the line of sight through the solar atmosphere. For small velocities we write

\[ V_0 = \frac{c}{\lambda_0} \int w(z) \Delta \lambda(z) \, dz, \]  

where \( \Delta \lambda(z) \) is the line center shift at a point \( z \) along the line of sight, \( \lambda_0 \) is the unperturbed line center wavelength, and \( V_0 \) is the inferred velocity. The form of \( w(z) \) may be found from solar atmospheric model parameters (cf. Canfield 1976). Using a computer code developed by Keil (1981) we have estimated the velocity formation height for these lines. Defining

\[ h = \int w(z) \, dz, \]  

where the integral is along the line of sight, we find the position-dependent height of formation shown in Figure 1. The two lines are very similar, with characteristic heights near the temperature minimum (at disk center) and a difference of only 50 km between them. These heights are measured relative to the height corresponding to unity optical depth at 500 nm. Since the calculated half-widths of the velocity contribution functions are 200 km, we conclude that both lines are measuring substantially the same velocity field.

The molecular oxygen lines have been identified by Babcock and Herzberg (1948). Van de Hulst (1945) calculated atmospheric line profiles for similar transitions and found that the line center is zenith angle dependent. Consequently, because the measurements depend on these lines as fiducials, we expect a daily fluctuation in the mean solar line center position of a few meters per second. On the other hand, spatial variations in the telluric line center positions in a single exposure should be negligible. Evidence for spatial variations was looked for by (1) comparing the telluric line center frequency distribution in each exposure with the line center distribution expected by knowing the line center fitting errors and (2) correlating low-spatial frequency fit coefficients found from the data with terrestrial indicators, such as the zenith angle of the Sun at the time of the observation. There was no evidence of any variation in the telluric line center position along the slit to better than 10 m s−1. As will be seen below, such residual errors are easily distinguishable from the rotation signal and enter the data only as an additional noise source.

III. DATA ANALYSIS

This analysis procedure is aimed at detecting long-lived, large-scale velocity features on the Sun. The approach involves looking for the rotation signature of these features in the power spectra of solar velocity fields. To do this, daily velocity field maps are obtained from the spectroscopic data. Least-squares fitting procedures are used to eliminate nonphotospheric velocity

<table>
<thead>
<tr>
<th>Line No.</th>
<th>λ(nm)</th>
<th>Transition</th>
<th>Landé Factor</th>
<th>Levels (cm⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>630.251</td>
<td>(^5P_1 \rightarrow ^5D_0)</td>
<td>5/2</td>
<td>29469–45334</td>
</tr>
<tr>
<td>1</td>
<td>630.152</td>
<td>(^5P_2 \rightarrow ^5D_2)</td>
<td>5/3</td>
<td>29733–45595</td>
</tr>
</tbody>
</table>
line shifts from these data. Residual velocities are then Fourier transformed to obtain two-dimensional power spectra for 10 solar latitude bands.

Coarse velocity maps were constructed for each set of observations by (1) identifying the spatial location of each point across the dispersion of each exposure and (2) fitting line profiles to obtain relative line shifts. Spatial coordinates were obtained by fitting the number of points on the disk in the east-west direction to a circular limb profile. The latitude of each exposure was not accurately resettable from day to day but was well determined by fitting the complete day’s observations to a solar profile. Velocities were found by fitting film density to parabolic line profiles over a range of roughly $3 \times 10^{-3} \text{nm}$ near the line minimum. The atmospheric lines were used to calibrate the wavelength and dispersion at each point along the slit. Film curvature and tilt were also accounted for.

The analysis yields two sets of line shifts, one for each of the iron lines, in approximately 10 bands (equally spaced in the sine of the heliographic latitude) for each observation day. The velocity error found by cross-correlating the two line shifts at each spatial point was typically 65 m s$^{-1}$. This was consistent with the error in the parabolic fit to film density at the line cores. This error was also consistent with the distribution of telluric line center positions from a single exposure. The error in the line center determination at any point along the slit was 65 m s$^{-1}$, and there was no evidence of a trend in the telluric line center positions along the slit to better than 10 m s$^{-1}$.

Several velocity trends at the scale of a solar radius must be removed from the maps before smaller scale velocity features may be observed. The largest contributions are from (1) the Earth’s orbital velocity, (2) the solar differential rotation, (3) possible day-to-day variations in the fiducials, and (4) the solar limb effect (cf. Beckers and de Veger 1978). The finite angular size of the Sun, the tilt of the rotation axis from the normal to the ecliptic plane, and slit positioning errors also contribute to spatial velocity trends. The first three contributions can be approximately accounted for in the data by fitting the line shift $\delta \lambda(\theta, \phi, t)$ to

$$\delta \lambda'(\theta, \phi, t) = a(\theta, t) \sin \phi + b(\theta, t) + \delta \lambda''(\theta, \phi, t), \quad (3)$$

where $\theta$ is the heliographic colatitude, and $\phi$ is the central meridian angle. The $b$ term is independent of $\phi$ and includes effects from orbital velocity terms and fiducial variation. Note that $\theta$ and $\phi$ are the spherical coordinates of a point on the Sun in a coordinate frame with the polar axis parallel to the solar rotation axis and the $x$-axis along the radius vector from the Sun to the observer. Here $t$ represents the observation time. The remaining wavelength trends are dominated by the limb effect, but all are effectively described by fitting the residuals from equation (3) to

$$\delta \lambda''(\theta, \phi, t) = \sum_{j=0}^{4} c_j(\theta, t) \phi^j + \delta \lambda'''(\theta, \phi, t). \quad (4)$$

A small correlation between the $c$ and $a$, and $b$ coefficients is unimportant since the primary goal of the procedure is to obtain photospheric line shift residuals.

Figure 2 shows the form for the solar differential rotation found from the $a$ coefficients in equation (3). A least-squares fit of the differential rotation rate, $\Omega(\theta)$, to the first three even-order Legendre polynomials ($P_0$, $P_2$, $P_4$) is also indicated here. We note that the derived synodic rotation rate of

$$\Omega(\theta) = 2.53 \times 10^{-6} \times [1 \pm 0.017 - (0.187 \pm 0.003) P_2(\cos \theta) - (0.036 \pm 0.005) P_4(\cos \theta)] \text{ rad s}^{-1} \quad (5)$$

is consistent with the differential rotation measured by Howard and Harvey (1970).

The residuals $\delta \lambda'''$ in equation (4) are separated by $\cos \theta$ increments of 0.18 (0.28 at the poles) into 10 bins so that the number of latitude exposures in each bin is approximately equal. Bin numbers 1 and 10 correspond to the south and north polar regions, respectively. Each set of exposures in a given bin is considered as a time sequence of velocity measurements $\delta \lambda'''(\phi, t)$, where $n$ specifies a bin number. Velocity fields with spatial transverse wavelengths larger than the bin size will be only
slightly affected by the daily jitter in the latitude of each measurement within a bin. Each set of measurements, \( \lambda''''(\phi, t) \), is restricted to the central region \( |\phi| < 0.5 \) to reduce projection effects. Spatial discrete transforms, \( a_n \), are obtained from

\[
a_n(k_m, t) = \sum_{j=0}^{N-1} \delta \lambda''''(\phi_j, t) \exp(-ik_m \phi_j),
\]

with \( k_m = 2\pi m \). Note that \( k_1 = 2\pi \) corresponds to a spatial wavelength of 1 solar radius. Restricted to \( |\phi| \leq 0.5 \), the sampling domain \( \phi \) deviates from equal intervals by less than 30%. Since the range of \( \phi \) is 1 radian and the increment in the transform variable is \( 2\pi \), the effect of the uneven domain on the transform coefficients is negligible. We have measured the spatial point spread of the spectrometer and densitometer system to have a half-width of about 30 \( \mu \)m. This attenuates the higher spatial frequency response of the data approximately as a Gaussian filter with half-width in \( k \) of about 185 rad\(^{-1}\) at disk center. Consequently, spatial frequency terms beyond 100 are significantly attenuated.

Temporal transforms of \( a_n(k, t) \) are more difficult to obtain because the time sample rate is uneven. Since nearly all of the observations were obtained near the same time of day, the observation times, \( t_e \), were first reduced to integer values (observation day numbers). A technique for transforming data sampled at uneven integral spacings (Kuhn 1982) was then applied to find \( b_n(k, \omega) \), so that

\[
a_n(k, t_e) = \sum_{j=0}^{127} \exp(-i\omega_j) b_n(k, \omega_j),
\]

where \( \omega_j = -\pi + j(2\pi/128) \). Here \( \omega_j \) has units of radians per day and \( \omega_0 = -\pi \) implies a frequency of one-half cycle per day. Power spectra \( P_n(k, \omega) = |b_n(k, \omega)|^2 \) were obtained for each bin and both iron lines. Figure 3 shows examples of the power spectra obtained from bin 5. Spectra from each of the bins have been studied for significant peaks or ridges in the \( k-\omega \) spectra. Although there are isolated 3 \( \sigma \) power peaks (as is expected), none of the structure shows any obvious significant features.

The cross-correlation of the peak locations in these spectra between the two data sets obtained from the two iron lines is good. Out of 424 peaks observed from all latitude bands in each data set with \( 0 < k \leq 20 \pi, 205 \) are common to both data sets. Here a peak is defined as a 2 \( \sigma \) deviation from the mean noise level, and agreement in peak location means agreement to within one-half increment in the transform variables, \( 1/2(\omega_j - \omega_{j-1}) \) and \( 1/2(k_j - k_{j-1}) \).

IV. DISCUSSION

Some caution must be applied to the interpretation of these residual line shifts as being purely due to large-scale motions. It is difficult to rule out the possibility of a contribution from a long-lived, large-scale convective...
FRAUNHOFER LINE SHIFTS

689K

630.25 nm

\( k = 20\pi (\lambda = 7.0 \times 10^4 \text{ km}) \)

630.15 nm

\( k = 23\pi (\lambda = 7.0 \times 10^5 \text{ km}) \)

line shift residuals in the following discussion must be tempered with the understanding that hitherto-undetermined velocity-intensity line profile correlations may contribute to small line shift residuals as do the velocity fields. We first consider the effect of random, small-scale velocity fields on the spatial data.

Figure 4 shows the mean spatial power spectrum derived from bin 5 and the 630.15 nm line. Bin 5 data are shown here because they correspond to an equatorial band and are least subject to projection corrections. Frequencies above \( k = 150 \) are attenuated by the spatial response of the spectrometer and densitometer apertures and are not plotted here. Note, as Labonte, Howard, and Gilman (1981) found, there is no apparent evidence of large-scale convection at any wavelength.

Most of the power in the mean spectrum can be explained in terms of a random, but large-scale, pattern of smaller velocity structures, e.g., supergranules. We take a simple model for the small-scale velocity structures measured at a given solar latitude along the spectrometer slit by suggesting that the residual photospheric velocity observed has the form of a random square wave. The probability of a velocity cell boundary, or analogously a zero crossing of the square wave, in an element \( d\phi \) along the slit will be given by \( \int d\phi \). So \( 1/\mu \) represents the mean separation between zero crossings and is analogous to the small-scale "cell" length. If the slit length is much longer than the mean cell size, then the mean power spectrum \( P(k) \) can be calculated (cf. Rice 1954)

\[
P(k) = \frac{4A^2\mu}{k^2 + 4\mu^2},
\]

where \( 2A \) is the peak-to-peak velocity amplitude of the square wave. Of course, this is a simplified model, but it does serve to show how small-scale features may affect the long-wavelength spectrum. A least-squares fit of the data in Figure 4 to equation (8) gives \( A = 60 \text{ m s}^{-1} \) and \( 1/\mu = 4.1 \times 10^{-2} \). These values imply a mean size of 29,000 km and a peak-to-peak velocity amplitude of 120 m s\(^{-1}\). This is in remarkably good agreement with other measurements of the supergranule scale size (cf. Allen 1973) of 32,000 km. The velocity amplitude is lower than other peak supergranule velocity measurements of 400 m s\(^{-1}\), but this is not unexpected, since the square wave velocity structure model has 120 m s\(^{-1}\) as a cell average velocity. It appears very likely that most of the power at low frequencies that is observed in mean spatial power spectra is dominated by incoherent small-scale features like the supergranulation. It is not surprising, therefore, that earlier attempts to find large-scale velocity fields at small wavenumber have been limited by this background of about 12 m s\(^{-1}\) amplitude.

An alternative is to look for the rotation signature of long-lived global velocity fields in the spatial and temporal power spectra of the velocity residuals derived from an equatorial band (bin 5) of both solar lines. The velocity power scale is indicated in the lower left of each set of transforms.

shift, much like that discussed by Beckers and Nelson (1978) in their explanation of the limb shift. Large-scale spatial correlations between the intensity and velocity formation profiles may produce asymmetric and displaced Fraunhofer lines.

Recently Livingston (1982) has found evidence that magnetic fields may affect Fraunhofer line profiles by affecting the granular convection. The corresponding change in the spatial intensity-velocity correlation can produce a spatial line shift field. More recently Foukal, Miller, and Keil (1982) have observed line profile changes associated with the network magnetic field on granular convection. In these data line shifts were determined from line core profiles so that such non-Doppler residuals are minimized, yet the possibility of a convective contribution to the data at levels near 2 m s\(^{-1}\) cannot be eliminated. As a result, references to

Fig. 3.—Representative spatial and temporal power spectra of the velocity residuals derived from an equatorial band (bin 5) of both solar lines. The velocity power scale is indicated in the lower left of each set of transforms.
temporal power spectra. If we let \( V(\phi) \) be the stationary velocity field and allow that it grows and dies exponentially with a lifetime of \( 1/\alpha \), then we take

\[
V(\phi - \Omega t)e^{-\alpha|t|}
\]

(9)
as a model for the field as it rotates across the solar disk with local angular velocity \( \Omega \). For a long time series of observations the two-dimensional power spectrum \( P(\omega, k) \) of equation (9) is

\[
P(\omega, k) = \frac{|V_k|^2 4\alpha^2}{((k\Omega - \omega)^2 + \alpha^2)^2},
\]

(10)

where \( V_k = \int V(\phi) \exp(i k\phi) \, d\phi \) is the spatial transform of the stationary velocity field. Since we may expect several large-scale fields (or "cells") to appear at time \( t_j \) and locations \( \phi_j \), the mean net power spectrum is expected to have the form

\[
P(\omega, k) = \frac{|V_k|^2 4\alpha^2}{((k\Omega - \omega)^2 + \alpha^2)^2}
\times \sum_{j=0}^{N-1} \exp\left[i(k\phi_j - \omega t_j)\right]^2,
\]

(11)

where \( t_j \) and \( \phi_j \) indicate the initial conditions for the appearance of \( N \) different cells during the observation time. The last term that includes \( t_j \) and \( \phi_j \) is probably of order unity but cannot be determined from this analysis.

Similarly, \( \alpha \) and the dependence of \(|V_k|^2\) on \( k \) is difficult to measure here (although \( \alpha \leq 1/3 \) day\(^{-1} \) for such cells to be observed from these data). At a given \( k \) it is expected from equation (11) that \( P(\omega, k) \) peaks at values of \( \omega \) such that \( \omega/k = \Omega \). Note that although the rotation rate, \( \Omega \), may be unknown, we expect rotation in, at least, the same direction as the photosphere. This implies that \( \omega \) will be negative for such features in these data and forms the basis for a simple statistical test for such global velocity fields, since any nonsolar noise source will not show an asymmetry in the distribution of power spectrum peaks at positive and negative \( \omega \).

We first identify peaks in each \( k-\omega \) spectrum, \( P_n(\omega, k) \). A peak was defined as a 2 \( \sigma \) deviation from a white noise background. Precise \( \omega \) values for the peaks were found by quadratic interpolation around the point of maximum power. The value of \( \omega/k \) for each peak was then considered in the histograms of Figure 5. The data from both lines and all of the latitude bins have been considered together. This has been done to improve the signal-to-noise ratio in the histograms. The peaks that are considered in Figure 5 are correlated to the extent that each peak found from both sets of spectra are accurately measuring a photospheric velocity. The two sets of histograms in Figure 5 were derived by considering two different upper cuts in allowed \( k \) for inclusion in the distribution. The first set (Figs. 5a, 5b) is derived for \( 0 < k \leq 8\pi \), while the second (Figs. 5c, 5d) is for \( 0 < k \leq 20\pi \). The first cut corresponds to the most conservative limit on \( k \) which still allows sufficient data for meaningful statistics. Because the daily jitter in the solar latitude of each observation is of order one bin,
we may expect only the lowest spatial frequencies to show coherence from day-to-day. Adopting a mean bin size of 0.3 radians gives an upper limit of \( k = 8\pi \), on the assumption that the solar latitudinal and longitudinal spatial scales are comparable.

The second limit of \( k = 20\pi \) was chosen as a more liberal upper bound to possible wavelength scales that contribute to the data in Figures 5c and 5d, but which does not include the effects of especially long-lived second subharmonics of supergranules (the equatorial wavelength corresponding to \( k = 20\pi \) is 70,000 km).

Note that because the effective temporal Nyquist frequency for these data is 0.5 day\(^{-1}\), the data from the spectral region \( k \geq 5\pi \) cannot contribute at \( |\omega/k| \geq 0.2 \) rad day\(^{-1}\) (11.6 day\(^{-1}\)), which is the photospheric rotation rate. It can be seen from equation (11) that if a complex dependence on \( \omega \) or \( k \) in \( V_k \) or the last multiplicative term in equation (11) is allowed, then peaks at frequencies other than \( \omega/k = \Omega \) may occur. Nevertheless, we expect even larger power amplitudes when \( k\Omega \) is near \( \omega \), and in the case of these data this occurs with \( \omega < 0 \). The implication is that although each of the peak locations measured from the power spectra may not imply a rotation rate of \( \omega/k \) directly, their sign will tend to favor the same sign as the photospheric rotation. Thus, even though the power spectra peaks at large \( k \) cannot contribute to the histogram at \( |\omega/k| \geq 0.2 \), any distribution asymmetry between positive and negative values of \( \omega/k \) (even at small absolute magnitude) is relevant to the search for large-scale fields.

Consider the data of Figure 5. A uniform white noise component to the power spectra must appear in the \( \omega/k \) distributions as a background that falls approximately as \( (\omega/k)^{-2} \). This is evident in Figures 5a and 5c. Because the minimum \( k \) value considered was \( 2\pi \) and \( \omega_{\max} = \pi \) day\(^{-1}\), the distributions must be zero beyond \( |\omega/k| = 0.5 \).

Evidence of a rotational contribution to these distributions was looked for by subtracting a background distribution from Figures 5a and 5c. Since the \( k \) dependence of some of the factors in equation (11) is not known, we choose to subtract the largest possible background distribution that is consistent with a nonrotating velocity signal. The only constraints are that the background is symmetric about \( \omega/k = 0 \) and that it yield no negative excess in the residual histograms. This amounts to finding the number excess between corresponding positive and negative \( \omega/k \) bins. Excess counts in negative and positive bins are plotted to the left of \( \omega/k = 0 \), and vice versa. The corresponding background-subtracted distributions from Figures 5a and 5c are shown in Figures 5b and 5d. It is clear that an asymmetry exists, with more counts at \( \omega/k < 0 \).

The statistical significance of this result can be estimated from the hypothesis that such an excess could come from a random distribution of power spectrum peaks, with positive and negative values of \( \omega/k \) equally likely. In Figure 5d the probability of obtaining the observed excess of 81 negative \( \omega/k \) peaks out of 848 events is approximately 0.25%, corresponding to a 3 \( \sigma \) confidence level (Brandt 1970). The corresponding confidence level for the data in Figures 5a and 5b with fewer points is at the 2.5 \( \sigma \) level. The data sets from the two lines are correlated to the extent that the measurement noise may not be dominated by the film noise. Thus, these are somewhat optimistic estimates, since
they assume complete independence between the data sets. Alternatively, the probability of obtaining at least as many bins to the left of $\omega/k = 0$ in Figures 5b and 5d, as is observed, is not strongly sensitive to correlations between the data sets. In Figure 5d the probability of observing 12 or more bins at negative $\omega/k$, out of a total of 14 bins, is less than 1.8%. The corresponding probability of obtaining 10 or more from 14 possible bins in Figure 5a is roughly 5%.

Figure 5b is based on data from the longest wavelengths and may be the most likely scale for which power spectrum peaks have $\omega/k$ values corresponding to the physical rotation frequency. Peaks at larger $k$ cannot contribute to the histogram at rotational velocities near $-0.2$ rad day$^{-1}$. There are insufficient data to constrain the parameters in equation (11), yet the dependence of the peak excess on $\omega/k$ may be further analyzed to show that it is, at least, not inconsistent with a realistic rotation rate.

Letting $C(\omega/k)$ be the distribution excess in the bin including peaks near $\omega/k$, the data in Figure 5b have been fit to the Gaussian function

$$C(\omega/k) = \exp \left( -\frac{(\omega/k - \Omega_0)^2}{2\Delta^2} \right),$$

(12)

where $\Omega_0$ is meant to approximate a mean rotation rate, and $\Delta$ is the width of the distribution. A counting error of $N_i^{1/2}$ is assigned to each bin, $i$, where $N_i$ is the number of counts in the $i$th bin of the symmetric background distribution. Figure 6 shows the excess distribution and the best least-squares fit to equation (12). The $\chi^2$ value was 9.1 with 12 degrees of freedom with $\Omega_0 = -0.17 \pm 0.09$ rad day$^{-1}$ and $\Delta = 0.09 \pm 0.04$ rad day$^{-1}$. This $\chi^2$ indicates a statistically sound fit to the distribution excess. The interpretation of $\Omega_0$ and $\Delta$ is somewhat uncertain, although we note that this mean rate is consistent with a mean photospheric rotation rate derived from the data displayed in Figure 2 of $-0.201 \pm 0.003$ rad day$^{-1}$.

The large width, $\Delta$, of the excess distribution may be due in part to an intrinsic range of rotation rates. It is unlikely that it results entirely from measurement uncertainty in the $\omega$ and $k$ location of peaks in $P_n(\omega, k)$. A conservative estimate of the $k$ and $\omega$ uncertainty to the peak locations of one-half increment $\frac{1}{2}(\omega_j - \omega_{j-1})$ and $\frac{1}{2}(k_j - k_{j-1})$ produces an error in the rotation rate of approximately $(\omega/k)((0.02/\omega) + (\pi/k))$. This was calculated to be less than 0.001 rad day$^{-1}$ over these distributions.

The projection of the photospheric vector velocity along the line of sight can cause a smearing of the $k$ spectrum of a global-scale field. Because only one component of the vector velocity is measured, the observed shape of the velocity field along the slit may change as the field rotates across the photosphere. This effect may also broaden the rotation rate distribution.

We believe that the consistency of the excess distribution in Figure 5b with the mean photospheric rotation rate lends additional support to the claim of detection of long-lived, large-scale flows.

The rms velocity of the data for $\omega/k < 0$ is 2 m s$^{-1}$ per 0.13 day$^{-1}$ bandwidth. By comparison, on the assumption of a 65 m s$^{-1}$ line center error per measurement, the expected noise amplitude for the same $\delta \omega \delta k$...
bandwidth is roughly 0.3 m s$^{-1}$. From equation (11) it is evident that the signal power may be modulated by velocity field parameters such as the lifetime of the velocity features and their density on the photosphere. Thus the actual velocity amplitude $|V_\nu|$ of the field is somewhat uncertain, but it is of order 2 m s$^{-1}$. There is little hope of observing such a small signal in averaged photospheric velocity maps without determining the rotation rate of these features more accurately.

Contributions to the rotation signal from sunspots or active regions must be small. Long-wavelength subharmonics to the sunspot or active region velocity field were looked for by cross-correlating the latitude distribution of the $\omega/k$ values near the photospheric rotation rate of $-0.2$ rad day$^{-1}$ with the latitude distribution of active regions. Table 2 shows a representative latitude distribution of flare events from Smith and Smith (1963) and of $\omega/k$ peaks between $-0.256$ and $-0.160$ rad day$^{-1}$. Note that the rotation signal is evenly distributed in latitude, and that the correlation coefficient is $-0.08$—indicating no significant correlation. Further, we note that the mean synodic sunspot rotation rate from Ward (1966) is $-0.231 \pm 0.005$ rad day$^{-1}$. This is somewhat higher than the mean found in Figure 6. Also, if the rotation signal was due entirely to sunspots, the width of this distribution should be smaller.

Contributions from supergranulation are also small, unless there is a heretofore undiscovered, long-lived, long-range coherence to the supergranule velocity field. Rogers (1970) and Worden and Simon (1976) measured supergranule lifetimes to be 36 hr or less. Since the mean interval between observations was 2.5 days, it is unlikely that low-frequency subharmonics of a random supergranule field will show much correlation from observation to observation. This is true for even smaller scales approaching the supergranule size, since the solar latitude of the region along the spectrometer slit varied by as much as or more than 9° between observations. The low-spatial frequency coherence time must be very short.

Then even at the shortest spatial wavelengths of the data in Figure 6 (five supergranule lengths), the probability of measuring the same magnitude and phase to the velocity field at this wavelength during two average sequential observations is less than $10^{-5}$. Giant-cell convection or a long-range coherence to the supergranule network (which might be generated by giant-cell flows) could easily explain the relatively long-lived, large-scale line shift fields evident in these data.

V. CONCLUSIONS

We find no obvious evidence of a simple ridge structure to the diagnostic $k-\omega$ diagrams at low spatial and temporal frequencies. On the other hand, statistical analysis of the phase rotation rate implied by isolated peaks in the spectra, shows a 2.5–3 $\sigma$ detection of line shift structure rotating in the direction of the photosphere, with significant power near the mean photospheric rotation rate. The rotation signal is approximately evenly distributed in projected solar latitude. These features must have lifetimes of several days to appear in the temporal transforms. Their mean velocity amplitude is near 2 m s$^{-1}$, and their spatial scale is between $10^5$ and $10^6$ km. Large-scale, long-lived magnetic fields which locally affect convection may produce line shift fields which contribute to these data.


REFERENCES


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