COMPARISON OF OBSERVED \textit{Ca} x\textit{IX} AND \textit{Ca} xv\textit{III} RELATIVE LINE INTENSITIES WITH CURRENT THEORY

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(Received 22 September, 1981)

Abstract. A comparison is made between \textit{Ca} x\textit{IX} and \textit{Ca} xv\textit{III} line ratios observed in solar flares with the Bent Crystal Spectrometer (BCS) on the Solar Maximum Mission (SMM) satellite and currently available atomic data. Close agreement is found with the excitation rates recently published by Pradhan \textit{et al.} (1981). The observations show little dependence of line ratios on electron temperature, supporting a further conclusion that cascade contributions to the $^2P$ and $^2S$ levels are not significant.

1. Introduction

The purpose of the present note is to compare spectra of calcium obtained during flares with the X-ray Polychromator (XRP), Bent Crystal Spectrometer (BCS) on the Solar Maximum Mission (Acton \textit{et al.}, 1980), with published calculations of atomic data for the $n = 1$ to $n = 2$ transitions in \textit{Ca} x\textit{IX} and relative intensities of \textit{Ca} xv\textit{III} satellite lines.

The basic theory for the analysis of lines from the $n = 2$ levels of the \textit{He} i-like ions and associated satellite lines has been established for some time (Gabriel \textit{et al.}, 1969; Freeman \textit{et al.}, 1971; Gabriel and Jordan, 1972). In recent years considerable effort has been made to improve the theory and atomic data for high \textit{Z} ions, in particular for iron (Bhalla \textit{et al.}, 1975; Vainstein and Safranova, 1978; Safranova \textit{et al.}, 1978; Bely-Dubau \textit{et al.}, 1979a, b). Calculations of relative intensities based on earlier atomic data are also available from the work of Mewe and Schrijver (1978a, b). New calculations for excitation rates in a number of \textit{He} i-like ions have recently been published by Pradhan \textit{et al.} (1981), including results for both iron and calcium, and it is with these that comparisons are made.

2. Observational Data

The BCS instrument and its sensitivity have been discussed by Acton \textit{et al.} (1980) and Rapley \textit{et al.} (1977). Some early results showing the scope of the instrument may be found in papers by Acton \textit{et al.} (1981), Culhane \textit{et al.} (1981), and Gabriel \textit{et al.} (1981).

The emission lines of \textit{Ca} x\textit{IX} and their associated \textit{Ca} xv\textit{III} satellite lines lie between $\sim 3.165$ Å and $3.231$ Å. The spectrum is illustrated in Figure 1. The position of the various lines is marked, adopting the notation used by Gabriel (1972). The wavelengths from that paper have sufficient accuracy for the present purposes. Table I summarizes the transitions discussed.

Calcium spectra obtained during two particular flares are discussed here, although similar analyses have been carried out on several others. The flare on 1980 April 7 at
Fig. 1. Example of calcium spectrum showing position of helium-like and lithium-like satellite lines. The notation is from Gabriel (1972).

### TABLE I

Lines considered in the analysis

<table>
<thead>
<tr>
<th>Ion</th>
<th>Transition</th>
<th>$\lambda$ (Å)*</th>
<th>Key symbol*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ca xix</td>
<td>$1s^2 1S_0 - 1s2p 1P^+_1$</td>
<td>3.176</td>
<td>w</td>
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<tr>
<td>Ca xix</td>
<td>$1s^2 1S_0 - 1s2p 3P^+_2$</td>
<td>3.189</td>
<td>x</td>
</tr>
<tr>
<td>Ca xvii</td>
<td>$1s^22p^2 2P_{5/2} - 1s2p^2 2S_{1/2}$</td>
<td>3.189</td>
<td>m</td>
</tr>
<tr>
<td>Ca xvii</td>
<td>$1s^22s^2 2S_{1/2} - 1s2p 2s(3P)^+ 2P_{3/2}^+$</td>
<td>3.191</td>
<td>s</td>
</tr>
<tr>
<td>Ca xvii</td>
<td>$1s^22s^2 2S_{1/2} - 1s2p 2s(3P)^+ 2P_{1/2}^+$</td>
<td>3.192</td>
<td>t</td>
</tr>
<tr>
<td>Ca xix</td>
<td>$1s^2 1S_0 - 1s2p 3P^+_1$</td>
<td>3.192</td>
<td>y</td>
</tr>
<tr>
<td>Ca xvii</td>
<td>$1s^22p^2 2P_{3/2}^+ - 1s2p^2 2P_{3/2}^-$</td>
<td>3.199</td>
<td>b</td>
</tr>
<tr>
<td>Ca xvii</td>
<td>$1s^22s^2 2S_{1/2} - 1s2p 2s(1P)^+ 2P_{3/2}^+$</td>
<td>3.200</td>
<td>q</td>
</tr>
<tr>
<td>Ca xvii</td>
<td>$1s^22s^2 2S_{1/2} - 1s2p 2s(1P)^+ 2P_{1/2}^+$</td>
<td>3.202</td>
<td>r</td>
</tr>
<tr>
<td>Ca xvii</td>
<td>$1s^22p^2 2P_{3/2}^+ - 1s2p^2 2P_{3/2}^+$</td>
<td>3.203</td>
<td>a</td>
</tr>
<tr>
<td>Ca xvii</td>
<td>$1s^22p^2 2P_{1/2}^+ - 1s2p^2 2P_{1/2}^+$</td>
<td>3.203</td>
<td>d</td>
</tr>
<tr>
<td>Ca xvii</td>
<td>$1s^22p^2 2P_{1/2}^+ - 1s2p^2 2D_{3/2}$</td>
<td>3.207</td>
<td>k</td>
</tr>
<tr>
<td>Ca xvii</td>
<td>$1s^22p^2 2P_{3/2}^+ - 1s2p^2 2D_{5/2}$</td>
<td>3.210</td>
<td>j</td>
</tr>
<tr>
<td>Ca xix</td>
<td>$1s^2 1S_0 - 1s2p 3S^+_1$</td>
<td>3.210</td>
<td>z</td>
</tr>
</tbody>
</table>

* Gabriel (1972).
\( \sim 01 \) hr was chosen because in spite of being a substantial \( M \) class event, there was only a very weak blue shifted line broadening. Other events, such as those on 1980 April 30th and the X-class event on 1980 May 21st (at \( \sim 21 \) hr) show substantial broadening which, as will be seen below, can affect the interpretation of line ratios. The event of May 21 is discussed as a comparison with that of April 7. The spectra were obtained with a time resolution of between 3 s and 11 s, but have been summed over 60 s and sampled at about 1 min intervals. Use has been made throughout of the standard XRP data reduction and data analysis programs. These give the observed fluxes in counts per sec per bin. Only relative fluxes are required here, avoiding any remaining uncertainties in the calibration.

3. Analysis

Calculations of the wavelengths and relative intensities of the Ca \( \text{xviii} \) \( n = 2 \) satellite lines are available from Bhalla et al. (1975) and Safronova et al. (1978) but similar calculations for satellites with \( n \geq 3 \) have not yet been published explicitly for calcium. The \( n = 3 \) contribution on the long wavelength side of the Ca \( \text{xix} \) resonance line is obvious and can be excluded. The \( n \geq 3 \) contributions to the \( 1s^2 \, ^1S_0 - 1s2p \, ^3P_1 \) line have been allowed for in an approximate way by excluding the obvious asymmetry and using the calculations for iron by Bely-Dubau et al. (1979b) and the \( Z \)-scaling discussed by Gabriel and Jordan (1972). Over the range of \( T_e \) observed in the flares discussed, the corrections to the adopted flux are smaller than 10%.

The procedure was as follows: (i) \( T_e \) was found from the \( k \)-satellite and apparent resonance line intensity. The correction to the resonance line and intersystem line was estimated from this \( T_e \), and then an improved \( T_e \) found. (ii) The published ratio for the \( k \) to \( j \) satellite intensities was used to find the \( j \)-satellite intensity, which was then subtracted from the observed total of the \( j + \) forbidden line \( z \). (iii) Similarly, the \( m \) and \( s \) contributions to the Ca \( \text{xix} \) magnetic quadrupole line and the \( t \) contribution to the apparent intersystem and quadrupole lines were removed. (iv) The \( q + b \) and \( r + a + d \) groups of satellites were also measured relative to \( k \), but the accuracy of these ratios is not high because the lines are weak.

The background was chosen from the spectrum at wavelengths shorter than that of the resonance line and longer than that of the forbidden line. It will be seen below that this is adequate when the lines do not have the blue-wing asymmetry which is often observed, but leads to a substantial overestimate of the \( k \)-line intensity and inaccuracies in other line intensities when the asymmetry is clearly present in the resonance line.

The line ratios \( z/w, x/y, (x + y)/z, (x + y + z)/w, (q + b)/k \) and \( (r + a + d)/k \) have been examined as a function of time and \( T_e \). These ratios are shown in Figures 2, 3, 4, and 5.

4. Comparison with Theory

A. Helium-Like Lines

For a plasma in ionization equilibrium and at low values of \( N_e \), the forbidden to resonance
line intensity ratio (temperature-dependent) is given by:

\[
\frac{I_z}{I_w} = \frac{C(1^1S \rightarrow 2^3P_0) + \frac{C(1^1S \rightarrow 2^3P_2)A(2^3P_2 \rightarrow 2^3S)}{A(2^3P_2 \rightarrow 1^1S) + A(2^3P_2 \rightarrow 2^3S)} + C(1^1S \rightarrow 2^3S)}{C(1^1S \rightarrow 2^1P)},
\]

(1)

where the collision rates, \(C\), indicate the total effective rates including cascade contributions. The transition probability from \(2^3P_1\) to \(1^1S_0\) in Ca xvi greatly exceeds that to \(2^3S_1\).

Early observations (cf., Gabriel and Jordan, 1972, and references therein) and theoretical cross-sections indicated that cascades may be important in populating the \(1s2s\,^3S\), and \(1s2p\,^3P\) levels. Approximations to these cascade contributions have been proposed by Blumenthal et al. (1972) and by Mewe and Schrijver (1978a). Blumenthal et al. suggested that recombination might cause a significant temperature dependence in the line ratios but Gabriel and Jordan (1973) concluded that recombination was less important than cascades. In the following analysis, effective collision rates will be derived from the observations and compared to the theoretical values. The radiative transition probabilities are taken from Lin et al. (1977); it is worth noting that the value for

![Graph showing the forbidden to resonance line ratio \(z/w\) in Ca xix as a function of time and \(T_e\).](image)

Fig. 2. The forbidden to resonance line ratio \(z/w\) in Ca xix, as a function of time and \(T_e\), (a) during the 1890 May 21 flare and (b) during the 1980 April 7 (01 hr) flare. The starting times for all the plots are 20–57 UT and 00–50 UT, respectively.
$A(2^3P_2 \rightarrow 1^1S)$ is significantly different to that given by Safranova et al. (1978) and used by Bhalla et al. (1975) in their calculations. Recently, Pradhan et al. (1981) have published excitation rate coefficients taking into account effects of autoionizing resonances. Their results can be compared to the observed values if the rates to $2^3P_2$, $2^3P_1$, and $2^3P_0$ are taken in the ratios found by Jones (1974) for the cross-sections near threshold.

Comparisons are also made with the excitation rates used by Bhalla et al., at a temperature corresponding to $E/kT_e = 3$ ($E$ – excitation energy of transition). Figures 2a and 2b show the temperature and forbidden/resonance line ratio during the May 21 and April 7 flares. In Figure 2a, the variation in the forbidden/resonance ratio in the early part of the May 21 flare can be accounted for by the effects of the strong blue-wing asymmetry present in the spectra. This can be taken into account through the fractional contribution apparent for the resonance line. The similar variations in other line ratios discussed below would then be removed. The electron temperature resulting from a corrected $k$ intensity would then be essentially constant at $\sim 1.8 (\pm 0.1) \times 10^7$ K during the first eight minutes of the flare. The same line ratio in April 7 flare, shown in Figure 2b shows only a small variation, comparable with the uncertainties of $\sim \pm 10\%$.

The average ratio after the first six minutes of the May 21 flare is 0.42 ± 0.01. During the April 7 flare the average is 0.39 ± 0.02.

There is no apparent dependence of the ratio on $T_e$, even when $T_e$ decreases quite rapidly during the May 21 event. This suggests that transient recombination effects (as discussed by Mewe and Schrijver, 1978b) are not important.

Adoption of the cross-sections calculated by Pradhan et al. (excluding cascades), leads to a value for the ratio of 0.385. The small (and statistically insignificant) difference between this and the observed mean of 0.40 could be attributed to cascade to $2^3S$ and/or $2^3P$ following excitation to levels with $n > 3$ and also to uncertainties in the cross-sections or a small contribution from radiative recombination.

Using the effective rates adopted by Bhalla et al. would, on the other hand, lead to the smaller value of 0.26 for the $x/w$ ratio, giving a larger difference to be accounted for by the possible causes above.

The quadrupole to intersystem line intensity ratio can be found from:

$$\frac{I_x}{I_y} = \frac{C(1^1S \rightarrow 2^3P_2)}{C(1^1S \rightarrow 2^3P_1)} \frac{A(2^3P_2 \rightarrow 1^1S)}{A(2^3P_2 \rightarrow 1^1S) + A(2^3P_2 \rightarrow 2^3S)}. \quad (2)$$

Calculating this ratio using the same rates as above leads to a value of 0.96 for a temperature of $1.5 \times 10^7$ K, close to the value found using rates proportional to the statistical weights. At the higher energy discussed by Jones (1974) the ratio is 0.85.

Satellite lines contribute to both $x$ and $y$. On the basis of the wavelengths of the lines, all of the $m$ satellite is subtracted from the quadrupole line, all of the $t$ satellite from the intersystem line, $\frac{1}{4}$ of the $s$-satellite from the quadrupole line and $\frac{2}{3}$ of the $s$ satellite from the intersystem line. This results in a ratio of 0.91 (May 21, ignoring first few minutes) or 1.1 (April 7) – not significantly different from the theoretical value.
Fig. 3. The ratio of the Ca xix intersystem plus magnetic-quadrupole lines to the forbidden line \((x + y)/z\), during the 1980 May 21 and 1980 April 7 (01 hr) flares.

The quadrupole plus intersystem line intensity to that of the forbidden line is density dependent above a critical \(N_e \approx 8 \times 10^{14} \text{ cm}^{-3}\). Below this density, collisional excitation from \(2^3S\) to \(2^3P\) is unimportant and the ratio is given by:

\[
I_{x+y} = \frac{C(1^1S \rightarrow 2^3P_2)}{I_z} = \frac{A(2^3P_2 \rightarrow 1^1S)}{A(2^3P_2 \rightarrow 1^1S) + A(2^3P_2 \rightarrow 2^3S)} + C(1^1S \rightarrow 2^3P_1)
\]

\[
+ C(1^1S \rightarrow 2^3P_0) + \frac{C(1^1S \rightarrow 2^3P_2)A(2^3P_2 \rightarrow 2^3S)}{A(2^3P_2 \rightarrow 1^1S) + A(2^3P_2 \rightarrow 2^3S)} + C(1^1S \rightarrow 2^3S)
\]

(3)

This ratio is \(1/R_0\) in the notation of Gabriel and Jordan (1969). According to the calculations of Bhalla et al. (1975) or Jones (1974), the ratio is insensitive to \(T_e\) but does depend on the branching ratio for transitions from \(2^3P_2\). Using the calculations of Lin et al. (1977) and Pradhan et al. (1981) and neglecting cascades etc., the ratio is predicted to be 1.02. The excitation rates adopted by Bhalla et al. would give a larger value of 1.39. Figure 3 shows the observed ratio for the May 21 and April 7 flares. The variation at early times is due to the blue wing asymmetry discussed above. The ratios from the two flares agree well and average to 1.12 (May 21) and 1.13 (April 7) respectively, a little larger than the value predicted by the excitation rates of Pradhan et al.

If the cascade contributions are treated as unknown factors by which the adopted rates to \(2^3P\) and \(2^3S\) must be multiplied then empirical corrections for the cascade contribution can be calculated from the observed ratios. It is found that cascades are negligible in populating the \(2^3S\) level but the Pradhan et al. value for the collisional rate to the \(2^3P\) levels needs to be corrected by a factor of 1.15 to produce an effective excitation rate of \(C(1^1S - 2^3P) = 4.8 \times 10^{-13} \text{ cm}^3 \text{ s}^{-1}\) at \(T_e = 1.5 \times 10^7 \text{ K}\). Adoption of the earlier rates used by Bhalla et al. would, on the other hand, result in rather larger cascade contributions to \(2^3P\) and \(2^3S\) of factors of 1.3 and 2.1, respectively.
Fig. 4. The ratio of the total triplet lines to the resonance line \((x + y + z)/w\) in Ca xix, during the 1980 May 21 and 1980 April 7 (01 hr) flares.

Fig. 5. The temperature dependence of the excitation rate coefficients.
Figure 5 shows the temperature variation of the rate coefficients from Pradhan et al. with the observed values superimposed. A comparison of the derived rates with those of other authors is given in Table II.

<table>
<thead>
<tr>
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<th></th>
<th></th>
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<tbody>
<tr>
<td>$1^1S - 2^1P$</td>
<td>7.2 (−13)</td>
<td>3.9 (−13)</td>
<td>5.4 (−13)</td>
</tr>
<tr>
<td>$1^1S - 2^3P_2$</td>
<td>2.3 (−13)*</td>
<td>2.1 (−13)</td>
<td>1.6 (−13)*</td>
</tr>
<tr>
<td>$1^1S - 2^3P_1$</td>
<td>1.4 (−13)*</td>
<td>1.3 (−13)</td>
<td>1.0 (−13)*</td>
</tr>
<tr>
<td>$1^1S - 2^3P_0$</td>
<td>4.6 (−14)*</td>
<td>4.2 (−14)</td>
<td>3.2 (−13)*</td>
</tr>
<tr>
<td>$1^1S - 2^3S$</td>
<td>1.4 (−13)</td>
<td>6.4 (−14)</td>
<td>4.6 (−14)</td>
</tr>
</tbody>
</table>

* Total $1^1S \rightarrow 2^3P$ divided in ratio according to calculations of Jones.

(b) Transition probabilities

<table>
<thead>
<tr>
<th>Transition</th>
<th>Å (Lin et al., 1977)</th>
</tr>
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<tbody>
<tr>
<td>$3P_2 - 3S_1$</td>
<td>4.98 (8)</td>
</tr>
<tr>
<td>$3P_2 - 1S_0$</td>
<td>7.55 (8)</td>
</tr>
<tr>
<td>$3P_1 - 1S_0$</td>
<td>4.85 (8)</td>
</tr>
<tr>
<td>$1P_1 - 1S_0$</td>
<td>1.65 (14)</td>
</tr>
<tr>
<td>$3S_1 - 1S_0$</td>
<td>1.42 (7)</td>
</tr>
</tbody>
</table>

It should also be noted at this point that the corrected value of $F = C(1^1S - 2^3S)/C(1^1S - 2^3P) = 0.29$ is slightly lower than the previously adopted value of 0.35 (Gabriel and Jordan, 1969).

A final comparison for the helium-like lines can be made between the observed and theoretical ratios for the total triplet intensity to that of the resonance line — $G$ in the notation of Gabriel and Jordan:

$$\frac{I_{x+y+z}}{I_w} = \frac{C(1^1S \rightarrow 2^3S) + C(1^1S \rightarrow 2^3P)}{C(1^1S \rightarrow 2^1P)}. \quad (4)$$

The observed ratios are shown in Figure 4. They average to 0.83 and 0.88 for the April 7 and May 21 flares, respectively.

Using the cross-sections of Pradhan et al., a ratio of 0.78 is expected but with the corrected value for $C(1^1S - 2^3P)$ determined above, the ratio becomes 0.86, in excellent agreement with the observed ratio, showing a very satisfactory consistency in the observations and calculations. A summary of these various ratios is given in Table III.
TABLE III
Comparison of observed and calculated line ratios

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Observed</th>
<th>Pradhan et al. (1981)</th>
<th>Bhalla et al. (1975)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_x/I_w$</td>
<td>0.41</td>
<td>0.385</td>
<td>0.26</td>
</tr>
<tr>
<td>$I_x/I_y$</td>
<td>1.0</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>$I_{x+y+z}/I_x$</td>
<td>1.1</td>
<td>1.02</td>
<td>1.39</td>
</tr>
<tr>
<td>$I_{x+y+z}/I_w$</td>
<td>0.86</td>
<td>0.78</td>
<td>0.63</td>
</tr>
</tbody>
</table>

B. LITHIUM-LIKE SATELLITES

Throughout the above analysis the satellite line intensities have been accepted as correct. The only sets of lines which allow a direct comparison with the theory are $(q + b)$, $(r + a + d)$ and $k$, as shown in Figure 1. Because the lines are weak, there is a large scatter in the observed ratios.

The $(q + b)$ blend is further complicated by the presence of the Ar XVII $1s^2 - 1s4p$ line at 3.1996 Å as previously noted by Veck (1980) and Doschek and Feldman (1981). The contribution due to this transition can be estimated from the ratio of collision strengths, $1s^2 \rightarrow 1s2p: 1s^2 \rightarrow 1s4p$ in Ar XVII and the ion abundance ratio Ar XVII: Ca XIX at the correct temperature. It is found that the intensity of the argon line is roughly 5% of that of the calcium resonance line and thus the observed ratios of $(q + b)/k$ should be corrected by this amount. The corrected ratio $(q + b)/k$ then averages 0.45 and 0.52 (April 7 and May 21, respectively) whilst $(r + a + d)/k$ averages 0.62 and 0.67 showing reasonable agreement between the two flares. The only trend is a very slight decrease with decreasing $T_e$ or increasing time possibly due to the imperfect removal of the contamination from Ar XVII. A rise in the ratios in the early part of the May 21 is attributable to the varying line width discussed above.

These pairs of lines are of interest because, according to the calculations by Bhalla et al., $q$ and $r$ depend not only on dielectronic recombination from the helium-like ion but also on inner shell excitation from the lithium-like ion. Denoting the Li-like ion number density as $N(\text{Li})$ and that of the He-like ion as $N(\text{He})$ and treating the ratio $N(\text{Li})/N(\text{He})$ as an unknown, it is simple to show that using the observed values of $T_e$ the two ratios cannot be satisfied by the same value of $N(\text{Li})/N(\text{He})$. The deduced value of the ratio $(q + b)/k$ implies an ionizing plasma—as noted by previous authors, including Doschek et al. (1980) and Feldman et al. (1980) from their SOLFLEX data. The ratio $(r + a + d)/k$ would imply even more extremely ionizing conditions, and this ratio is therefore suspect. As pointed out by Bhalla et al., the $q$ and $r$ intensities are sensitive to small changes in the atomic data—and it was these intensities which changed most when improvements were made between the work by Gabriel (1972) and Bhalla et al. (1975). The agreement between the two ratios is closer using the calculations by Safronova et al. (1978) but a strongly ionizing plasma is still implied. The small changes in the ratios during the flares discussed—whether in the rising or falling parts of the calcium flux strongly suggest that the theory for these lines bears closer examination.
5. Conclusion

The new spectra of helium and lithium-like calcium allow comparisons to be made between observed line ratios and those predicted from theoretical excitation rates. If the effective excitation rate to $2^3P$ is increased by 15% above that calculated by Pradhan et al. then there is excellent agreement between the observations and other calculated line ratios based on the rates by Pradhan et al. This small difference may well be due to a contribution from cascades. The lack of dependence on $T_e$ in the observed line ratios in the decaying part of the flares support the conclusion that cascade contributions are not significant. The spectra indicate discrepancies between observations and theoretical values for the ratios of some of the weaker dielectronic satellite lines.

Acknowledgements

We wish to acknowledge the work of the principal investigators, L. W. Acton, J. L. Culhane, and A. H. Gabriel and all in the XRP team whose efforts have led to availability of the data used. We wish in particular to thank J. Sherman, D. Mathur, and R. Bentley who have worked on the data handling computer software.

Note added in proof: Dr L. Steenman–Clark has drawn to our attention a paper by Pradhan (1981) in which he points out that the contributions to the excitation rates from resonances, included in Pradhan et al. (1981), may be overestimated due to neglect of the branching ratio to radiative decay (dielectronic recombination) rather than autoionization (Presnyakov and Urnov (1975) have also discussed this effect). Calculations specifically for Ca xix are not available but any reduction in the theoretical rates to $2^3S$ and $2^3P$ would increase the contribution attributed to cascades. The effective rates determined from the observations are of course totals and remain unchanged. If the rates used by Bhalia et al. (1975) are treated as lower limits then the contributions from cascades of factors of 1.3 ($2^3P$) and 2.1 ($2^3S$) derived above from these rates, can be considered as upper limits.

References


