THERMODYNAMICAL PROPERTIES OF UNRESOLVED MAGNETIC FLUX TUBES

I: A Diagnostic Method Based on Circular Polarization Ratios in Line Pairs

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Abstract. We propose a diagnostic method, based on the observation of circular polarization signals in line pairs, to derive the thermodynamical properties of unresolved magnetic elements in the solar atmosphere. The concept of response function for the ratio of circular polarization signals in two lines is introduced and its main properties are analyzed. Some detailed calculations for suitably selected line pairs are presented.

1. Introduction

It is nowadays well known that solar magnetic fields outside sunspots are fragmented in small scale elements. These magnetic fragments or magnetic flux tubes (MFT) are highly concentrated both in facular areas and in the network (Harvey, 1977). According to observations (Frazier and Stenflo, 1973; Stenflo, 1973) and theoretical models (Spruit, 1976), these flux tubes have subarcsec linear dimensions and magnetic field strengths of the order of 1500–2000 G. A suitable method has indeed been developed to obtain an approximate measurement of the magnetic field strength inside the tubes (Stenflo, 1973). This method is based on the simultaneous observation of the circular polarization signals in line pairs. It provides the possibility of obtaining a measurement of the magnetic field irrespectively of the fraction of area covered by the MFTs over the observed area on the solar surface. It is well known, in fact, that usual magnetograph observations fail in obtaining appropriate values for the magnetic field in the MFTs due to the mixture of magnetic and non-magnetic regions over the observed area.

While the amplitude of the magnetic field seems to be fairly known in MFTs, very little information is available about their thermodynamical properties. Up to now, most of the work aimed to derive the thermodynamical properties of MFTs has been mainly based on observations of facular contrast in the continuum and of line weakenings. From these observations an empirical model for the facular element is generally obtained by fitting the theoretical results to the observed line profiles or facular contrast. An example of this procedure is given by Chapman (1977). A major difficulty in interpreting these observations is due to the fact that the MFTs do not cover the entire observed area, but an unknown fraction \( f \) of it. \( f \) has to be considered an additional parameter in the overall problem and it is often difficult to disentangle the effects of the thermodynamical structure of the MFTs from those due to their reduced filling of the observed area.
This difficulty can however be avoided by means of polarimetric observations. A Stokes parameter profile of a certain feature of the solar spectrum will in fact contain, as far as the linear and circular polarization are concerned, information referring to the MFT only. The non-magnetic atmosphere surrounding the unresolved magnetic structure will in fact produce an unpolarized spectrum which will not influence the observations of $Q$, $U$, and $V$. The amplitude of the polarization signals will however still depend on the fraction $f$ with a relationship in general unknown, that can be considered linear if the MFTs are supposed to share the same thermodynamical and magnetic properties.

In the following we will make this general assumption concerning the MFTs and we will refer to the properties of the single tube as representative of each of them. Even if the amplitude of the polarization signal is a function of $f$, we expect that amplitude ratios, for instance in different lines, will, on the contrary, reflect intrinsic properties of the MFT, irrespectively of the value $f$. Needless to say that polarization signals that are to be used to derive meaningful ratios, should be obtained simultaneously for the same observed region. Extreme care has also to be devoted to proper calibration if the features to be measured are not sufficiently close in the spectrum that a unique observation can cover both of them.

2. Outline of the Method

Following this approach, in order to derive a suitable diagnostic method for the thermodynamics of MFTs, we have concentrated our attention on circular polarization signals in line pairs having the specific properties outlined in the following.

We consider two lines of the same element, having the same effective Landé factor ($\tilde{g}_1 = \tilde{g}_2$), and having the same intensity profile in the mean photosphere. In a magnetic region having a uniform magnetic field and the same thermodynamical structure as the mean photosphere we should expect the two lines to show almost identical circular polarization profiles. The profiles are indeed exactly the same in the limiting case of faint magnetic fields where the circular polarization profile is given by the expression (Landi Degl' Innocenti and Landi Degl' Innocenti, 1973)

$$V = \tilde{g}\nu_L \cos \psi \frac{dI_0}{d\nu},$$

where $\nu_L$ is the Larmor frequency, $\psi$ is the inclination angle of the magnetic field with respect to the line of sight and $I_0(\nu)$ is the intensity profile across the spectral line. If two lines have the same $\tilde{g}$ and the same $I_0(\nu)$, then according to Equation (1), they give the same $V$-profile in a faint magnetic field. The limiting value of the magnetic field for Equation (1) to hold is typically 100 G for a line at 5000 Å, having $\tilde{g} = 2$. For values of the magnetic field of the order of 1500 G, which are believed to be typical of MFTs, detailed calculations show that the $V$-profiles of the two lines are still very close as can be argued by general physical arguments.
As a consequence, any observed difference in the circular polarization signals of the two lines should be ascribed to the thermodynamical structure of the MFT resulting in a differential weakening of the two lines. In fact, as the two lines have the same Landé factor and the same intensity profile in the mean photosphere, the only reason which can give rise to a different behaviour of the circular polarization profile is their different response to the atmosphere of the MFT. It is then convenient to choose the two lines in such a way that their lower energy-levels are well separated in energy; alternatively the two lines can belong to different ionization stages of the same atom.

A search has been made on the solar atlas in the attempt of finding line pairs obeying the above requirements. The search has been restricted to line pairs of the same element with a wavelength separation less than about 5 Å. We have deliberately avoided pairs composed of lines of different elements in order to deal, as much as possible, with the same broadening mechanisms for both lines. Moreover, the requirement of the closeness in wavelength has been set to avoid calibration problems; if the two lines are sufficiently close they can in fact be measured simultaneously with, for instance, the High Altitude Observatory Stokes Polarimeter (Baur et al., 1980, 1981). The selected line pairs are shown in Table I. Needless to say that the requirements of the same Landé factor and the same photospheric intensity profile for the two lines is only approximately satisfied.

Once a particular pair has been selected, the problem arises of how to deduce information concerning the thermodynamics of the MFT, from the observed ratio of the circular polarization signals in the two lines. Two different approaches can in principle be followed. The first consists in computing the circular polarization in the two lines from a grid of models describing the atmosphere of the MFT; the model which produces the best agreement would be considered the most representative one. The other approach, which we have followed in this paper, is based on the general idea of response function. Since the existing models of MFTs are far from being well established, we consider that our approach is to be preferred at this stage.

3. Theory

The concept of response function has already been introduced by several authors (Beckers and Milkey (1975) and Caccin et al. (1977) for non-magnetic lines and Landi Degl’Innocenti and Landi Degl’Innocenti (1977) for magnetic lines). The theory presented in this last paper could be suitably employed for our present purposes. However we prefer to deal with a simplified version of that theory which results from the approximation of considering the magnetic field in the MFT parallel to the line of sight. This approximation will be more appropriate for the interpretation of observations of MFTs obtained near the center of the solar disk. We then disregard linear polarization and concentrate only on the Stokes parameters $I$ and $V$. 

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<table>
<thead>
<tr>
<th>Pair No.</th>
<th>Line</th>
<th>Wavelength$^a$</th>
<th>Multiplet$^a$ No.</th>
<th>Equivalent$^a$ width (mA)</th>
<th>Central$^b$ depression</th>
<th>Transition$^c$</th>
<th>Low$^c$ excitation potential</th>
<th>Effective Landé factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fe I</td>
<td>5079.230</td>
<td>66</td>
<td>100</td>
<td>0.81</td>
<td>$a^3P_2 \rightarrow y^3P_1^e$</td>
<td>2.19</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Fe I</td>
<td>5079.745</td>
<td>16</td>
<td>87</td>
<td>0.82</td>
<td>$a^3F_2 \rightarrow z^3F_1^e$</td>
<td>0.99</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>Fe I</td>
<td>5196.065</td>
<td>1091</td>
<td>78</td>
<td>0.70</td>
<td>$y^3F_3^e \rightarrow f^3P_2$</td>
<td>4.24</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>Fe II</td>
<td>5197.576</td>
<td>49</td>
<td>80</td>
<td>0.72</td>
<td>$a^3G_{5/2} \rightarrow z^3F_{3/2}^e$</td>
<td>3.22</td>
<td>0.7</td>
</tr>
<tr>
<td>3</td>
<td>Fe I</td>
<td>5543.199</td>
<td>926</td>
<td>61</td>
<td>0.59</td>
<td>$b^1G_4 \rightarrow x^3G_3^o$</td>
<td>3.68</td>
<td>1.38</td>
</tr>
<tr>
<td></td>
<td>Fe I</td>
<td>5543.944</td>
<td>1062</td>
<td>63</td>
<td>0.60</td>
<td>$y^3D_1^o \rightarrow g^3D_2$</td>
<td>4.20</td>
<td>1.5</td>
</tr>
<tr>
<td>4</td>
<td>Fe I</td>
<td>5952.726</td>
<td>959</td>
<td>68</td>
<td>0.54</td>
<td>$z^3F_2 \rightarrow e^3F_2$</td>
<td>3.97</td>
<td>0.67</td>
</tr>
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<td></td>
<td>Fe I</td>
<td>5956.706</td>
<td>14</td>
<td>60</td>
<td>0.55</td>
<td>$a^3F_3 \rightarrow z^3P_2^e$</td>
<td>0.86</td>
<td>0.7</td>
</tr>
</tbody>
</table>

$^a$ Moore et al. (1966).
$^b$ Delbouille et al. (1973).
$^c$ Moore (1959).
The general equations describing the transfer of polarized radiation for the Stokes parameters in a magnetic field can be found in Stenflo (1971) and Landi Degl'Innocenti and Landi Degl'Innocenti (1972). In the LTE approximation, for a magnetic field directed along the line of sight and for an observation at disk center, such equations reduce to the following:

\[
\frac{dI_{r, l}}{d\tau} = (k_c/L_c + (k_L/L_c) \Phi_{r, l})(I_{r, l} - B/2),
\]

(2)

where \( I_r \) and \( I_l \) are the intensities for right and left circular polarization respectively, \( k_L \) and \( k_c \) are the line absorption coefficient and the continuous absorption coefficient at the line wavelength, \( L_c \) is the continuous absorption coefficient at the reference wavelength where the optical depth is measured (\( d\tau = -L_c \) ds), \( \Phi_{r, l} \) are the absorption profiles for right and left circular polarization, respectively, and, finally, \( B \) is the local Planck function. The connection between \( I_{r, l} \) and the Stokes parameters is

\[
I_{r, l} = (I \pm V)/2.
\]

(3)

while the absorption profiles can be written in the form (for a normal Zeeman triplet)

\[
\Phi_{r, l} = H(a, v \mp v_H),
\]

(4)

where \( H(a, v) \) is the Voigt function, \( a \) is the damping constant, \( v \) is the wavelength separation from line center expressed in terms of the Doppler broadening,

\[
v = (\lambda - \lambda_0)/\Delta \lambda_D,
\]

and, finally, \( v_H \) is the ratio between Zeeman splitting and \( \Delta \lambda_D \), which can be expressed through the magnetic field intensity \( B \) in the form

\[
v_H = \frac{e\lambda_0^2 B}{4\pi mc^2 \Delta \lambda_D}.
\]

In the case of anomalous Zeeman effect, formula (4) has to be generalized to take into account the strengths and splittings of the various Zeeman components.

Equation (2) can be solved to give

\[
I_{r, l}(\tau) = \frac{1}{2} \int_{\tau}^{\infty} \eta_{r, l}(\tau') B(\tau') \exp \left\{ -\int_{\tau'}^{\tau} \eta_{r, l}(\tau'') d\tau'' \right\} d\tau',
\]

(5)

where we have introduced the symbol

\[
\eta_{r, l} = k_c/L_c + (k_L/L_c) \Phi_{r, l}.
\]

(6)

We choose a particular model atmosphere, like for instance the HSRA model (Gingerich et al., 1971), to describe the thermodynamical structure of the mean photosphere, and we assume this atmosphere, plus a specified value of the magnetic field (say 1500 G) as the zero-order approximation to represent the structure of
the MFT. We then suppose that, at any given value of the reference optical depth \( \tau \), the temperature \( T \) and electron pressure \( P_e \) inside the MFT are given by

\[
T^{\text{MFT}}(\tau) = T(\tau) + \delta T(\tau) = T(\tau)\{1 + x(\tau)\},
\]

\[
P_e^{\text{MFT}}(\tau) = P_e(\tau) + \delta P_e(\tau) = P_e(\tau)\{1 + y(\tau)\},
\]

where \( T(\tau) \) and \( P_e(\tau) \) are the corresponding zero-order values, and \( x(\tau) \) and \( y(\tau) \) represent the fractional variation of temperature and electron pressure inside the tube with respect to the values of the outer atmosphere. If \( x(\tau) \) and \( y(\tau) \) are much smaller than unity at any \( \tau \)-value, then we can write:

\[
\delta(k_c/\bar{k}_c) = \left( \frac{T}{\bar{k}_c} \frac{\partial k_c}{\partial T} \right) x + \left( \frac{P_e}{\bar{k}_c} \frac{\partial k_c}{\partial P_e} \right) y,
\]

where the variation of \( \bar{k}_c \) has not been explicitly considered, consistently with the fact that we continue to use the unperturbed optical depth scale in our perturbative calculations. Analogously, we have:

\[
\delta(k_L/\bar{k}_c) = \left( \frac{T}{\bar{k}_c} \frac{\partial k_L}{\partial T} \right) x + \left( \frac{P_e}{\bar{k}_c} \frac{\partial k_L}{\partial P_e} \right) y,
\]

\[
\delta B = \left( T \frac{\partial B}{\partial T} \right) x.
\]

Substituting these expressions into Equation (2) and equating the first order terms in \( x \) and \( y \), we obtain for the first order corrections \( \delta I_{r,l} \) the equations

\[
\frac{d}{d\tau} \delta I_{r,l} = \eta_{r,l}(\delta I_{r,l} - \delta \tilde{B}_{r,l}),
\]

where \( \eta_{r,l} \) is the same symbol introduced in Equation (5) and \( \delta \tilde{B}_{r,l} \), which formally represents the source function for the \( \delta I_{r,b} \) is given by

\[
\delta \tilde{B}_{r,l} = \frac{1}{2}\delta B - \frac{\delta(k_c/\bar{k}_c) + \Phi_{r,l} \delta(k_L/\bar{k}_c)}{(k_c/k_c) + \Phi_{r,l}(k_L/\bar{k}_c)} (I_{r,l} - B/2),
\]

\( I_{r,l} \) being explicitly given by Equation (5).

To obtain this last expression we have made the further assumption of neglecting the variation \( \delta \Phi_{r,l} \) of the profiles. This approximation is justified by the slight dependence of the profiles on the parameters \( T \) and \( P_e \), together with our poor knowledge of the line-broadening mechanisms in the solar atmosphere.

Equation (9) can be formally solved to give the emerging values of the first-order corrections \( \delta I_{r,l} \) to the right and left circular polarization intensities. In terms of Stokes parameters we can then write:

\[
\delta V = \delta I_r - \delta I_l = \int_0^\infty \{RFV_T(\tau)x(\tau) + RFV_{P_e}(\tau)y(\tau)\} \, d\tau,
\]
where we have introduced the response function for the $V$-Stokes parameter corresponding to the percentage variations of temperature and electron pressure, which are given by

$$RFV_T(\tau) = \alpha_r(\tau) \exp \left( - \int_0^\tau \eta_r(\tau') \, d\tau' \right) - \alpha_l(\tau) \exp \left( - \int_0^\tau \eta_l(\tau') \, d\tau' \right),$$

$$RFV_{P_e}(\tau) = \beta_r(\tau) \exp \left( - \int_0^\tau \eta_r(\tau') \, d\tau' \right) - \beta_l(\tau) \exp \left( - \int_0^\tau \eta_l(\tau') \, d\tau' \right),$$

where

$$\alpha_{r,l} = \frac{1}{2} \eta_{r,l} T \frac{\partial B}{\partial T} - \frac{T}{k_c} \left( \frac{\partial k_c}{\partial T} + \frac{\partial k_L}{\partial T} \Phi_{r,l} \right) (I_{r,l} - B/2),$$

$$\beta_{r,l} = -\frac{P_e}{k_c} \left( \frac{\partial k_c}{\partial P_e} + \frac{\partial k_L}{\partial P_e} \Phi_{r,l} \right) (I_{r,l} - B/2).$$

The response functions now derived clearly depend on the wavelength distance from line center. It is easy to show that they become identically zero both at line center and at infinite distance from line center. This is an obvious consequence of the fact that the $V$-profile vanishes at such wavelengths irrespectively of the thermodynamical structure of the atmosphere. A representative wavelength can be chosen to calculate the RFVs; in the following we will refer to the response functions calculated at the wavelength which gives the maximum $V$-value in the zero-order model.

When two different lines are involved, we can calculate for each of them the corresponding RFVs according to Equations (12a, b). If $V_1$ and $V_2$ are the circular polarization signals corresponding to the zero-order model, the variation of their ratio, due to the MFT's model described by Equation (7) is given by

$$\frac{\delta(V_1/V_2)}{V_1/V_2} = \int_{-\infty}^{\infty} \{\text{RFR}_{T}(\tau) x(\tau) + \text{RFR}_{P_e}(\tau) y(\tau)\} \, d(\ln \tau),$$

where the response functions for the ratio $V_1/V_2$ are

$$\text{RFR}_{T}(\tau) = \tau \left( \frac{RFV^{(1)}_T(\tau)}{V_1} - \frac{RFV^{(2)}_T(\tau)}{V_2} \right)$$

with analogous expression for $\text{RFR}_{P_e}(\tau)$.

In the integral in Equation (13) we have preferred to change the integration variable from $\tau$ to $\ln \tau$ for computational convenience.

4. Numerical Computations

We have explicitly calculated the response functions for the ratio $V_1/V_2$ relative to three line pairs, namely line pair number 1, 2, and 3 in Table I. The HSRA
TABLE II
Spectral line parameters as derived from the fit

<table>
<thead>
<tr>
<th>Line</th>
<th>Wavelength</th>
<th>gfA</th>
<th>Δλ_D(mÅ)</th>
<th>a</th>
<th>W_d/W_0^a</th>
<th>r_i/r_o^b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe i</td>
<td>5079.230</td>
<td>6.0 × 10^{-8}</td>
<td>30</td>
<td>0.24</td>
<td>1.011</td>
<td>0.982</td>
</tr>
<tr>
<td>Fe i</td>
<td>5079.745</td>
<td>3.8 × 10^{-9}</td>
<td>30</td>
<td>0.10</td>
<td>1.009</td>
<td>1.003</td>
</tr>
<tr>
<td>Fe i</td>
<td>5196.065</td>
<td>3.5 × 10^{-6}</td>
<td>30</td>
<td>0.24</td>
<td>0.996</td>
<td>0.990</td>
</tr>
<tr>
<td>Fe ii</td>
<td>5197.576</td>
<td>7.2 × 10^{-8}</td>
<td>30</td>
<td>0.09</td>
<td>0.997</td>
<td>0.994</td>
</tr>
<tr>
<td>Fe i</td>
<td>5543.199</td>
<td>5.6 × 10^{-7}</td>
<td>32</td>
<td>0.26</td>
<td>1.001</td>
<td>1.002</td>
</tr>
<tr>
<td>Fe i</td>
<td>5543.944</td>
<td>2.02 × 10^{-6}</td>
<td>32</td>
<td>0.23</td>
<td>1.006</td>
<td>1.008</td>
</tr>
</tbody>
</table>

^a Ratio between theoretical and observed equivalent widths.
^b Ratio between theoretical and observed line center depressions.

model, with a constant, vertical magnetic field of 1500 G has been assumed as zero-order model for the MFT. The relevant properties of the lines, namely the oscillator strength f, the damping constant a, and the Doppler broadening Δλ_D (both assumed constant with depth), have been derived by fitting the theoretical profiles, computed with the HSRA model without magnetic field, to the observed profiles of a solar atlas (Delbouille et al., 1973). Particular care has been devoted to reproduce, as accurately as possible, the central depression and the equivalent

Fig. 1. Best fit to the observed profiles for the lines λ5079.230 and λ5079.745 (line pair number 1). The solid line is taken from Delbouille et al. (1973); the dots show the theoretical results obtained for the parameters in Table II.
width of the line; these quantities are more relevant for our purposes than the
detailed shape of the line wings. The results of the fit are listed in Table II, while
in Figure 1, a typical fit is shown. The resulting $gfA$ values for the single lines
agree, within an order of magnitude, with the theoretical estimates that can be
obtained from the $gf$ values listed by Kurucz and Peytremann (1975) and the iron
abundance $A = 1.6 \times 10^{-5}$ quoted by Chapman (1977). With the line parameters
so deduced we have then calculated the zero-order $V$-profiles, separately for each
pair of lines. The numerical code is described elsewhere (Landi Degl'Innocenti,
1976). A typical result is shown in Figure 2. The difference between the $V$-profiles

![Graph showing circular polarization profiles](image)

**Fig. 2.** Circular polarization profiles for lines $\lambda 5543.199$ and $\lambda 5543.944$ (line pair number 3) calculated in the zero order atmosphere (HSRA plus a constant vertical magnetic field of 1500 G) for the line parameters in Table II.

of the two lines is indeed fairly small as has been anticipated in Section 2. The
response functions are finally computed by means of Equations (14), (12), and (8).
Particular attention has to be devoted to the explicit calculations of the partial
derivatives of the quantities $k_e$ and $k_L$ (Equations (8)). In fact, as we consider $T$
and $P_e$ as the only independent variables to describe the thermodynamical properties
of the atmosphere, any dependence of $k_e$ and $k_L$ on other quantities, like for
instance the gas pressure, has to be taken properly into account. This is done by
writing explicit relationships of the form $P_g = P_g(T, P_e)$. The RFR response functions
for the three line pairs are shown in Figures 3a and 3b.
5. Remarks and Conclusions

Some general remarks can be made about the response functions so obtained:

(a) As shown in Figure 3, the RFRs of the single line pairs are approximately of the same order of magnitude; however the RFR for pair number 2 is the most pronounced while the one for pair 3 is the least pronounced. This is due to the fact that the two lines of pair 2 belong to different ionization stages while, between pairs 1 and 3, the difference arises from the larger separation between the excitation potentials of the lines of pair 1. This result confirms the intuitive idea that the RFR relative to a given pair will be larger if the difference between the excitation potentials of the two lines is larger.

(b) A numerical quadrature of the RFRs,

$$\mathcal{I}_T = \int_{-\infty}^{\infty} RFR_T(\tau) \, d(\ln \tau), \quad \mathcal{I}_{P_e} = \int_{-\infty}^{\infty} RFR_{P_e}(\tau) \, d(\ln \tau),$$

(15)
brings for the three line pairs the values given in Table III. For constant values of \( x(\tau) \) and \( y(\tau) \), Equation (13) simply becomes

\[
\frac{\delta(V_{1}/V_{2})}{V_{1}/V_{2}} = J_{T} x + J_{P_{e}} y
\]  

(16)

which, together with the values quoted in Table III, shows that a given value of \( x \) results in a value of the same order of magnitude for \( \delta(V_{1}/V_{2})/(V_{1}/V_{2}) \). This

<table>
<thead>
<tr>
<th>Line pair</th>
<th>( J_{T} )</th>
<th>( J_{P_{e}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.40</td>
<td>-0.11</td>
</tr>
<tr>
<td>2</td>
<td>3.15</td>
<td>-0.23</td>
</tr>
<tr>
<td>3</td>
<td>1.08</td>
<td>-0.09</td>
</tr>
</tbody>
</table>
means that the method presented in this paper is fairly sensitive to the temperature structure of the atmosphere of the MFT. The sensitivity to the electron pressure is however about one order of magnitude lower than the sensitivity to the temperature.

(c) It has however to be remarked that the wide range in \( \ln \tau \) where the RFRs are significantly different from zero, strongly hampers our ability in obtaining detailed information concerning the structure of the MFT from observations. In other words, only if the RFR would be a very peaked function of \( \ln \tau \), observations could provide a good diagnosis for the temperature or electron pressure at the point of the peak. Unfortunately this is not the case for the RFRs that we have computed. The situation is even worse due to the sign inversion that the RFRs experience along the optical depth scale. If, for instance, we would have to interpret a positive value of \( \delta(V_1/V_2)/(V_1/V_2) \) observed for line pair number 2, this could be produced either by a temperature increase in the upper layers or by a temperature decrease at deeper optical depths (see Figure 3a). Only with a good combination of fairly different RFRs, this kind of ambiguity can in principle be removed.

(d) A simple comparison between Figures 3a and 3b shows that between the two functions \( \text{RFR}_{P_e}(\tau) \) and \( \text{RFR}_{T}(\tau) \) relative to each pair, exists an approximate relationship of the form

\[
\text{RFR}_{P_e}(\tau) = \xi \text{RFR}_{T}(\tau),
\]

where \( \xi \) is a constant whose value is about \(-0.1\), independently of the pair considered. This fact may seem surprising but can be understood by the following approximate argument. Considering Equations (12b), it can be shown that the leading terms in the expressions for \( \alpha_{r,1} \) and \( \beta_{r,1} \) are

\[
\begin{align*}
\alpha_{r,1} & = -\frac{T}{k_e} \frac{\partial k_L}{\partial T} \Phi_{r,i}(I_{r,i} - B/2), \\
\beta_{r,1} & = -\frac{P_e}{k_e} \frac{\partial k_L}{\partial P_e} \Phi_{r,i}(I_{r,i} - B/2)
\end{align*}
\]

so that for the ratio \( \xi = \beta_r/\alpha_r = \beta_i/\alpha_i \) we have

\[
\xi = \frac{\partial (\ln k_L)}{\partial (\ln P_e)} / \frac{\partial (\ln k_L)}{\partial (\ln T)}.
\]

On the other hand, for a Fe \( i \) line, \( k_L \) is proportional, apart from a factor independent of \( T \) and \( P_e \), to the quantity

\[
k_L \sim \frac{\exp(-\chi_i/kT)N_H}{1 + (N_1/N_0) + (N_2/N_0) + \cdots}.
\]
denominator of Equation (20) is \((N_i/N_0)\), which can be expressed through the Saha equation. The strongest dependence of \(k_L\) on \(T\) and \(P_e\) is so contained in the expression.
\[
k_L \sim P_e T^{-5/2} \exp \left( (\chi_I - \chi_l)/kT \right),
\]

where \(\chi_I\) is the ionization potential of neutral iron.

From Equation (18) we obtain:
\[
\xi \approx -\left( \frac{5}{2} + (\chi_I - \chi_l)/kT \right)^{-1}
\]

which is a number of order \(-0.1\).

The value \(\xi\) now deduced for the ratio \(\beta_e/\alpha_e\) (and \(\beta_I/\alpha_I\)) is then transmitted to the ratio of the response functions through Equations (12a) and (14).

As a consequence of the approximate relation (17), we can rewrite Equation (13) in the form:
\[
\frac{\delta(V_1/V_2)}{V_1/V_2} \approx \int_{-\infty}^{\infty} RFR_T(\tau)(x(\tau) + \xi y(\tau)) \, d(\ln \tau)
\]

which shows that the method presented in this paper cannot give independent information on \(T\) and \(P_e\), but only on their combination \(z(\tau) = x(\tau) + \xi y(\tau)\).

(e) The actual deduction of the depth dependence of the combination \(z(\tau)\) from the observations is a problem in itself which requires further investigation. However if \(N\) values of \(\delta(V_1/V_2)/(V_1/V_2)\) are observed for \(N\) different line pairs, one can in principle solve the inversion problem by parametrizing the depth dependence of the function \(z(\tau)\) with a suitable number of parameters less or equal to \(N\). These parameters may then be obtained by a standard least-square technique.

References


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