ON CHANGES OF THE ROTATION VELOCITIES
OF STABLE, RECURRENT SUNSPOTS AND THEIR
INTERPRETATION WITH A FLUX TUBE MODEL*

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Abstract. The angular rotation velocities of stable, recurrent sunspots were investigated using data from
the Greenwich Photoheliographic Results 1940 until 1968. We found constant rotation velocities during
the passages on the solar disk with errors of about ±4 m s⁻¹. During their lifetime these spots show a
decreasing braking of their rotation velocities from 0.8 to 0.3 m s⁻¹ per day. A plausible interpretation
is found by assuming the spots to be coupled to a slowly rising subsurface flux tube and a rotation
velocity which increases with depth.

1. Introduction

In a recent paper Koch et al. (1981) claim for a very constant rotation velocity of
stable sunspots, which they observed using a computer-controlled tracing technique.
In this work we try to verify their results using published data (Greenwich Photo-
heliographic Results) and, in addition, investigate changes of the rotation velocities
of stable, recurrent sunspots during their lifetime. The behaviour of rising flux tubes
in the upper convection zone is used to interpret the results.

An investigation related to our work was given in a communication by the
Astronomer Royal (1925): He gave changes of the positions of bipolar spots during
their lifetimes observed 1878 until about 1925. His results for bipolar spots are
only partly in agreement with ours for single spots. The same arguments hold for
the results of Godoli and Mazzuconi (1979), who investigated all sunspot groups
with one to four passages 1944 until 1954.

2. Data and Reduction

The most important data of sunspot groups observed within 0.85 of the solar radius
from the Greenwich Photoheliographic Results 1940 until 1968 and the
classification of these groups from Zürich publications were stored in computer-
readable form (Balthasar and Wöhl, 1980). From these 8352 groups we selected
those of types H or J which were observed at least 6 times during one passage.

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Among this sample of 511 passages we found 39 groups which fulfilled the more important demand to have at least two passages of the type described. This sample was again divided into two classes:

(1) Sunspots which were observed only during these two passages (15 spots).
(2) Groups which were observed for three or more passages (24 groups).

For all passages of the groups we used the data for the positions of the groups from the Greenwich Photoheliographic Results to determine their mean angular rotation velocities during these passages. We assumed a constant angular velocity during each passage and determined the errors for these fits. For the spots of class 1 a histogram statistics of the errors is given in Figure 1.

![Histogram statistics of the mean errors of the fitted rotation velocities of the spots of class 1.](image)

Fig. 1. Histogram statistics of the mean errors of the fitted rotation velocities of the spots of class 1.

In addition we determined the dates of the central meridian passages of all groups using the described fits. The data of the groups were then used to determine changes of the angular velocities in between the passages.

We also calculated parabolic fits of the positions to investigate possible changes of the velocities during the passages.

In addition we used the data to determine the differential rotation of the groups according to the law \( \omega = \omega_0 + \omega_1 \sin^2 \phi \) (with \( \phi \) = latitude, \( \omega_0 \) and \( \omega_1 \) = fit parameters).

### 3. Results

The average error of the fitted angular rotation velocities is 0.0363°/d for all 30 passages of the spots of class 1. Omitting one extremely high value we get 0.0305°/d,
which equals to about 4 m s\(^{-1}\). This is in good agreement with the value of 2 to 5 m s\(^{-1}\) given by Koch \textit{et al.} (1981). Our minimum value is also 2 m s\(^{-1}\).

For the groups of class 2 the mean error of the angular rotation velocity is 0.0484°/d for a total of 90 passages. This higher value can be ascribed to the fraction of active groups which are not of type H or J, but are included in this special sample. The range is 0.007 to 0.318°/d for the mean error of the fits.

The changes of the rotation velocities of the spots of class 1 from the first to the second passage have a range of −0.432°/d to +0.436°/d, the mean value is −0.0589°/d, with a standard error of the mean of 0.056°/d. This equals to a deceleration of the rotation velocity of about 8 m s\(^{-1}\) per rotation or 0.3 m s\(^{-1}\) per day.

Since the class 2 contains also developing groups only averages of the first and second half of the lifetimes of the groups were used to determine changes of the rotation velocities as compared to their mean values. From the 24 groups 17 rotated more rapid in the beginning than in the end. The differences for all groups are +0.1601°/d and −0.1189°/d as compared to the averaged values for the total lifetimes for the first phase and the second phase, respectively. The standard errors of the mean are 0.0316°/d and 0.0314°/d, respectively. The distribution of these differences is given in Figure 2. The results of a \(t\)-test show that the difference of the mean values is significant on a 1\% level.

The mean difference between the two phases is 2.08 rotations (there are 3 to 5 passages per group available), therefore the reduction of the velocity is 0.1341°/d.

![Histogram statistics of the velocity differences of rotation velocities in the first phases and the last phases of the lifetimes of sunspot groups of class 2 as compared to the mean rotation velocities.](image)

\(\text{Fig. 2. Histogram statistics of the velocity differences of rotation velocities in the first phases and the last phases of the lifetimes of sunspot groups of class 2 as compared to the mean rotation velocities.}\)
per rotation. The more pronounced deceleration is observed in the first phase, when it is about 21.5 m s\(^{-1}\) per rotation or about 0.8 m s\(^{-1}\) per day, while in the second phase we have about 16 m s\(^{-1}\) per rotation or 0.6 m s\(^{-1}\) per day.

In contrast to the highly significant deceleration of the spots of class 2 we did not find a significant result for class 1; this may be due to the short time interval between the two rotation values.

The sample of spots of class 1 show an increase of the mean latitude from 12.74\(^\circ\) to 13.37\(^\circ\). For the groups of class 2 this increase is from 15.525\(^\circ\) to 15.574\(^\circ\) in the first phase and from 15.574\(^\circ\) to 15.576\(^\circ\) in the second phase, respectively (absolute value of the latitude used). The differential rotation of the groups of class 1 is in the 1st passage (\(\phi =\) latitude, sidereal rotation velocities given)

\[
\omega(\phi) = (14.33 \pm 0.06) - (3.36 \pm 0.67) \sin^2 \phi [^\circ/d]
\]

and

\[
\omega(\phi) = (14.22 \pm 0.05) - (2.61 \pm 0.56) \sin^2 \phi [^\circ/d]
\]

in the 2nd passage.

For the groups of class 2 the differential rotation is:

\[
\omega(\phi) = (14.35 \pm 0.10) - (0.76 \pm 1.16) \sin^2 \phi [^\circ/d],
\]

\[
\omega(\phi) = (14.18 \pm 0.06) - (2.13 \pm 0.60) \sin^2 \phi [^\circ/d]
\]

in the first and second phase, respectively.

The changes of angular rotation velocities determined from the data of the central meridian passages are similar to those determined from the linear fits during the disk passages. The absolute rotation velocities are higher by about +0.05\(^\circ\)/d and +0.03\(^\circ\)/d for the groups of class 1 and class 2, respectively.

Fitting of a parabola \(y = x_0 + x_1 t + x_2 t^2\) to the position for each passage yields no results concerning the changes of rotation velocities in the case of class 1: The mean value of the parameter \(x_2\) is +0.0005\(^\circ\)/d\(^2\), but with a standard error of 0.005\(^\circ\)/d\(^2\). In the case of the groups of class 2 the mean is -0.006\(^\circ\)/d\(^2\) with a standard error of 0.008\(^\circ\)/d\(^2\). By this method of reduction about the same amount of deceleration is obtained as given above, but the result is much more uncertain.

4. Interpretation of the Results

As already shown by Koch et al. (1981) stable sunspots of types H and J have very constant rotation velocities during their passages on the solar disk with typical mean errors of the fits of 2 to 5 m s\(^{-1}\). In addition we have shown that recurrent spots of these types show a small deceleration which is about 0.8 m s\(^{-1}\) per day in the first phase after the group becomes visible (− and may still be of another type than H or J) and which is later only 0.6 m s\(^{-1}\) per day or even 0.3 m s\(^{-1}\) per day for another sample of spots (with questionable significance, see Section 3).

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This finding of decreasing deceleration is—so far as we understand—not supported by the result, that the rotation velocities of recurrent sunspots determined from central meridian passages are slightly higher than the averages of the fits from the first and second disk passages. But it is supported in the case of groups of class 2 by the fits of parabola to the daily positions during the passages on the disk.

It could be assumed that these decelerations might be caused by systematic motions of the spots in latitude: Indeed there is an increase of the averaged latitude by +0.63° for the spots of class 1 from one passage to the next. But using the law of differential rotation for these spots the deceleration which could be due to a meridional motion is only one third of the amount observed. The spots of class 2 show also an increase of the mean latitudes but with a much smaller amount of less than +0.1°. The corresponding deceleration caused by differential rotation is more than a factor of 100 smaller than the observed values. There are two alternative ways to interpret the observed deceleration:

(a) Spots originate with fast rotation due to coupling with deeper layers of the convection zone, decouple afterwards and are decelerated then by aerodynamic drag and turbulent viscosity until they move with the photospheric plasma.

(b) Spots or groups are always connected with a subphotospheric flux tube whose rotation velocity they show. This tube rises slowly towards the photosphere while everywhere floating with the local plasma due to efficient viscous interaction. If the rotation velocity increases with depth, this shows up in the deceleration of the spots while the basic flux tube rises.

Let us examine (a) first. Consider, for simplicity, a circular cylinder with radius \( R \) and uniform density \( \rho_1 \) which moves with velocity \( u \) relative to a medium with density \( \rho_e \). The velocity as a function of time can be calculated by considering a braking force per unit length. This could be either aerodynamic drag \( F_D \) or the force due to turbulent viscosity, \( F_v \) (Parker, 1979):

\[
F_D = \frac{1}{2} \rho_e u^2 R C_D \\
F_v = \frac{4 \pi \rho_e \nu_t}{Q}
\]

with drag coefficient \( C_D = 0(1) \) and \( Q = \ln (4/\text{Re}) \) where \( \text{Re} \) is the effective Reynolds number \( \text{Re} = u R / \nu_t < 1 \) with turbulent kinematic viscosity \( \nu_t \). Equating (1) and (2) with the momentum change per unit length we obtain

\[
u_D(t) = \frac{u_D(0)}{(1 + \gamma u_D(0) t)} , \quad \gamma = \frac{C_D(2 \pi R)}{(\rho_e/\rho_1)}
\]

for aerodynamic drag and

\[
u_v(t) = u_v(0) \exp(-\beta t) , \quad \beta = \frac{(4 \nu_t)}{(QR^2)}(\rho_e/\rho_1)
\]

for turbulent viscosity.

Using \( \nu_t = 10^{-6} \) to \( 10^{10} \, \text{m}^2 \text{s}^{-1} \), \( R = 10^6 \) to \( 10^7 \, \text{m} \) and \( C_D(\rho_e/\rho_1) = 0(1), (4/Q) (\rho_e/\rho_1) = 0(1) \) we get \( \gamma = 10^{-7} \) to \( 10^{-8} \, \text{m}^{-1} \) and \( \beta = 10^{-6} \) to \( 10^{-2} \, \text{s}^{-1} \). Differentiating (3) and (4) after \( t \) we can calculate the initial velocity \( u(0) \) if we insert the observed
deceleration of $-0.8 \text{ m s}^{-1}$ per day, which is about $-10^{-5} \text{ m s}^{-2}$:

$$u_D(0) = (-\dot{u}_D(0)/\gamma)^{1/2} \approx 10 \text{ to } 30 \text{ m s}^{-1},$$

$$u_v(0) = (-\dot{u}_v(0)/\beta) \approx 10^{-3} \text{ to } 10 \text{ m s}^{-1}$$

for drag and viscosity, respectively. $u_D(0)$ has a plausible value for the initial velocity difference, while $u_v(0)$ is only reasonable for $\beta = 10^{-6}$, i.e. $\nu = 10^8 \text{ m}^2 \text{ s}^{-1}$ which is a rather small value for the turbulent viscosity. Let us look at the velocity differences later, using the higher values for $u_D(0)$ and $u_v(0)$: See Table I.

**TABLE I**

Relative velocity and deceleration for viscous ($u_v$, $\dot{u}_v$) and aerodynamic ($u_D$, $\dot{u}_D$) braking, respectively, as a function of time. Both mechanisms are far too effective

<table>
<thead>
<tr>
<th>$t$</th>
<th>[days]</th>
<th>0</th>
<th>25</th>
<th>50</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_D(t)$ [m s$^{-1}$]</td>
<td>30</td>
<td>17.1</td>
<td>12</td>
<td>9.2</td>
<td></td>
</tr>
<tr>
<td>$\dot{u}_D(t)$ [m s$^{-1}$ per day]</td>
<td>0.78</td>
<td>0.25</td>
<td>0.12</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>$u_v(t)$ [m s$^{-1}$]</td>
<td>10</td>
<td>1.2</td>
<td>0.13</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>$\dot{u}_v(t)$ [m s$^{-1}$ per day]</td>
<td>0.86</td>
<td>0.1</td>
<td>0.01</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

It is clear from these values that both mechanisms do not fit the observations that give decelerations of $0.3 \text{ m s}^{-1}$ per day (class 1) and $0.6 \text{ m s}^{-1}$ per day (class 2) even after two rotations. If we fit the deceleration at the beginning to the observed value of $0.8 \text{ m s}^{-1}$ per day we cannot reproduce the values observed later. Consequently, interpretation (a) has to be dropped.

Possibility (b) cannot be checked as easily. Imagine the following sequence of events: A horizontal magnetic flux tube rises due to its magnetic buoyancy. After reaching a depth of about $10000 \text{ km}$ below the photosphere it gets unstable to loop formation because of downflow along the fieldlines (Schüssler, 1980).

The loops break through the photosphere and build an active region. The material flowing down is cooled relative to the exterior due to the superadiabatic effect (Parker, 1978; Spruit, 1979) and gathers in a ‘bag’ in the subsurface parts of the flux tube; this effect decreases the buoyancy of the tube, possibly together with topological pumping due to supergranulation (Drobychevskii and Yuferev, 1974; Meyer et al., 1979). The ‘bag’ material gets heated by convection and radiation and consequently the tube rises slowly due to buoyancy (our results lead to a mean rising velocity of $2 \text{ m s}^{-1}$). Because of the efficiency of viscosity (see Table I) the tube always floats with the ambient gas and the spots therefore show a deceleration if $\partial\Omega/\partial r < 0$. We cannot make a simple estimate like for case (a) because we do not know enough about the dynamics of the buoyant rise in these regions.
The well known fact that the rotation velocity of stable sunspots (of types H and J) is about 0.2 to 0.3°/d smaller than the rotation velocity of more active groups (particularly of types B, C, and D) was e.g. demonstrated by Balthasar and Wöhl (1980) for sunspot groups observed 1940 to 1968. At the end of their lifetimes the groups of classes 1 and 2 show even equatorial rotation velocities which are 0.11 to 0.16°/d smaller than those of all spots of the same types (see Table 2 of Balthasar and Wöhl, 1980).

The equatorial rotation velocity of 14.22 and 14.18°/d is about the same as that of the photospheric plasma measured by Pérez Garde et al. (1981).

However it should be mentioned that this equatorial rotation velocity for the solar plasma is one of the highest values published within the last decade. The 'classical' value of Howard and Harvey (1972) was only 13.76°/d. On the other hand Scherrrer et al. (1980) found an equatorial rotation velocity of 14.45°/d for the photospheric plasma. Without entering into the very complicated problem of possible variations and/or instrumental effects which may have caused false results in the determination of the plasma velocity, we believe that very stable, recurrent sunspots have rotation velocities of the same amount as the photospheric plasma during the last phases of their lifetimes. Using again the flux tube model we must then conclude that active groups (e.g. of types B, C, D, and others) are coupled to the plasma in deeper layers of the convection zone. According to the changes of the differential rotation of the groups investigated we find no significant result: In the case of the spots of class 1 there seems to be a decrease of the gradient of the differential rotation, but the amount of the gradient seems to be the same as expected for spots of these types (see again Table 2 of Balthasar and Wöhl, 1980). The groups of class 2 on the other hand show a smaller amount of the gradient which is increasing during the lifetime of the groups. It seems that the rotation of the selected groups is in total slightly more rigid than for all groups observed.

5. Conclusions

We find for two selected classes of stable, recurrent sunspots that these spots show a deceleration of their rotation velocities during their lifetimes. As already stated in the introduction our sample of spots is quite different from that of Godoli and Mazzuccconi (1979). Nevertheless their indication for a deceleration of the rotation velocities with the ages of the sunspots is in agreement with our results. In addition we gave quantitative results which lead to a more sophisticated interpretation by the flux tube model.

The interpretation of the behaviour of the sunspot motions in time with a flux tube model leads to a hypothesis about the differential rotation in depth: We state an equatorial rotation of about 14.34°/d in a depth of about 10 000 km below the photosphere. From the mean duration of 2.08 rotations (about 56 days) for the deceleration of the groups of class 2 to the photospheric velocity we find a mean velocity of the rising flux tubes of about 2 m s⁻¹. The result that the amount of
deceleration is higher in the beginning than in the end of the lifetimes is either an indication for a varying gradient of differential rotation in depth or could be due to variations of the rising velocity of the flux tubes.

We believe that these results and their interpretation by the flux tube model are able to improve the qualification of sunspots (at least of stable sunspots) as tracers of solar dynamics. Nevertheless more precise and simultaneous observations of individual motions of sunspots, the plasma within and around them are needed to obtain a more detailed understanding of the flux tube dynamics. In addition theoretical models of the flux tubes and their behaviour within the upper convection zone and photosphere must be improved.

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References

Greenwich Photoheliographical results, Greenwich 1940–1968.