Comment on the Paper

'A NEW RESONANCE IN THE SOLAR ATMOSPHERE'

by Joseph V. Hollweg

(Research Note)

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Abstract. In the absence of genuine forcing terms, there is no resonance between linear fast mhd and gravito-acoustic waves.

Recently Hollweg (1979) reported a new 'resonance' in the solar atmosphere wherein a fast mhd wave 'matches' a gravito-acoustic wave. We propose to show in what follows that this resonance is spurious.

Following Hollweg, we write the linearized equations of continuity, motion and magnetic induction as

\[
\frac{\partial}{\partial t} \delta \rho + \nabla \cdot \rho_0 \delta \mathbf{v} = 0 ,
\]

\[
\rho_0 \frac{\partial}{\partial t} \delta \mathbf{v} = -\nabla \delta \rho + \delta \mathbf{p} - \nabla \left( \frac{\mathbf{B}_0 \cdot \delta \mathbf{B}}{4\pi} \right) + \frac{\mathbf{B}_0 \cdot \nabla}{4\pi} \delta \mathbf{B} ,
\]

\[
\frac{\partial}{\partial t} \delta \mathbf{B} = \nabla \times (\delta \mathbf{v} \times \mathbf{B}_0) ,
\]

where \( \delta \rho, \delta \rho, \delta \mathbf{v}, \) and \( \delta \mathbf{B} \) are the first order variations in density, pressure, velocity and magnetic field while \( \rho_0 \) and \( \mathbf{B}_0 \) are the zero-order density and magnetic field. We confine our discussion to the isothermal case (the same arguments being valid for the adiabatic case as well). The energy equation will be, therefore,

\[
\delta p = T_0 \delta \rho ,
\]

where \( T_0 \) is the zero-order temperature. Using expressions of the form \( a(z) \exp(i(k_x x + \omega t)) \) for the first order quantities we can write Equations (1a) through (1d) as

\[
i \omega \delta \rho + \rho_0 \frac{\partial}{\partial z} \delta v_z + \delta v_z \frac{d \rho_0}{dz} = -i k_x \rho_0 \delta v_x ,
\]

\[
i \omega \rho_0 \delta v_x = \frac{B_{0z}}{4\pi} \left[ \frac{\partial}{\partial z} \delta B_z - i k_x \delta B_x \right] - i k_x \delta \rho ,
\]

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\[ i \omega \delta v_z = -g \frac{\delta \rho}{\rho_0} - \frac{1}{\rho_0} \frac{\partial}{\partial z} \delta \rho, \quad (2c) \]

\[ i \omega \delta B_x = B_{0z} \frac{\partial}{\partial z} \delta v_x, \quad (2d) \]

\[ \omega \delta B_z = -k_x B_{0z} \delta v_x. \quad (2e) \]

The quantities \( \delta \rho, \delta \rho, \) and \( \delta \mathbf{B} \) can be eliminated from Equations (2a) through (2e) to yield the two equations

\[ \left\{ V_A^2 \frac{\partial^2}{\partial z^2} + (\omega^2 - V_A^2 k_x^2) \right\} \delta v_x = -ik_x T_0 \left( \frac{\partial}{\partial z} - \frac{g}{T_0} \right) \delta v_x + k_x^2 T_0 \delta v_x \quad (3a) \]

and

\[ \left( \frac{\partial^2}{\partial z^2} - \frac{g}{T_0} \frac{\partial}{\partial z} + \frac{\omega^2}{T_0} \right) \delta v_z = -ik_x \frac{\partial}{\partial z} \delta v_x. \quad (3b) \]

Hollweg neglected the entire r.h.s. of Equation (3a). Such a neglect is valid at best for the term \( k_x^2 T_0 \delta v_x \) under the approximation \( V_A^2 \gg T_0 \) (strong magnetic field). However, the neglect of the term containing \( \delta v_x \) is an inconsistency if one is expecting a resonance to increase \( \delta v_z \) without bound. Thus one must retain that term. Under the approximation \( V_A^2 \gg T_0 \) Equation (3a) is modified to

\[ \left\{ V_A^2 \frac{\partial^2}{\partial z^2} + (\omega^2 - k_x^2 V_A^2) \right\} \delta v_x = -ik_x T_0 \left( \frac{\partial}{\partial z} - \frac{g}{T_0} \right) \delta v_x. \quad (3a') \]

Hollweg's solution for \( \delta v_x \) is given by

\[ \delta v_x = c_1 J_p(\Lambda) + c_2 J_{-p}(\Lambda), \quad (4) \]

where

\[ p = 2|k_x| T_0/g \]

and

\[ \Lambda = \frac{2\omega T_0}{gV_A} e^{-gz/T_0}. \]

If one substitutes (4) in (3a') we get

\[ k_x T_0 \left( \frac{\partial}{\partial z} - \frac{g}{T_0} \right) \delta v_z = 0. \quad (5) \]

We thus see that the quantities \( \delta v_x \) and \( \delta v_z \) become decoupled and can be determined independently. The fallacy in Hollweg's treatment of the problem was in substituting expression (4) for \( \delta v_x \) in Equation (3b) (where it appears as a forcing term) forgetting that Equation (4) implies Equation (5). Such a fallacy will certainly lead to spurious resonances.
This can be discerned more clearly in the following illustration where we use the Equations (3a′) and (3b) neglecting gravity:

Under the approximation \( g = 0 \), Equations (3a′) and (3b) reduce to

\[
\left\{ V_A^2 \frac{\partial^2}{\partial z^2} + (\omega^2 - k_x^2 V_A^2) \right\} \delta v_x = -i k_x T_0 \frac{\partial}{\partial z} \delta v_z \tag{6a}
\]

and

\[
\left( \frac{\partial^2}{\partial z^2} + \frac{\omega^2}{T_0} \right) \delta v_x = -i k_x \frac{\partial}{\partial z} \delta v_x , \tag{6b}
\]

where \( V_A \) is now constant.

The procedure followed by Hollweg is tantamount to assuming

\[
\delta v_x = c_1 e^{i q z} + c_2 e^{-i q z} , \tag{7a}
\]

where

\[
q = \left( \frac{\omega^2 - V_A^2 k_x^2}{V_A^2} \right)^{1/2} .
\]

This leads to

\[
 i k_x T_0 \frac{\partial}{\partial z} \delta v_z = 0 \tag{7b}
\]

by virtue of Equation (6a). If we ignore Equation (7b) temporarily, then substitution of (7a) in Equation (6b) yields the particular integral

\[
\delta v_x = \frac{i k_x}{(q^2 - \omega^2 / T_0)} \{ c_1 e^{i q z} + c_2 e^{-i q z} \} . \tag{8}
\]

This gives a spurious ‘resonance’ when \( \omega^2 = q^2 T_0 \). We call this spurious because Equation (8) is in no way compatible with (7b) except when \( k_x = 0 \). In this case, however, there is no resonance and instead we see that \( \delta v_z \) vanishes!

In general it must be remembered that resonances are possible in a linear system only when there are external forcing terms in the equations. In the absence of such external forcing terms, the equations can only yield a dispersion relation from which it is possible to determine the natural modes of the system.

Reference