CONVECTIVE DYNAMOS FOR ROTATING STARS

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I. Introduction

Let me first set some bounds on my topic, since I have only thirty minutes. First, I feel it appropriate to focus primarily on the global stellar dynamo problem, as opposed to the problem of how individual magnetic flux tubes are formed and behave. One reason I do this is that we are now confronted with rather clear evidence that some stars have field reversing dynamos, and others do not. I believe global dynamo theory is currently better able to provide clues as to why this occurs. I do not mean to imply that flux tube dynamics is not important for stellar dynamo theory; on the contrary, it may be crucial. However, no one has yet attempted to incorporate flux tube dynamics into dynamo models beyond very crude parameterizations. This problem is difficult enough for the sun, for which at least we can see some of the properties of flux tubes at the top of the convection zone. Clearly the uncertainties rise significantly for other stars.

The second reason I choose to focus on the global dynamo problem is that I feel I know more about it, and it gives me the (selfish) excuse to say something about my own recent work.

My current view is that convectively driven dynamos are the most likely source of magnetic fields in stars that have convection zones. Dynamo theory applied to the sun and stars still has numerous serious difficulties yet to be cleared up, and it could, in the end, still fail, but on balance I find the case for dynamo origin of fields much more convincing than any other explanation. Certainly dynamo origin has been explored much more extensively and carefully. Competing "theories," if we can call them that, usually involve torsional or some other kind of oscillations acting on a perhaps primordial field buried under the convection zone. Such oscillations can always be postulated, but rotation observations made at the surface, e.g., Howard and Labonte, (1980); Labonte and Howard, (1981), indicate perturbations on the differential rotation of less than 0.5%. As we will illustrate later in more detail, differential rotation plays a central role in stellar dynamo theory, because of its ability to induce toroidal magnetic fields from poloidal ones. It's very hard for me to see how such weak oscillations can compete in inducing fields. The same argument applies to acoustic oscillations in a star competing with convection to generate and maintain either differential rotation or magnetic fields, and yet such effects have been proposed.

Even more extreme is the notion that fields in the solar or a stellar convection zone are primordial, having never been dissipated, as has been argued principally by Piddington (see, e.g., Piddington, 1978, and earlier references cited therein). I feel the crucial point that has been missed here is that, however large the electrical conductivity, turbulence is quite capable of cascading magnetic energy down to length scales on which ohmic dissipation takes place, on a time scale which is essentially the eddy turnover time for the energy containing eddies. Frisch (1977?) has made this argument by analogy with ordinary turbulence, and I find it quite convincing. I believe the

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difficulty with stellar dynamo theory is not that there is no induction of fields (which would have to be the case if the fields were primordial, since they are not growing without limit) but that the theory predicts too much induction. As I will argue later, we are currently probably underestimating the magnitude of processes which bound the field.

Tidal forcing from the planets (principally Jupiter in the solar system) has also been proposed numerous times (particularly in the Soviet literature) to explain the solar cycle. I have not attempted a calculation of the tidal flows expected in the sun, but, again, they seem very unlikely to be able to compete with convection and differential rotation. So far, all we have here is the apparent near coincidence of solar cycle and Jupiter orbital periods. But of course the true magnetic period of the sun is 22 years, not 11. The planetary tidal hypothesis is amusing to contemplate in one respect, in that, if it held for stars generally, then we should expect that stars observed to have Call, and therefore magnetic, cycles also have planetary systems whose major planets have orbital periods somewhere near a decade, while stars with non-cyclic Call emission apparently do not. I leave it to others, if they wish, to ponder that possibility further.

II. Brief sketch of solar dynamo theory

Dynamo theory as it is currently applied to the sun and stars has its roots principally in the work of Parker (1955). Parker demonstrated that the combination of a) differential rotation b) "cyclonic" or helical motions, and c) turbulent diffusion of magnetic fields, could give rise to amplification of large scale toroidal and poloidal magnetic fields, as well as migratory dynamo waves which could explain the migration of the sunspot zones toward the equator. New toroidal fields are produced from the poloidal field by the shearing of differential rotation; new poloidal fields are generated from the toroidal field by the lifting and twisting of the helical motions; and turbulent diffusion allows coalescence of small scale magnetic structures into a net global pattern.

Important papers by Babcock (1961) and Leighton (1964, 1969) followed, which were closer to the solar dynamo problem in a phenomenological sense, but were mathematically and physically less rigorous, more ad hoc. But all involve an oscillating feedback between poloidal and toroidal fields, and a dissipation mechanism to keep the fields from growing without bounds.

Greater rigor and formality returned to the dynamo problem with the development, principally by Steenbeck, Krause, and Radler, of "mean field electrodynamics" in the late 1960's (for reviews, see Krause 1976, Radler, 1976). This theory attempts to predict the behavior of a global scale average field from the action of small scale fluid motions on small scale magnetic fields. In mathematical terms, the induction equation for this mean field $\mathbf{B}$ takes the form

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B} + \alpha \mathbf{B}) - \lambda \nabla \times \nabla \times \mathbf{B}. \quad (1)$$

In equation (1) $\mathbf{V}$ represents the global motion present, larger in scale than the averaging element (which in practical terms is a volume element larger than an individual eddy but smaller than the dimensions of the fluid body, or perhaps an axisymmetric ring of fluid of small cross-section). The parameter $\alpha$ incorporates Parker's
cycloic motions, and in the theory is proportional to the helicity \( H = \mathbf{v} \cdot \nabla \times \mathbf{v} \) of the fluid flow \( \mathbf{v} \) in the form

\[
\alpha = -\frac{1}{3} \mathbf{v} \cdot \nabla \times \mathbf{v} \tau
\]

in which \( \tau \) is a correlation time which measures the persistence of the small scale motion and the overbar denotes an ensemble or volume element average. \( \lambda \) is a turbulent diffusivity for magnetic flux. \( \alpha \) and \( \lambda \) are usually taken to be scalars although in general we should expect them to be tensors.

When \( \mathbf{V} \) is taken to be an axisymmetric differential rotation \( \omega(r, \phi) \), where \( r \) is radius and \( \phi \) latitude, and \( \alpha/\omega \ll 1 \), then the \( \alpha \) effect is ignored in generation of toroidal fields, and we have the so-called \( \alpha - \omega \) dynamos, which are a more formal and general version of Parker's migratory dynamo. \( \alpha - \omega \) dynamos have been applied most extensively to the sun, because they generally give field reversing solutions with migratory toroidal and poloidal fields. The long series of dynamo models by Yoshimura (see, e.g., Yoshimura 1975a) are also of this type. However, Yoshimura has identified \( \alpha \) (his "regeneration action") with global or "giant cell" convection on the sun, while most other solar dynamo modelers have been less specific about the scale of motion principally responsible for \( \alpha \).

In simple \( \alpha - \omega \) dynamo models, the frequency of field reversals is proportional to the square root of the product of \( \alpha \) and the angular velocity gradient, so the larger the helicity, or shear in the differential rotation, the shorter the dynamo period. With "tuning" of the parameters \( \omega, \alpha, \) and \( \lambda \), \( \alpha - \omega \) dynamos can simulate many features of the solar cycle. Results from a particularly simple example from Stix (1976) are illustrated in Figure 1. This model has angular velocity increasing with depth in a step function, and an \( \alpha \sim \cos \phi \). An 11 year period is seen in the migration of toroidal and poloidal fields toward the equator.

![Diagram showing evolution of solar field](image)

Figure 1. \( \alpha - \omega \) dynamo solution from Stix (1976) showing evolution for one complete "solar cycle." Closed contours on left in each cross section are toroidal field magnitudes (solid and dashed opposite signs); Poloidal field lines are on the right. Reprinted by permission of the publisher.
Dipole symmetry of the fields, such as is reflected in Hale's sunspot polarity law and polar fields, is slightly favored in dynamo models where $\alpha$ and $\omega$ gradients are concentrated in low latitudes, but it is not really understood why. More highly tuned models involving more complex functions for $\alpha$ and $\omega$, such as that of Yoshimura (1975a), can give poleward migration of poloidal fields near the poles. Still further elaborations involving introduction of ad hoc nonlinear feedbacks with time delays (e.g. Yoshimura 1978a,b) allow generation of successive solar cycles of varying amplitude and period.

Yoshimura (1975b) demonstrated that in $\alpha-\omega$ dynamos the migration of toroidal field with time is along isorotation surfaces. Which direction the field propagates is determined by the sign of the product of the helicity or $\alpha$ and the sign of the angular velocity gradients. Propagation toward the equator with the right phase relation between toroidal and poloidal fields requires the angular velocity increase with depth, as well as $\alpha > 0$ (helicity left handed or negative) in the northern hemisphere, $\alpha < 0$ in the southern hemisphere. We will illustrate in more detail how helicity and differential rotation interact to produce migration of the toroidal field when we sketch some of our own recent results below.

A large part of the "success" of $\alpha-\omega$ dynamos in simulating solar cycle behavior is clearly due to the number of free parameters that can be tuned. In addition, with few exceptions, only the kinematic dynamo problem is being solved. That is, the motions are assumed, not calculated, and feedbacks from the induced electromagnetic body forces are ignored. But even in the kinematic dynamo context there are serious difficulties. The turbulent diffusivity $\lambda$ is a considerable oversimplification of what must really be happening; it is undoubtedly a time varying tensor, perhaps even changing sign, and is dependent in some nonlinear fashion on the scale and magnitude of motions included. The angular velocity gradient with depth in the sun is not known beyond inferences from the difference between sunspots and doppler rotations that it increases inward in the outermost 10 - 20 x $10^3$ Km. But perhaps most important, mixing length arguments (Kohler, 1973) and consistent global convection models such as our own predict helicity and $\alpha$ which are 2 - 3 orders of magnitude too large to give the correct solar cycle period. (The sign of $\alpha$ does not appear to be a serious problem) An $\alpha$ this large implies that helicity is competing on equal terms with differential rotation in determining the toroidal field, so that we no longer are dealing with a pure $\alpha-\omega$ dynamo. If $\alpha$ is large enough, it determines both poloidal and toroidal field, and we have an "$\alpha^3$" dynamo. Most dynamos of this type are not field reversing! This point may be very important for explaining the existence of stars with strong magnetic activity but no apparent cyclic behavior.

In the Yoshimura dynamos, $\alpha$ is small in the limit of weak influence of rotation upon convection that he assumes. But in this limit it is impossible to maintain a differential rotation of the magnitude and even sign that the sun has with convection of the amplitude he assumes. We return to this point later when considering our own model results.

III. Some current nonlinear dynamo calculations.

In part to try to get away from the relatively large number of free parameters and functions present in kinematic dynamo theory, we have for the past three years been doing nonlinear full MHD dynamo calculations, starting from a nonlinear model for convection in a rotating spherical shell. This model was originally developed to study what amplitude and profile of differential rotation is generated by global convection, as a function of such basic input parameters as the rotation rate of the system and
the amount of heating at the bottom of the shell. The mathematical structure of this model has been described in detail in a number of previous publications, e.g. Gilman 1975, 1977; Gilman and Miller, 1981. In brief, it represents a stratified but incompressible shell of fluid heated uniformly from inside and rotated at a specified rate. Stress free boundary conditions confine the angular momentum to the shell in the absence of magnetic stresses acting across a boundary. The fluid is assumed to have constant diffusivities for momentum and temperature. Solutions are found for the three dimensional nonaxisymmetric velocity, temperature and pressure patterns using a Fourier expansion in longitude, and a grid in the meridian plane. Typically between 16 and 25 longitudinal wave numbers are included. The nonlinear interactions among all these modes are explicitly calculated.

We have used this model primarily to determine the conditions under which differential rotation similar in profile and magnitude to that of the sun are obtained, when solar values are used for most of the physical parameters of the model. In general, we have found that large amplitude, broad equatorial acceleration occurs only when the convection zone is relatively deep, say 1/3 of the radius or more, and the influence of rotation upon the convection is strong (the opposite limit to that used by Yoshimura, 1972). Under these circumstances the amplitude or kinetic energy of the convection is comparable to that of the differential rotation, the precise ratio depending sensitively upon such parameters as the eddy viscosity of the fluid.

We have generalized the hydrodynamic code to a full MHD code using the same solution procedures (see Gilman and Miller, 1981) to find a number of dynamo solutions, mostly for the case when the model differential rotation surface profile is at least initially similar to that of the sun. In these calculations, we are, as with the $\alpha-\omega$ dynamos, forced to use a turbulent diffusivity for magnetic field, but there is no assumed $\alpha$ as in equation (1). The full induction effects of the convection and differential rotation are instead explicitly calculated, and the full feedback of the electromagnetic body force arising from the dynamo generated fields is retained. Under certain circumstances, its effects can be quite important, as we shall see.

Usually, we find a dynamo solution by first allowing the hydrodynamic solution for convection and differential rotation to become fully established, through a time integration of the equations starting from an initial random temperature field. Then we add a toroidal magnetic field and follow its subsequent development or decay. In some cases, we use as initial conditions for a new calculation the end solution for a previous case with somewhat different parameter values, partly in order to speed convergence, but particularly to find certain finite amplitude dynamo solutions for conditions under which a small amplitude magnetic field would decay.

Although our model is not an $\alpha-\omega$ dynamo in the sense that we do not lump induction effects into a parameter $\alpha$, our solutions behave in many ways similar to such dynamos, and it is useful to describe their behavior in terms of the separate effects of differential rotation and helicity. Let me take you through a series of schematic drawings that describe the induction process going on in this model, which repeats many of the arguments given in Parker (1955) made specific to the properties of convection and differential rotation we find in the model.

Figure 2 gives a schematic of the global convection pattern found in the model and the differential rotation it drives. I call your attention particularly to the structure of the convection. The horizontal velocity vectors are tilted relative to the east-west direction, and the whole flow pattern has a spiral structure. The first feature implies angular momentum transport by the convection toward the equator, which is responsible for maintaining the equatorial acceleration. The second feature indicates
the flow has helicity, which together with the differential rotation, drives the dynamo. Both of these properties of the convection arise due to the influence of coriolis forces. This particular drawing of the convection is for a case of moderate influence of rotation upon the convection, for which the axis of the convective roll bends toward the pole with latitude. With stronger influence, the axis cuts the outer surface at a finite latitude, as the roll attempts to align with the axis of rotation. The convection pattern seen in Figure 2 illustrates how closely linked the mechanism for differential rotation maintenance (angular momentum transport) and dynamo action (helicity) are. The physics of the system really does not allow them to be varied independently, as has been done in kinematic dynamo calculations.

![Diagram of convection and differential rotation](image)

**Figure 2.** Schematic of global convection patterns which drive the differential rotation in a spherical shell convection model.

**Figure 3** shows typical meridional cross sections of differential rotation and helicity found in solutions with strong influence of rotation upon the convection. The angular velocity $\Omega$ is constant on cylinders concentric with the axis of rotation, and therefore $\Omega$ decreases with radius as well as latitude. Outside the tangent cylinder to the inner boundary (dashed line) the helicity is negative (left-handed) in the northern hemisphere, positive in the southern hemisphere. (The shell is cut off at 60 degrees N and S to avoid computational instability due to convergence of the meridians.)
In our dynamo calculations, we have found that it is the helicity outside the tangent cylinder that is primarily responsible for the dynamo action (the model clearly does not handle high latitude helicity and dynamo action as well, either, something which needs further study). What that helicity does to a toroidal flux tube or ribbon is shown in Figure 4, which is a cross-section of the shell at a northern hemisphere low latitude in the plane perpendicular to the axis of rotation. Clockwise flow is associated with fluid, rising out of the plane, counterclockwise with sinking fluid. An initially straight toroidal flux ribbon will be lifted in a direction parallel to the rotation axis and pushed out toward the outer boundary, sunk and pushed toward the inner boundary, taking on the structure shown. (The solid line part of the ribbon lies above the plane, the dashed part below the plane.) The end result of this lifting and twisting process is schematically shown for one loop in Figure 5. A new loop in the meridian plane has been formed from the original toroidal field.
Left Handed Helicity of Northern Hemisphere from Global Convection

Figure 4. Schematic of convective flow field (heavy solid arrows and words "rising" and "sinking") that gives rise to left-handed helicity in the northern hemisphere, along with superimposed magnetic flux ribbon (light, broad, arrows drawn in perspective). Flux ribbon is shown deformed from purely toroidal ribbon by the action of helicity pattern shown. Solid part of ribbon above the plane, dashed ribbon below the plane.

With rotation constant on cylinders, the angular velocity gradient is in the direction perpendicular to the axis of rotation (z direction in Figure 5). This gradient then shears the meridional flux loop again out into the toroidal direction. With angular velocity increasing outward, the outer part of the loop will be pushed forward in longitude, the inner part backward. This results in new toroidal field near the top of the loop of the same sign as original, and toroidal field of the opposite sign near the bottom. The net effect is to move the pattern of toroidal field upward, and therefore poleward, roughly parallel to the rotation axis. In the southern hemisphere, due to the change of sign in helicity, the toroidal field pattern would move downward. This propagation direction is opposite to what we want for the sun, and we return to this point below.

What we have illustrated is what a single convective mode whose structure looks like that in Figure 2 would do. In reality in the model, there is always present a whole spectrum of convective modes of different longitudinal wave numbers, different amplitudes, and somewhat different structures. But their net effect is similar to what we have shown.
IV. Nonlinear dynamo results

For a given hydrodynamic solution, whether we get dynamo action (growing magnetic fields sustained at finite amplitude) or not depends upon what magnitude we assume for the turbulent diffusivity $\lambda$. Obviously, the smaller is $\lambda$, the more dynamo action we should get. In every case, there is a threshold value of $\lambda$ above which (magnetic Reynolds number below which) an initial perturbation field decays, below which it grows. This behavior is similar to that for the kinematic dynamo problem. But the kind of dynamo action we get depends sensitively on the relative amplitudes of differential rotation and convection. We find it convenient to measure these amplitudes by the total kinetic energies integrated over the whole volume of convecting fluid.

The first dynamo calculations we performed (Gilman and Miller, 1981) were for flow patterns for which the kinetic energy of the convection was $2/3$ or more of the total, the differential rotation $1/3$ or less. We found what I would call “random” dynamo action, in which amplified fields were maintained, but no clear pattern of field reversals emerged, nor was any dominant symmetry in the magnetic field about the equator established even if one was assumed initially. The toroidal field was a small percentage of the total magnetic field energy ($\sim 5 - 10\%$), and its maintenance was primarily by the helicity of the convection, not by differential rotation. Thus, in this case our dynamo was more of the $\alpha^2$ type than the $\alpha - \omega$ type. Initially, this was a discouraging result, but it seemed clear we needed to find other hydrodynamic solutions for the same amplitude differential rotation (since that is constrained by observations) but smaller convection amplitude, in order to reduce the relative role of convection in maintaining toroidal field. A second reason for doing this is that the convection spectrum significantly exceeded the upper limits estimated for the sun by LaBonte, Howard and Gilman (1981).
We have recently found such solutions, by reducing the eddy viscosity and thermal conductivity in our model by a factor of 10. In these solutions, the differential rotation without magnetic field is now as much as 80% of the total energy of the system, the sum of all the convection energy as small as 20%. The reason this is possible is that with smaller viscosity, a smaller angular momentum transport rate by the convection can maintain the same amplitude differential rotation.

The dynamo action we are finding from these hydrodynamic solutions is quite different. Within about 10% above the threshold for dynamo action to occur, an initial toroidal field antisymmetric about the equator (such as the sun is believed to have) remains predominantly antisymmetric, leading to poloidal fields of dipole type, and undergoes clear, regular reversals. The toroidal field energy is a much larger fraction of the total (up to 30%), and it is maintained mostly by shearing of the axisymmetric poloidal field by the differential rotation. Thus we are obtaining solutions very much like an \( \alpha - \omega \) dynamo. We are still calculating solutions for an initially symmetric toroidal field, for which reversals may not last beyond an initial transient. But the threshold for both symmetries is about the same, and both initial symmetries seem to be preserved in this parameter range.

For slightly smaller \( \lambda \), or larger magnetic Reynolds number, we find the fields grow to the point where feedbacks from the electromagnetic body force cause fundamental changes in the dynamo itself. In particular, the convection is relatively unaffected, but the differential rotation can be severely reduced. For an axisymmetric dynamo only 15% or so above the threshold, the feedback is strong enough to reduce the differential rotation kinetic energy by more than a factor of 2. At 60% above the dynamo threshold, the differential rotation kinetic energy reduction is a factor of 5. In the latter case, the field reversals appear to cease, (more like the \( \alpha^2 \) dynamo again) and the dominant symmetry changes from antisymmetric to symmetric. In the former case, the solutions are tetering on the edge of the same behavior. Presumably in solutions still further above the dynamo threshold, the differential rotation would be damped even more.

The preferential suppression of differential rotation compared to convection is, we believe, a rather general result. The reason it occurs is that, even when the differential rotation is substantially larger in amplitude than the convection, the work it does against the viscous force is much less, because it is a much larger scale flow pattern and its velocity gradients are weaker. Consequently, a given electromagnetic body force is, compared to the viscous force, a larger brake on the differential rotation than on the convection. With the convection undiminished, the angular momentum transport by it remains the same, but the differential rotation drops in magnitude until the sum of work done against the viscous force and the electromagnetic body force roughly equals the work against viscosity alone in the absence of a magnetic field.

Presumably, if we calculate dynamos for even smaller turbulent magnetic diffusivity \( \lambda \), we would generate fields of such magnitude that the convection itself would start to be suppressed. This would provide the final upper limit on dynamo field strength since the strength of the entire induction process would be bounded by its own feedbacks.

Even though the differential rotation is damped, the average magnetic energy sustained is higher in the solutions we have studied. We appear, therefore, to have found at least two distinctly different classes of solutions: field reversing dynamos with somewhat lower levels of magnetic fields, and non-reversing dynamos with a higher level of field. We suggest these two types of solutions might provide an explanation for the two types of variability seen in calcium emission, e.g., Vaughan and
Preston, (1980). The strong, irregular emitters have generated a strong enough field to greatly reduce their differential rotation, while the weaker emitters have not. It is not clear how a change in rotation rate in the model would affect when field reversals occur and what the field amplitude is, because it would be felt in both the differential rotation maintenance and in the helicity. We would need to do a lot more calculations to sort this out, but it may be that for higher rotation rates, the range of parameter values over which the dynamo action suppresses differential rotation is expanded, making it more likely to occur in a sample population of stars.

Durney, Mihalas and Robinson (1981) have also proposed that the strong, non-cyclic Call emitters are dynamos operating further above the dynamo threshold. They argue that as a result several different dynamo modes are excited which interact and interfere, obscuring a simple cyclic variability. In a nonlinear dynamo model such as ours it is not possible to separate out different dynamo "modes" but it is clear that when differential rotation is suppressed, more of the magnetic energy goes into smaller scale, and the field does take on a less global character, as they suggest. Our mechanism is different from theirs, however, in that it employs a nonlinear feedback from the induced fields to suppress the global, reversing component of the total pattern - it is not just obscured, it is not present.

Returning to the solar case, when we do get field reversals in our model, the period is too short for the sun by about a factor of 10. Our reversing toroidal field is quite broad in spatial scale compared to what may be the case on the sun as evidenced by sunspots. I continue to suspect the short period occurs because the model does not capture the true small scale interaction between an intermittent solar field and global convection. The solar field may "escape" a lot of the helicity if, for example, the field is continually pushed into the corners of giant convection cells, contrary to the idealized picture we drew in Figures 4 and 5. Indeed, Childress (1978) has illustrated with a very simple model how this might happen. Alternatively, or perhaps as well, the global convection on the sun may be even smaller in amplitude than even in our present model calculations.

V. Role of compressibility

An additional difficulty is that when field reversals occur, the migration of the toroidal fields is toward the poles (along the isorotation surfaces) rather than toward the equator. Currently, the only way we see around this difficulty is through the addition of several scale heights of density variation with radius in our model convection zone, something we obviously need to add anyway. The reason compressibility may help is that we have early indications (Glatzmaier and Gilman, 1981a) that global convection modes that extend all the way from top to bottom in a rotating compressible convection zone will generate a differential rotation nearly constant on cylinders with equatorial acceleration only near the bottom of the zone where the scale height is large. The rotation will decrease with height in the outer part. The helicity profile will remain similar to that in Figure 3 at all levels (Glatzmaier and Gilman, 1981b). Figure 6 shows a schematic differential rotation profile we might expect from a full nonlinear compressible calculation (not yet attempted), showing a maximum angular velocity somewhere in the middle of the layer.

Given such a profile, the migration of the toroidal field with time would be toward high latitudes and toward the outer boundary deep in the convection zone until the field reaches the level of angular velocity gradient reversal. Thereafter migration would be back toward the equator. Thus a new magnetic cycle would be thought of as starting near the equator deep down, where synchronization of the two hemispheres would be a natural consequence. Toroidal field, i.e., sunspots, of a new cycle would be seen first in a mid latitude, as observed. This scheme is consistent with an informal

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suggestion made by Raymond Hide more than a decade ago, that the latitude where
new cycle spots first appear may be a measure of the depth of the convection zone.
This assumes whatever dynamo action maintains them is operating primarily outside
the tangent cylinder to the inner convection zone boundary due to rotational effects,
as is the case with our dynamo.

Figure 6. Contours of differential rotation predicted to be produced by compres-
sible global convection when individual convection cells reach from bottom to top
of the convection zone, as calculated by Glatzmaier and Gilman (1981a).

This picture is rather different from present $\alpha - $ $\omega$ dynamo models applied to the
sun. It should be regarded as an hypothesis to be tested with a full compressible
dynamo model, certainly not as a result already obtained. We should remark that in a
compressible dynamo model, magnetic buoyancy will also be present, and we expect
this effect to be particularly important in the top part of the convection zone where a
field of given strength results in a much larger buoyancy force than near the bottom.

VI. Other feedback effects.

In addition to the feedback effects already described, there are two others also
worth mentioning. The first is that even if the induced magnetic field is weak, say hav-
ing magnetic energy no more than $10^{-3}$ or $10^{-4}$ of the kinetic energy of the system, the
cumulative effect can still be significant. In particular, the fluid flow is instable to
small perturbations, so that the time histories of two solutions, one with no magnetic
field and one with a small field, diverge away from each other after a few thousand
timesteps. At the same elapsed time in the two cases, the same convective modes, i.e.,
the same longitudinal wave numbers, will have different amplitudes and phases. The two solutions eventually become uncorrelated with each other. This limits the predictability of such flows, and argues against any deterministic feedback with a long, but essentially fixed, time delay, such as has been invoked by Yoshimura (1975 a,b) to explain the amplitude meanderings in the envelope of the solar cycle.

Another, quite different form of feedback we have encountered is one which gives rise to a finite amplitude dynamo instability. For a certain range of magnetic turbulent diffusivity $\lambda$ a small amplitude magnetic field decays, but a large amplitude field, which has already partially suppressed the differential rotation, is sustained, and has field reversals. The mechanism is that, for a small field, further decay does not amplify the differential rotation, so the decay continues. But a drop in a finite amplitude field results in some resurgence of the differential rotation, which prevents further decay. In this case we have essentially a servomechanism which keeps the field within a certain range. How important this mechanism might be in stars is unknown, as is the range of $\lambda$ for which it is true. Presumably, if $\lambda$ is large enough, the field will decay no matter how much differential rotation growth there is. And, as we have already mentioned, if $\lambda$ is small enough, a large amplitude field is maintained even though the differential rotation is almost completely suppressed. We are continuing to study these kinds of nonlinear dynamos.

VII. Questions concerning application of dynamo theory to the sun and stars

Let me close by summarizing some of the uncertainties and difficulties involved in applying dynamo theory to the sun and stars.

1. We do not know the profile of angular velocity with depth on the sun, and we know virtually nothing about differential rotation for any other star. There is some hope we can infer rotation with depth for the sun from frequency shifts in the 5 minute oscillations (see, e.g., Rhodes, Deubner and Ulrich, 1979). Measuring differential rotation in stars is an extremely difficult problem, likely to elude us for a long time yet, although Bruning (1981) may have pointed the way as to what to look for.

2. The form and magnitude of the helicity of the motions cannot be measured, and theoretical predictions of this quantity generally give values much too large when used in dynamo models applied to the sun.

3. Since global convection has not been measured even on the sun, we cannot pin down the ratio of differential rotation to convection. Our model, and kinematic dynamo theory before it, indicates this ratio is very important for determining what kind of dynamo a star will have.

4. The theory of turbulent diffusion as applied to stars, particularly diffusion of magnetic flux, is very uncertain.

5. The relative importance of magnetic buoyancy and convection in stellar dynamos is unclear, and probably can be assessed only in theoretical models which allow both to exist.

6. The "filling factor" for the magnetic field, that is, what fraction of the volume contains strong fields, is known for the surface of the sun, but not below; only a few very qualitative inferences have been made for other stars. How intermittent the field is will strongly affect the behavior of the dynamo, both induction processes and feedbacks.

7. Convection zone depth is somewhat uncertain for stars. It is an important parameter for dynamos, because it puts limits on the spatial scale of the largest convective elements, the turnover time for these elements, and it helps determine the
profile of differential rotation. Shallow convection zones are bound to have different differential rotation profiles than deep zones (Gilman, 1980). Convection zone depth will also affect the tendency of stars to form more complex differential rotation profiles, such as illustrated in Figure 6. Also, the region just under the bottom of the convection zone might be the seat of more regular motions favorable for dynamo action, as has been suggested by Galloway and Weiss (1980).

A final remark I wish to make is that even stars without convection zones might conceivably be acting as dynamos. This is most likely to be the case for rapidly rotating stars, where the violation of Von Zeipel's theorem would be greatest. In such stars, Eddington-Sweet circulations should be set up, due to nearly spherical pressure and temperature surfaces not coinciding with more rotationally distorted potential surfaces. However, Busse (1981) has recently argued that the end result of these circulations would instead be primarily a differential rotation, in the form of a baroclinic thermal wind. He demonstrates that such differential rotation profiles are likely to be unstable to non-axisymmetric disturbances in the form of baroclinic waves similar in many respects to those which occur in the terrestrial and other planetary atmospheres. In such a rapidly rotating system, there is no doubt such disturbances would contain helicity, so that the resulting mix of reduced differential rotation and finite amplitude waves could easily act as a dynamo. Baroclinic instability in stars is virtually unexplored at present, but may prove to be important, and is worth further study. It may also occur above convective cores, and possibly beneath convective envelopes.

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REFERENCES