THE IONIZATION STATE IN A GAS WITH A
NON-MAXWELLIAN ELECTRON DISTRIBUTION

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The inferred degree of ionization of a gas is often used in astrophysics as a
diagnostic of the gas temperature. In the solar transition region and corona, in
the outer atmospheres of cool stars, and in some portions of the interstellar
medium), photoionization can be neglected, and the ionization state is fixed by
the balance between ion-electron collisional ionization and dielectronic and/or
radiative recombination. Under these conditions, higher degrees of ionization
result from higher energy ion-electron collisions which are common in a high
temperature gas, and the degree of ionization is thus a reasonable temperature
diagnostic.

Actually, ionization occurs through collisions with electrons that have
kinetic energies greater than the ionization potential of the given ion, and so
the ionization rate depends on to the number of such high-energy electrons in
the tail of the electron velocity distribution (Roussel-Dupre 1979). High-
velocity electrons move across large distances between effective coulomb colli-
sions, and, in a strong temperature or density gradient, the tail can be overpop-
ulated relative to Maxwell-Boltzmann distribution of equivalent energy density
(Scudder and Olbert 1979, 1981; Shoup 1981). Under these circumstances, the
ionization rate can also be greatly increased. We illustrate these effects for a
parameterized form of the electron distribution function with an enhanced
high-velocity tail, namely the "kappa distribution".

Ionization in the Kappa Parameterization

Scudder and Olbert (1981) have shown that expected variations in the
non-equilibrium properties of the electron distribution function f can often be
well represented by changes in a generalized Lorentz function with a single
parameter \( \kappa \), i.e.,

\[
f_\kappa = \frac{A_\kappa}{\pi^{3/2} v_p^3} \left(1 + \frac{v^2}{\kappa v_p^2}\right)^{-(\kappa+1)}
\]

(1)

where \( v_p \) is the most probable speed and \( A_\kappa \) is a normalization constant given by,

\[
A_\kappa = \frac{\Gamma(\kappa + 1)}{\kappa^{3/2} \Gamma(\kappa - 1/2)}
\]

(2)

and where \( \Gamma(\kappa+1)=\kappa! \) is the gamma (factorial) function (cf. Abramowitz and
Stegun 1972, p. 255). This normalized "kappa" distribution is plotted in fig. 1
for various values of the parameter \( \kappa \); note that for \( v \leq \sqrt{\kappa v_p} \) the number of par-
ticles can be closely fit by a Maxwellian distribution, whereas for \( v >> \sqrt{\kappa v_p} \) the
distribution declines as a velocity power law of index \( \alpha = 2\kappa + 2 \). The prominence
of the power law tail diminishes with increasing \( \kappa \), and it can be shown that, as
\( \kappa \to \infty \), the distribution approaches a Maxwellian.

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Fig. 1. Kappa electron velocity distribution function $f_\kappa$ vs. electron speed $v$ in units of the most probable speed $v_p$.

The electron-impact collisional-ionization rate in a given distribution of electrons can be generally written in the form $n_e \langle \sigma v \rangle$, where $n_e$ is the overall number density of electrons, the angular brackets denote an average over the electron distribution in particle speed $v$, and the ionization cross section $\sigma$...
varies with speed as \( \sigma(u) \sim \frac{\ln(u/u_0)}{u_0^2} \), where \( u_0 \) is the ionization threshold speed (see e.g. Lotz 1987). We can then define the rate enhancement \( \beta \) in a given isotropic, but non-equilibrium velocity distribution \( f \) relative to the rate in a Maxwellian distribution \( f_m = \left( \frac{m_e}{2\pi k T} \right)^{3/2} e^{-\left( m_e u^2 / 2 k T \right)} \) at a temperature \( T \).

\[
\beta = \frac{\langle \sigma v \rangle_{\text{non-th}}}{\langle \sigma v \rangle_{\text{th}}} = \frac{\int u \ln(u/u_0) f dv}{u_0} = \frac{\int u \ln(u/u_0) f_m dv}{u_0} = 4\pi \frac{\sqrt{2\pi k T/m_e}}{E_1(\varepsilon_0)} \int u \ln(u/u_0) f dv,
\]

where \( k \) is Boltzmann's constant, \( E_1 \) is the first exponential integral (cf. Abramowitz and Stegun 1972, p.228), and \( \varepsilon_0 = \chi/kT \) is the ratio of ionization potential to thermal energy of the Maxwellian.

Performing the integral in eqn. (3) for a kappa distribution with a temperature \( T \) (defined in terms of the second velocity moment of the distribution), we find,

\[
\beta_\kappa = A_\kappa \frac{x_0}{\sqrt{\varepsilon_0}} \left[ \frac{\kappa}{x_0^2} \right]^\kappa \frac{1F_2(\kappa,\kappa+1,;-x_0^2)}{\kappa E_1(\varepsilon_0)},
\]

where \( 1F_2 \) is the Gauss hypergeometric function (cf. Abramowitz and Stegun 1972; p. 556), and \( x_0 = u_0/u_p \) is the ratio of ionization threshold speed to the most probable speed of the kappa distribution. For integer kappa, the definite integral represented by the hypergeometric function is most easily evaluated as a finite series through repeated integration by parts (cf. Gradshteyn and Ryzhik 1979; eqn. 2.111.3, p. 58).

In fig. 2 the kappa distribution rate enhancement \( \beta_\kappa(T) \) relative to a Maxwellian with the same kinetic temperature \( T \) is plotted versus the ratio of ionization potential to thermal energy \( \varepsilon_p = \chi/kT \). Note that the enhancements can be very large for high ionization potentials (i.e. \( \beta_\kappa \gg 1 \) when \( \varepsilon_0 \gg 1 \)), but that there can also be rate reductions for moderate ionization potentials (i.e. \( \beta_\kappa \ll 1 \) when \( \varepsilon_0 \approx 1 \)). This rate reduction reflects the fact that the electrons in the enhanced high-velocity tail far above threshold are actually less effective at ionization than those of lower energy closer to the distribution core.

**Effect on Specific Ionization Temperature Diagnostics**

We thus see that ionization rates in a plasma with a non-equilibrium electron distribution can be substantially higher than in an equilibrium plasma with same density and temperature, and this implies that the possibly non-equilibrium nature of the plasma should be taken into account when using information on the ionization state to infer the gas temperature. As a specific example, we consider here the errors in inferred ionization temperatures that can occur for the balances \( O^{18} \leftrightarrow O^{17} \) and \( Fe^{+11} \leftrightarrow Fe^{+12} \) under conditions representative of those in the solar corona. Although the lower level of each exchange is the most abundant stage in an equilibrium balance at a typical coronal temperature of \( 1.5 \times 10^6 \) K (Jordan 1969, 1970), the oxygen ionization threshold energy \( \chi_{O^{18}} = 739 eV \) is actually more than twice that of iron \( \chi_{Fe^{+11}} = 331 eV \).
Fig. 2. a. Ionization rate enhancement $\beta_\kappa(T)$ of a kappa distribution relative to a Maxwellian with the same temperature $T$. b. Ionization rate enhancement $\beta_\epsilon(\upsilon_p)$ of a kappa distribution relative to a Maxwellian with the same most probable speed $\upsilon_p$. 
In Table 1 are given the errors $\Delta T$ in inferring the true kinetic temperature $T=1.5 \times 10^6 \, K$ of a kappa distribution from the oxygen and iron ionization state if the non-equilibrium character of the distribution is neglected. Because of its higher threshold potential, the ionization of oxygen is much more sensitive to the high-velocity electrons and thus shows generally much larger temperature errors. In calculating the ionization temperature error $\Delta T$, we have included any changes in recombination rate, although these changes were never large because recombination is most sensitive to low-energy electrons in the relatively unaltered distribution core. These results indicate that the oxygen ionization state is not as good a temperature diagnostic as that of iron for plasmas of coronal temperatures.

**Table 1: Ionization Temperature Errors for Kappa Distribution at $1.5 \times 10^6 \, K$**

<table>
<thead>
<tr>
<th>Kappa</th>
<th>Temperature Error $\Delta T$ ($10^6 , K$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Fe^{1+1_f} \leftrightarrow Fe^{1+2}$</td>
</tr>
<tr>
<td>2</td>
<td>-0.19</td>
</tr>
<tr>
<td>3</td>
<td>-0.08</td>
</tr>
<tr>
<td>4</td>
<td>-0.02</td>
</tr>
<tr>
<td>5</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>0.02</td>
</tr>
<tr>
<td>20</td>
<td>0.01</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Summary**

Since collisional ionization in a plasma occurs through ion-electron collisions at energies above a given ionization threshold potential, ionization rates can be greatly increased by an enhancement in the number of electrons in the high-velocity tail of the electron velocity distribution. In astrophysical plasmas with strong gradients in temperature and/or density, such high-velocity tail enhancements may be common, and so ionization by non-equilibrium electrons may be important to interpreting properly the inferred ionization state in many astrophysical contexts. Specifically, the ionization rate in a non-equilibrium "kappa distribution" can be enhanced by as much as a factor of $10^{13}$ relative to a Maxwell-Boltzmann distribution of equivalent energy density, and errors in inferred ionization temperatures can be as large as a factor of two. The relative importance of the high-velocity tail to a given ionization balance depends on the relative magnitude of the appropriate ionization potential and the mean thermal energy of electrons. In particular, the rather high ionization threshold for the oxygen balance $O^{4+8} \leftrightarrow O^{4+7}$ implies that the oxygen ionization in the solar corona is more sensitive to a non-equilibrium electron distribution than the ionization state of other elements (e.g. iron) with lower ionization potentials. A comparison of the ionization state of species with different ionization potentials (e.g. oxygen and iron) could therefore yield information on the form of the electron distribution function in plasmas at coronal temperatures.
REFERENCES


Roussel-Dupre, R. 1979, Ph. D. Thesis, Univ. of Colo., Boulder

