A STUDY OF CONVECTIVE MODEL STELLAR ATMOSPHERES USING A MODIFIED MIXING-LENGTH THEORY

JOHN B. LISTER
David Dunlap Observatory and Erindale College, University of Toronto
M. C. LANE
Department of Astronomy, University of Toronto
AND
ROBERT L. KURUCZ
Harvard-Smithsonian Center for Astrophysics

Received 1981 October 2; accepted 1982 March 12

ABSTRACT

We have modified the model atmosphere program ATLAS to incorporate two recently suggested changes to the standard mixing-length theory of convection. These are (1) the use of horizontally averaged opacity in place of the opacity at the average horizontal temperature and (2) the use of a variable mixing length. We have studied the resulting changes in the structure of the atmosphere by systematically varying effective temperature, surface gravity, and composition.

We find that the modified model atmospheres show the following changes: (1) convection transports a smaller fraction of the total flux; (2) the convection zone is generally narrower, and the variation of $F_{\text{conv}}/F_{\text{total}}$ with depth is smoother; (3) the temperature is higher in the convective region; (4) our model is brighter in the spectral region $1500 < \lambda < 2000 \, \text{Å}$; and (5) there are small, systematic flux differences in the visual spectral region.

We have also compared our models to the results of nonlocal mixing-length and non-mixing-length theories of convection. Our results agree well with these theories for $\log \tau_R < 0.6$, but at greater depths our models have less convection and higher temperatures.

In comparison to empirical solar models we also find good agreement for $\log \tau_{5000} < 0.4$, but our model is hotter at greater depths. We argue that the empirical determination of the solar temperature structure lacks the power to provide a strong test of convective theories at these great depths.

Subject headings: convection — stars: atmospheres

I. INTRODUCTION

For stars with effective temperatures less than $\sim 9000 \, \text{K}$, the total flux is carried by a combination of radiation and convection in some parts of the atmosphere. As the effective temperature decreases, the flux transported by convection increases, and, for solar-type stars, there is a large subphotospheric zone in which convection carries substantially more than $90\%$ of the energy. It is obvious that the convective regions merit continued study, and just two examples will serve to illustrate the importance of this task. First, the convective zone is the suspected source of the mechanical energy that powers the chromospheres and coronae in late-type stars. An inadequate description of convection would lead to an inaccurate model of this power source. The second example is the diffusion model for the metallic-line A stars which pictures the peculiar elemental abundances as being produced by element diffusion in the region below the hydrogen convection zone (Watson 1970). If the way we compute the convective zone is subject to error, the viability of the model might be compromised. Unfortunately our theoretical understanding of convection is still only poorly developed.

The most widely used approach for computing convective energy transport, the local mixing-length theory (Böhm-Vitense 1958), is primarily a phenomenological theory (see Cox and Giuli 1968) which cannot be applied to either of the important problems cited above. Recent results of work on nonlocal mixing-length or non-mixing-length theories of convection (Nordlund 1976; Nelson 1980; Latour, Toomre, and Zahn 1981) hold considerable promise of providing a more realistic approach, but they have not yet been widely used.

In addition to a certain plausibility, the greatest attribute of the local mixing-length theory is its simplicity. Being easily programmed, variants of this theory have been employed in many of the widely used stellar atmosphere codes (e.g., Kurucz 1970; Gustafsson et al. 1975), and the resulting extensive grids of models have been employed by a large number of astronomers. Recently Deupree (1979) pointed out an inconsistency in the local mixing-length theory. In the standard theory each convectively unstable level in the star is pictured as consisting of rising hot elements and sinking cooler elements, from which an average temperature for that level can be defined. The physical properties of the gas...
needed to compute the convective flux are then found using this average temperature. Deupree noted that this is certainly not correct for the opacity, which must be changing rapidly and nonlinearly with temperature to produce the convective instability in the first place. To correct the mixing-length theory for this inconsistency it is necessary to compute a horizontally averaged opacity, taking into account the different temperatures present in the rising and sinking elements. This can be done by introducing an iterative procedure to calculate the horizontal temperature variations. The result of this modification is to alter the structure of the convective zone and bring it much closer to the structure predicted by the two-dimensional hydrodynamic computations of Deupree (1977). Building on this modification, Deupree and Varner (1980) have proposed a scheme to allow the ratio of mixing-length to pressure scale height to vary with the horizontally averaged temperature in such a way that the mixing-length theory can match the average horizontal structure of the two-dimensional hydrodynamic calculations. As they point out, this approach is open to a number of criticisms. In addition to the ones they list, there is also the fact that the hydrodynamic calculations are subject to uncertainty at the very surface because of the use of the diffusion approximation. The sensitivity of convection to radiative transfer effects at the surface has been demonstrated by Nordlund (1976) and Nelson (1980). Nevertheless, the horizontally averaged opacity and the variable mixing length do represent steps toward a more physically realistic representation of convection. They introduce information about effects, particularly nonlocal ones, which were missing from the standard mixing-length theory, while still retaining its computational ease.

The work of Deupree has been directed toward improving our understanding of stellar interiors. In this paper we have investigated the changes in the structure of stellar atmospheres which result from using Deupree's modified form of the local mixing-length theory.

II. THE MODEL ATMOSPHERE PROGRAM

The widely used program ATLAS (Kurucz 1970) was employed for all of the computer runs to be reported in this paper. We have used the ATLAS6 version of the program, incorporating the modifications described by Kurucz (1979a), and, except where explicitly stated, all of the models included atomic line blanketing as represented by an opacity distribution function. There was one additional modification to the program not incorporated in the version described by Kurucz (1979a). As discussed in detail by Kurucz (1970), ATLAS uses the version of the local mixing-length theory presented in Cox and Giuli (1968). This formulation assumes that the convecting elements are optically thick, and so the horizontal radiative energy loss of an element was determined using the diffusion approximation to radiative transfer. As Mihalas (1978) has described, an alternative expression for the efficiency of the convecting element can be derived if one assumes that it is optically thin, and a simple scheme based on the optical thickness of the element allows one to change smoothly between the optically thick and thin cases. Our version of ATLAS6 uses this modification to the standard mixing-length theory. It should be pointed out that in all cases which we have examined, the horizontal optical thickness of the convecting element has been greater than unity whenever there was a significant convective flux. Therefore, this modification has produced only a small portion of the structural changes to be presented in the following sections. A more fundamental concern is that, as mentioned earlier, the convection is sensitive to radiative energy losses to the stellar surface at small optical depths. The local mixing-length theory ignores this, and the use of the interpolation formula for the bubble's horizontal optical thickness does not remedy this effect.

For ease of reference, we have collected in the appendix the equations for the opacity averaging, the variable mixing length, and the variable optical depth of the convective element discussed above.

III. CHANGES WITH EFFECTIVE TEMPERATURE

Our approach is to explore the differences between our models and those described by Kurucz (1979b) which result from the altered form of the local mixing-length theory. The models of Kurucz incorporate the variable optical thickness of the convecting element discussed in §II, and, except for the modifications of Deupree, they are identical to the models we have computed. Because these LTE atmospheres are characterized by the parameters effective temperature, surface gravity, and composition, we allow only one variable to change at a time in making this comparison. In Figure 1 we show the differences as a function of $T_{\text{eff}}$, holding log $g$ fixed at 4.0 and using solar abundances. For each model we show both the fraction of the total flux carried by convection and the temperature as a function of the Rosseland optical depth, $\tau_R$.

A general conclusion from Figure 1 is that the modified local mixing-length theory predicts less convection. Physically this results from the nonlinear dependence of the opacity on the temperature. Each horizontal layer which is convectively unstable is pictured as containing rising and falling elements which are respectively hotter and cooler than the average horizontal temperature. The effective opacity is found from averaging the opacity of the hot and cool elements as given in equation (A5). This effective opacity is less than the opacity at the average horizontal temperature of the level. Therefore, radiation is able to carry a larger fraction of the flux. There is also a second physical reason for the reduced convective flux. In the standard mixing-length treatment one can compute the horizontal temperature fluctuation from equation (A4), although this is not generally needed. One finds, as mentioned by Deupree (1979), that the temperature fluctuation can be very large, even exceeding the average horizontal temperature at that depth. By using the effective opacity of equation (A5), the horizontal temperature fluctuation is reduced. Again picturing the layer as consisting of rising and falling elements, these elements have been able to approach equilibration.
LESTER, LANE, AND KURUCZ

Vol. 260

because the reduced opacity has facilitated a greater horizontal radiative exchange of energy. The reduced temperature difference between the rising and falling elements results directly in a lower convective flux.

The reduced convection is present at all values of the $T_{\text{eff}}$, and the model at 7500 K shows this very clearly. In the model of Kurucz (1979b) convection carries as much as 0.87 of the total flux at these temperatures, whereas our model peaks at 0.25. It is also apparent that the convective zone is considerably narrower in our models. The region where $\log \left( \frac{F_{\text{conv}}}{F_{\text{total}}} \right) > -2.0$ covers the optical depths $0.34 < \log \tau_R < 1.3$ in our model, while that of Kurucz spans $0.18 < \log \tau_R < 1.83$. While the optical depth depends on the physical conditions, which in turn depend on the convective theory, this narrowness of the convection zone is still as apparent if a geometrical depth or a mass column density is used as the independent variable.

Another aspect of the convection which is apparent in the 7500 K model is that the modified mixing-length theory yields a much smoother variation of $\log \left( \frac{F_{\text{conv}}}{F_{\text{total}}} \right)$ with $\log \tau_R$ than does the regular theory. This is particularly noticeable at $\log \tau_R \approx 1.8$ in the Kurucz model where $F_{\text{conv}}/F_{\text{total}}$ falls by a factor of 1000. Our model shows no such precipitous declines.

While the decreased convection is the dominant feature of Figure 1, the models with $T_{\text{eff}} < 7000$ K show regions where the convection is enhanced in our models. These all occur at relatively shallow depths ($\log \tau_R < 0.3$ in the deepest case), and they are a direct result of the variable mixing length proposed by Deupree and Varner. Their algorithm makes the ratio $l/H$ a function of the horizontally averaged temperature. When the temperature is hotter than 10,000 K, the ratio $l/H$ is allowed to go smoothly to some arbitrary value which for the models shown here was 1.0. Because of the temperature structure of our atmospheres, only a small portion of the outer convection zone in any model was affected by the variation in the value of the mixing length, but this is most apparent in these cooler models. However, we caution, as noted earlier, that the proposed variation of $l/H$ is subject to uncertainty at the surface.

The other major difference evident in Figure 1 is that at a particular optical depth the horizontally averaged temperature in our models can be up to 6000 K greater than given by the Kurucz (1979b) models. This is a direct consequence of the alteration of the energy balance which results from the decreased convection in our models. The actual amount of the temperature increase depends on the size and importance of the convection zone. In the hottest models the convection zone is not large or strong enough to enable a significant temperature difference to develop. In the coolest models we have not integrated through the convection zone in either model, and so we have not seen the largest temperature differences which these models possess. It is in the

Fig. 1.—Variation of $\log \frac{F_{\text{conv}}}{F_{\text{total}}}$ and of temperature as a function of the log of the Rosseland optical depth, $\tau_R$, for line-blanketed model atmospheres with $\log g = 4.0$ and solar abundances. Each panel is labeled with the value of the effective temperature. Filled circles are $F_{\text{conv}}/F_{\text{total}}$, and filled triangles are the temperature. The models of this paper are the solid curves, and those of Kurucz (1979b) are the dashed curves.
models of intermediate temperature that we see most clearly the differences in the temperature structure as we reach the depths at which the convective zone in our models begins to lose efficiency due to ionization. This effect, combined with the narrower extent of the convective zone, leads our models to predict significantly higher temperatures at shallower optical depths than found in the conventional models. Note that these large differences occur at the base of the hydrogen convection zone, which is at temperatures greater than 10,000 K in all cases. Therefore, these differences are due primarily to the horizontal averaging of the opacity, but the variable mixing-length algorithm might have some secondary influence through its introduction of nonlocal information.

Figure 1 shows the atmospheric structure as a function of \( \log \tau_g \). Because the temperature structure, which is the controlling parameter for these LTE atmospheres, is the same for \( \log \tau_g \leq 0.5 \) for the two sets of models, one might expect the emergent fluxes from the models to be nearly identical. Figure 2 shows that this is not the case at all. In this figure we have plotted the magnitude differences between our models and those of Kurucz (1979b) for all wavelengths greater than 1000 Å. For all effective temperatures we see that our models are brighter by up to 0.2–0.3 mag somewhere in the spectral interval 1500–2200 Å. At other wavelengths our models tend to be slightly fainter by up to 0.02 mag, except in the 4000–5000 Å region where the behavior is quite complex between the Balmer lines.

**Fig. 2.**—Differences between the emergent fluxes of our models and those of Kurucz (1979b), expressed in mag (this paper – Kurucz), as a function of \( \lambda^{-1} \). The models have \( \log g = 4.0 \), solar abundances, and the \( T_{\text{eff}} \) given in each panel.
We can understand the increased brightness of our models in the interval 1500-2200 Å with the help of Figure 3. In this figure we have plotted $T(\tau_A)$ and $B(\tau_A)$, the monochromatic Planck function, at two different wavelengths for the model of $T_{\text{eff}} = 7500$ K and $\log g = 4.0$. The top panel represents the behavior at 1649 Å, chosen as a representative wavelength at which our model is brighter, and the bottom panel shows the results at 2481 Å where our model is slightly fainter. We see that while our model is hotter at $\tau_A \geq 5$ at both wavelengths, the temperature increase is much steeper with $\tau_A$ at 1649 Å than at 2481 Å. The atmosphere at 1649 Å is quite transparent after reaching the depth of $\tau_A \approx 4$, so that the higher temperatures in our convection zone are reached with only a small further increase in $\tau_A$. This leads to a rapid divergence in the $T(\tau_A)$ structure at this wavelength. In turn, this produces a significant difference in the source function of the two models, as represented by $B$. In part, this represents the great sensitivity of $B$ to temperature at these wavelengths on the short wavelength side of the Planck maximum. Although the structural differences are occurring at $\tau_A > 5$, the resulting changes to the source function are large enough to produce the observed brightening.

This behavior can be contrasted to what we see at 2481 Å. Here the temperature gradient is much shallower for both models, so that the rate of divergence of the temperatures is much less. Also, because this wavelength is much closer to the Planck maximum, the temperature difference translates into only a modest change in the source function, which is not sufficient to affect the surface flux. The fact that our model is slightly fainter at this wavelength reflects a slight temperature difference ($<200$ K) in the opposite sense at shallower optical depths.

As is apparent from Figure 2, a test of the predictions of our models is possible using ultraviolet photometry of stars in the temperature range 6000–8000 K. A preliminary attempt to make such a test is in progress. Here we note that magnitude differences of opposite sign are also present in the visual spectral region. These will have an influence on conventional ground-based photometry. Relyea and Kurucz (1978) have already compared the models of Kurucz (1979a) to existing photometry, and they concluded that for stars with $T_{\text{eff}} < 8500$ K, the theoretical colors are not in good agreement with the data. Using the Strömgren system, the sense of the discrepancy is that at a given color in the interval $0.1 < b - y < 0.3$, the models predict $m_1$ too small by ~0.04. Using $u - b$ instead of $m_1$, the discrepancy is that the models are too blue by ~0.2 mag (see Fig. 7 and 8 in Relyea and Kurucz).

In Figure 4 we compare the synthetic colors of our models with those computed by Kurucz (1979a, b). We see that our models predict $m_1 \approx 0.02$ mag greater than Kurucz (1979a). Alternatively, our $u - b$ is larger by up to 0.05 mag. Both of these changes are, in the correct sense, to bring the computed colors closer to the observations, and in both comparisons our models go beyond the results achieved by Kurucz (1979b) to resolve the
problem. In both cases, however, the magnitude of the changes is not sufficient to remove the discrepancy entirely. The resolution of this problem may involve spatial temperature inhomogeneities such as predicted by Nelson (1980).

Before leaving the change with effective temperature, we note that at a particular $T_{\text{eff}}$ our models and those of Kurucz (1979a, b) predict different values of $b - y$. Attempts to use $b - y$ alone to fix the $T_{\text{eff}}$ of a star will yield model-dependent results. A given value of $b - y$ will select a model from the grid of Kurucz (1979a) which is $\sim 300$ K hotter than a model constructed using the modified mixing-length theory.

IV. CHANGES WITH SURFACE GRAVITY

The second major parameter of our atmospheres is surface gravity, and we want to explore what differences result from Deupree’s form of mixing-length theory when gravity is varied. In Figure 5 we compare our models with those of Kurucz (1979b) for the case of changing surface gravity while the effective temperature is held fixed at 7500 K. As before, the main difference is that our models have convection zones which are narrower, less efficient, and hotter. This is very apparent in the model with log $g = 4.5$, where our model has integrated through the convective zone to yield temperatures much greater than the comparison model which is still carrying $\sim 90\%$ of its flux by convection at log $\tau_R = 2.0$.

The variation of the convective zone with gravity is qualitatively similar for the two sets of models. Reducing the gravity decreases the density, which leads to less efficient convection. Both sets of models show quite dramatic changes in the strength and the profile of the convection zone as log $g$ is varied, but the associated changes in the temperature structure are quite different. Even in our highest gravity model, convection carries only $\sim 50\%$ of the flux at maximum, and then for only a very narrow range of depths. By the time log $g$ has been reduced to 3.5, the peak convective contribution to the flux transport has dropped to less than 10%. With radiation carrying such a large fraction of the flux in this and all lower gravity models, the effect of the convection zone on the temperature structure is very slight. Except for the highest gravity model, we find very little difference in the temperature structure with gravity at 7500 K. On the other hand, the models of Kurucz (1979b) do show substantial changes in the temperature structure with gravity because, as explained in § III, convection in those models carries a larger fraction of the total flux. As the gravity is lowered, the two temperature structures gradually come into coincidence, but this is primarily due to the Kurucz models conforming to our relatively constant temperature structure.

A comparison of the emergent fluxes from our models with those of Kurucz (1979b) is shown in Figure 6 for a range of gravities at $T_{\text{eff}} = 7500$ K. For log $g \leq 2.5$ the differences are not shown because they become insignificant, as we would expect from Figure 5 where we see that the temperature structures become nearly identical. At higher gravities we see basically the same behavior.

Fig. 5.—Same as Fig. 1 for model atmospheres with $T_{\text{eff}} = 7500$ K, solar abundances, and the log $g$ values shown in each panel
for all the models. The extent of the spectral region over which our models are brighter increases with increasing gravity, reflecting the growing differences in the temperature structure with gravity.

There are obviously differences in the visual fluxes of the two sets of models which will alter the synthetic colors on standard photometric systems. In Figure 7 we show \( m_1 \) and \( u - b \) as a function of \( b - y \) on the Strömgren systems. For comparison we also have included the predictions of the models by Kurucz (1979a, b). We see that at a constant \( b - y \) our models predict \( m_1 \) up to 0.03 mag larger than Kurucz (1979a), while \( u - b \) is up to 0.25 mag smaller. These changes are large enough to have significant effects. For example, Strömgren (1966) has pointed out that a change of 0.025 in \( m_1 \) is sufficient to classify a star's metallicity. Therefore, our results show that the models used to interpret photometry can have important consequences. The validity of these predictions should be checked.

V. CHANGES WITH COMPOSITION

The final parameter of our stellar atmospheres is composition. In Figure 8 we show the differences between our models and those of Kurucz (1979b) which result from changing the abundances of the elements heavier than helium while fixing \( T_{\text{eff}} = 7500 \) K and \( \log g = 4.0 \). Each panel is labeled by the composition expressed as the logarithm of the abundance relative to the solar composition.

For both sets of models the primary variation apparent in Figure 8 is the well-known (see Böhm-Vitense 1971) increase in convection with decreasing metallicity. The differences between the two sets of models again just follow from the reduced amount of convection and the smaller change in the convection with metallicity for our models. The strength and profile of our convective zones change quite slowly with composition. Over the range of metallicity shown in Figure 8 our convective zones carry from 14% to 35% of the flux at maximum and have a maximum width at the level of \( \log (F_{\text{com}}/F_{\text{tot}}) = -2.0 \) of \( 0.3 < \log \tau_R < 1.4 \). In comparison, the Kurucz models have peak efficiencies >80% and in the lowest metallicity case the integration has not emerged from the lower boundary of the convective zone even at \( \log \tau_R = 2.0 \).

The corresponding temperature structures also show substantial differences. At the highest metallicity the differences are smallest but still amount to \( \sim 3500 \) K.
at maximum. For the lowest metallicity the maximum temperature is >7000 K. Most of this variation is due to changes in the Kurucz models. Comparing our models with abundances +1.0 and −2.0, we see that the temperature structures differ by only ~600 K at maximum.

As with the two previous comparisons, the differences in the $T(\tau_t)$ structures between the two sets of models shown in Figure 8 lead to differences in the emergent fluxes which are shown in Figure 9. The largest differences between the two sets of models are again in the ultraviolet at $\lambda < 2000 \, \AA$. This time, however, we have an additional effect because the heavy elements are the major sources of opacity in this spectral region, contributing both bound-free and bound-bound absorptions to the total opacity. Because of this, the decrease in metallicity leads to an increased transparency which, in turn, enables the hotter convection zones in our models to make a larger contribution to the surface flux. Because of this, the maximum difference between the two sets of models grows from 0.2 mag for the +1.0 comparison to more than 0.6 mag for the −2.0 models. This is a combined effect of greater transparency and greater temperature difference for this low metallicity model.

A second effect of the variation of the metallicity is that the ultraviolet wavelength to which the atmosphere remains fairly transparent also varies. In the solar abundance models compared in the previous sections the
Fig. 9.—Same as Fig. 2 for model atmospheres with $T_{\text{eff}} = 7500$ K, log $g = 4.0$, and the abundances shown in each panel, expressed as the log of the metal abundance relative to the Sun.
ultraviolet flux drops discontinuously by factors of 10–100 at 1677 and 1520 Å, both due to bound-free edges of neutral silicon. Because of these edges, the emergent fluxes of the two sets of models are quite similar at shorter wavelengths which cannot penetrate to depths where the temperatures are significantly different. Also, any magnitude differences which might exist at shorter wavelengths would not provide a test of the models because the fluxes are unobservably faint. However, as the metallicity decreases, the importance of these bound-free edges diminishes. We see in Figure 9 that the spectral region over which the magnitude difference is very significant extends down to 1285 Å for the −1.0 and −2.0 models. We also find that the total amount of flux in this spectral band decreases smoothly with decreasing wavelength in these low metallicity models, rather than dropping discontinuously across edges. Because of this, the large magnitude differences present in these metal-poor models might be observable.

In the visual spectral region we again find small but significant differences in the emergent fluxes of the two sets of models. Figure 10 shows these in terms of the Strömgren indices used in previous sections. We see that the major difference is an offset in b − y between the synthetic colors of our models and those of Kurucz (1979a, b). This difference is actually present in Figure 4, so what we see is that the variation of the colors with metallicity is very similar in the various sets of models.

VI. OTHER COMPARISONS

a) Other Theories of Convection

Several investigators have attempted to develop non-local mixing-length theories of convection. In a careful study, Nordlund (1974) has used the same test model atmosphere to compare the predictions of standard mixing-length theory with the theories of Parsons (1969), Ulrich (1970a, b), and of Spiegel (1963) as implemented by Travis and Matsushima (1973a, b). To make use of Nordlund's results we have computed several unblanketed model atmospheres for \( T_{\text{eff}} = 5900 \) K, \( \log g = 4.44 \), and using Nordlund's standard values of the mixing-length parameters: \( \alpha = 1.5 \), \( v = 8.0 \), and \( y = 0.076 \) (see Henyey, Vardya, and Bodenheimer 1965 for the definitions and descriptions of these parameters). We have used \( \beta = 0.0 \) rather than the value of 0.5 employed by Nordlund, but both Nordlund and Henyey et al. show that the convective energy transport is insensitive to the value of \( \beta \) adopted. Using the traditional mixing-length theory, our model gives the fraction \( F_{\text{conv}}/F_{\text{total}} \) as a function of optical depth which agrees with the results shown in Figure 1 of Nordlund (1974) to within a few percent. The resulting \( T(t_{\odot}) \) relations are the same to the limits set by our ability to read Nordlund's Figure 2. This good agreement between two completely independent calculations gives us confidence that we can meaningfully compare our results to the tests conducted by Nordlund.

The comparison of the \( F_{\text{conv}}/F_{\text{total}} \) and the temperature structure of our model with the cases considered by Nordlund (1974) are shown in Figure 11. It is immediately apparent that, compared to the theories investigated by Nordlund, our model predicts significantly less convective flux for the atmosphere shallower than \( \log t_{\odot} = 1.2 \). As Nordlund noted, the results of Travis and Matsushima (1973a, b) and of Ulrich (1970a, b) predict significant convection shallower than \( \log t_{\odot} = 0 \), which leads to a temperature structure in disagreement with empirical solar atmospheres unless the quantity analogous to the mixing length becomes quite small in these layers. In contrast, our model predicts acceptably small values of the convective flux in these transparent regions of the atmosphere. However, our models show reduced convection until the depth of \( \log t_{\odot} \approx 1.2 \) is reached, resulting in the increased temperatures shown in the bottom panel of Figure 11. At \( \log t_{\odot} = 1.0 \) our model is 750 K hotter than the model using Parson's theory. The sensitivity to the convective constant "y" is shown by curves labeled with different values of this parameter. The value \( y = 0.076 \) is the standard of Nordlund, while \( y = 0.5 \) is the standard value used by the program ATLAS. It is clear that increasing the assumed value of \( y \) moves the predictions of our model toward the results produced using Parson's theory. Note also that these differences in the fraction of the flux carried by convection, which lead to the temperature differences, occur at depths where the average horizontal temperatures are less than 10,000 K. This means that our models are reflecting both the horizontal averaging of the opacity.

© American Astronomical Society • Provided by the NASA Astrophysics Data System
and the variable mixing length of Deupree and Varner. Errors in the algorithm for the variation of $l/H$ could also be affecting our models.

In addition to the nonlocal mixing-length results discussed above, Nordlund (1976) and Nelson (1980) have developed non–mixing-length theories of convection. Both of these theories were motivated by the desire to reproduce the solar granulation, but both authors have also applied their methods to effective temperatures appropriate for main-sequence F stars. We choose to compare our results to those of Nordlund because he has presented them in somewhat more detail and because he has determined the temperature structure and convective transport self-consistently.

Nordlund represented convection by two oppositely directed streams of material. Unlike the mixing-length theories, these two streams are able to penetrate the convectively stable, optically transparent layers of the photosphere. In Figure 12 we show the fraction of the convective flux and the temperature structure for one of Nordlund’s models, together with the results for our model of the same effective temperature and gravity. We have taken the two-stream temperature structure from Figure 3 of Nordlund (1976) by averaging the temperatures in the two components, taken as occupying equal horizontal areas, at a given value of the standard optical depth in the model. It is obvious that for $\log \tau_{5000} < 0.7$ the temperature structures of the two models are in very good agreement. The differences which are present in these layers could be reduced by an adjustment of the “$y$” parameter, as shown in Figure 11. As we go to greater optical depths, however, the two models yield significantly different temperature structures, with our model being hotter. The explanation of the difference follows directly from the top part of Figure 12, where we see that Nordlund’s theory predicts a much greater convection for the atmosphere at $\log \tau_{5000} < 1.2$. This difference in the amount of convection, which was precisely the aim of Nordlund’s theory, requires our model to be hotter to transport the same total flux by radiation. Again, our model reflects the variation in $l/H$ as well as the horizontal averaging of the opacity at the relatively cool temperatures involved in this comparison.

\section{b) Empirical Solar Models}

A final comparison is possible, not with the results of a computed model atmosphere, but with the empirically determined solar temperature structure. Several em-
Empirical solar atmospheres have been published in the last ten years, including Vernazza, Avrett, and Loeser (1976), Holweger and Müller (1974), and the HSRA (Gingerich et al. 1971). In Figure 13 we compare the results of our blanketed solar model with the temperature structures of these empirical models.

Figure 13 shows that, once again, our model predicts higher temperatures than the comparison models, and the differences are significant for $\log \tau_{5000} \gtrsim 0.5$. As before, this results from the reduced convection in our model. The lack of agreement at these depths might seem to be damaging to the acceptability of Deupree's version of the mixing-length theory until it is realized how weak a grasp the observations have on the temperature structure this deep in the solar atmosphere. The deep temperature structure is determined by solar observations in the near infrared, with the greatest penetration being achieved at a wavelength of $\sim 1.6 \mu$m. Our model predicts a brightness temperature of 6673 K for the intensity from the center of the solar disk at 1.63 $\mu$m. This is essentially identical to the prediction of the model of Vernazza, Avrett, and Loeser (1976; see their Fig. 16), even though our model is 1000 K hotter than their model at $\log \tau_{5000} = 0.9$. The explanation for this is that the radiation at 1.63 $\mu$m is dominated by material at $\log \tau_{5000} \lesssim 0.3$ where the temperatures of the two models are in very close agreement. In Figure 14 we have demonstrated this in another way by comparing the temperature structure of our solar model with those published by Kurucz (1974, 1979b). The three models give the following central disk brightness temperatures at 1.63 $\mu$m: 6673 K (this paper), 6638 K (Kurucz 1974 or 1979a), and 6686 K (Kurucz 1979b). Note that the Kurucz (1979b) model gives the highest brightness temperature even though our model is significantly hotter for $\log \tau_R > 0.6$. Our model is cooler than one or both of the Kurucz models for $0.0 < \log \tau_R < 0.6$, and this is what dominates the brightness temperature, even at the most transparent spectral region. Therefore, we feel that the empirical solar models lack sufficient power to rule decisively on Deupree's form of mixing-length theory.

VII. CONCLUSIONS

We have investigated the changes in the structure of stellar atmospheres which result from using the modifications to the standard local mixing-length theory proposed by Deupree (1979) and Deupree and Varner (1980). Compared to models of Kurucz (1979b), which are identical to ours except for the treatment of convection, we found the following:

1. Convection in our models transports a smaller fraction of the total flux.
2. Our convection zones are generally narrower, and the variation of $F_{\text{conv}}/F_{\text{total}}$ with depth is smoother.
3. The temperature is higher in our convection zones.
4. Our models predict observable flux differences for
the spectral range 1500 < \lambda < 2000 \text{ Å}, with our models being brighter.

5. Our models also predict small, systematic flux differences in the visual spectral region. These changes affect the synthesized Strömgren indices in the correct sense to bring them closer to the observed values, but the amount of the change is not sufficient to entirely remove the discrepancy found by Relyea and Kurucz (1978).

The conclusions listed above result almost entirely from the horizontal averaging of the opacity discussed by Deupree (1979) rather than the variable mixing length proposed by Deupree and Varner (1980). The variable mixing length does, however, have a significant influence on the remaining conclusions.

6. Compared to nonlocal mixing-length theories, our models predict less convective flux transport in the upper atmosphere, and this leads to our models being significantly hotter deeper than log \tau_r \approx 0.6.

7. Compared to Nordlund's two-stream convective theory, our model produces less convection for log \tau_{5000} < 1.2. The temperature structures are in good agreement shallower than log \tau_{5000} \approx 0.7, but our model is hotter at greater depths.

8. Compared to empirical solar models, we find satisfactory agreement between the temperature structures for log \tau_{5000} \leq 0.4, but our model is hotter at greater depths. We argue that the empirical determination of the solar temperature structure at log \tau_{5000} > 0.4 lacks the power to provide a strong test of this form of convection.

As a final point we wish to reiterate several of the qualifications stated throughout this paper. The local mixing-length theory, including the variant studied here, has a number of serious deficiencies as a physical theory. It is not applicable to problems involving convective turbulence and overshoot, such as those mentioned in § I. Also, because it treats radiative transfer using the diffusion approximation, it cannot address the radiative energy losses to space at small optical depths which control convection near the stellar surface. Nevertheless, this form of the local mixing-length theory does contain additional information, compared to the standard theory, which results in a step toward a more physically realistic treatment of convection which is still tractable in application.

We gratefully acknowledge that this work was supported in part by grants from the Natural Sciences and Engineering Research Council of Canada and by Erindale College. We thank Anna Pezacki for assistance with our computer.

APPENDIX

As discussed in the paper, we have modified the treatment of convection to include three effects. The first change concerns the optical thickness of the convecting element, \tau_e. In mixing-length theory we relate the various temperature gradients by defining an efficiency parameter

\gamma \equiv \frac{\text{excess energy content at time of dissolution}}{\text{energy lost by radiation during the element's life}}.

The denominator depends on the optical thickness of the element. Mihalas (1978) has shown that for an optically thin element we get

\gamma_{\text{thin}} = \frac{\rho C_p \bar{v}}{8 \sigma T^3 \tau_e}.

(A1)

For the optically thick case the diffusion approximation to radiative transfer leads to

\gamma_{\text{thick}} = \frac{1}{2} \left( \tau_e \rho C_p \bar{v} \right) \frac{1}{(8 \sigma T^3)}.

(A2)

To interpolate between these limiting cases we have followed Mihalas in adopting

\gamma = \frac{\rho C_p \bar{v}}{8 \sigma T^3} \left( 1 + \frac{1}{2} \frac{\tau_e^2}{\tau_e} \right).

(A3)

The second change was to allow horizontal averaging of the opacity as discussed by Deupree (1979). We start by assuming no horizontal temperature fluctuations, \Delta T_h = 0. After solving for the temperature gradient of the convecting element, \nabla_e, we determine the horizontal temperature fluctuation using

\Delta T_h = \frac{\rho g T (\nabla - \nabla_e)}{P},

(A4)

where \nabla is the temperature gradient of the surrounding medium (see Cox and Giuli 1968 for the motivation
of this equation). In general, $\Delta T_b \neq \Delta T_a$. Setting $\Delta T_b = \Delta T_a$, or using some kind of average to improve stability, we evaluate the Rosseland mean opacity at $T \pm VT_a$, from which we construct an effective opacity

$$\frac{1}{\tilde{\kappa}} = 1 \left[ \frac{1}{\kappa(T + \Delta T_a)} + \frac{1}{\kappa(T - \Delta T_a)} \right].$$

(A5)

Using this, we repeat the determination of $\nu_a$ and $\Delta T_b$. This procedure is iterated until $\Delta T_a = \Delta T_b$ to within 1 K.

The third change was to use the variable $l/H$ of Deupree and Varner (1980). Using $T_a \equiv T/10^4$ K, they give

$$\log \left( \frac{l}{H} \right) = -2.85(T_a - 0.7) + 0.3, \quad T_a < 0.746;$$

(A6)

$$\log \left( \frac{l}{H} \right) = 6.36(T_a - 0.97)^2 - 0.15, \quad 0.746 \leq T_a \leq 1.$$  

(A7)

For $T_a > 1$, we use either the value from equation (A7) or the specified limiting value of $l/H$, whichever is less.

REFERENCES


Robert L. Kurucz: Center for Astrophysics, 60 Garden St., Cambridge, MA 02138

M. C. Lane: Department of Astronomy, University of Toronto, Toronto, Ontario M5S 1A7, Canada

John B. Lester: Erindale College, University of Toronto, Mississauga, Ontario L5L 1C6, Canada

© American Astronomical Society • Provided by the NASA Astrophysics Data System