A NUMERICAL SIMULATION OF COOLING CORONAL FLARE PLASMA
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ABSTRACT
We have simulated the cooling of coronal flare plasma \( T_e > 10^7 \) K using a numerical model of a vertical magnetic flux tube containing an idealized flare chromosphere, transition region, and corona. The model solves the set of one-dimensional, two-fluid hydrodynamic equations. The cooling of the flux tube is calculated for a specific case beginning with an initial atmosphere in hydrostatic equilibrium and a maximum temperature of about \( 18 \times 10^6 \) K. The behavior of temperature, density, and velocity is calculated as a function of height as the system cools. Early in the cooling, energy is transported by conduction into the transition region and chromosphere where it is radiated away. Later, the transition region-corona interface moves upward into the tube at velocities of about 20 km s\(^{-1}\), while the chromosphere cools and the coronal component cools by both conduction and radiation. Coronal downflow velocities of about 60 km s\(^{-1}\) are evident during this phase. The expected spectral line emission from the system in X-ray lines of Fe xxv, Fe xxiv, Fe xxi, O xvii, and O xiv is also calculated and compared to recent observational results. Some observational results can be explained as a consequence of simple cooling of flare flux tubes. The expected spectral line emission from certain transition region lines is also briefly considered. The dependence of our results on flare size is discussed, and our results are compared with similar previous work.

Subject headings: hydromagnetics — plasmas — Sun: corona — Sun: flares

I. INTRODUCTION

Skylab observations of solar flares showed that soft X-ray flare emission apparently arises in magnetic flux tubes or loops (e.g., Cheng and Widing 1975; Kahler, Kreiger, and Vaiana 1975). The footpoints of these loops are anchored in chromospheric regions of opposite magnetic polarity, and the most intense X-ray emission arises from the region between the opposite polarities coincident spatially with the magnetic neutral line. The temperature of the X-ray emitting plasma confined in these magnetic flux tubes is typically \( > 10^7 \) K at times of peak X-ray emission. More generally, the Skylab observations have revealed that looplike structures are a common phenomenon in active regions throughout the solar atmosphere. (e.g., Tousey et al. 1973; Vaiana, Kreiger, and Timothy 1973).

There has consequently been considerable interest in the physics of plasma confined to magnetic flux tubes (e.g., Rosner, Tucker, and Vaiana 1978; Jordan 1980). In the case of solar flare flux tubes, extensive work has been done by Antiochos and colleagues (e.g., Antiochos and Sturrock 1976, 1978, 1982; Antiochos and Krall 1979; Antiochos 1979, 1980). This work on the dynamics of flare plasma in flux tubes is becoming increasingly the subject of numerical simulations as well as analytical investigations. Also, rough comparisons of some of the results have been made to observational data from Skylab. Recent idealized numerical simulations of both the heating and cooling of flare plasma in loops have been carried out by Nagai (1980). The numerical simulations of Antiochos and Krall (1979) and Nagai (1980) use different numerical techniques, as well as different boundary conditions and initial conditions. Some of their results are only weakly dependent on the particular idealizations and simplifying assumptions, while others are sensitive to the details of the calculations.

Particular problems in these and related calculations concerning quiet Sun phenomena center about the difficulty of resolving the chromospheric-coronal transition region (generally underresolved by a factor of 10 to 100), of treating the coupled two-temperature thermal conduction equations with radiation, of dealing accurately with shocks, of providing a physically reasonable lower boundary condition for the calculation, of treating the coupled nonequilibrium ionization chemistry of the moderate Z ions which provide almost all of our spectral diagnostic information, and, in the case of flares, of providing a reasonable heating function. Aside from the work mentioned above, these and related computational difficulties arise in the types of calculations carried out by Hollweg, Jackson, and Galloway (1981), Smith and Auer (1980), and Craig and McClymont (1981). The calculations presented below include improvements in several computational areas and avoid the heating function problem.
Since the heating function is unknown and almost any experimental result can be matched by the choice of a suitable heating function, these preliminary calculations consider an activated solar flare flux tube with no further source of heating. Specifically, we begin with a high temperature solar plasma ($T_e \leq 18 \times 10^6$ K) in hydrostatic equilibrium and investigate the hydrodynamic consequences of the cooling of this plasma. Spectroscopic line intensities for observationally important lines are calculated as the plasma cools. Because many flares require a continuous heat input well into their development, not all of these spectroscopic results are expected to agree exactly with observation. Nevertheless, the results of the calculations can be compared significantly with many observations. Further, the consideration of the dynamics of a cooling loop represent a crucial calibration before developing more comprehensive models.

It is appropriate to undertake numerical calculations of a high temperature flaring flux tube not only for a comparison with and extension of previous work by relaxing some of the simplifying assumptions, but also for a comparison to newer experimental results, made possible by incorporating self-consistent calculations of certain ion species, spectral line intensities, and line ratios into the numerical simulation. Since Skylab, observational flare data of very high quality in the X-ray and UV wavelength regions have been obtained from instruments on the P78-1 spacecraft and on the Solar Maximum Mission (SMM) spacecraft. For the purposes of this paper the most significant new results are:

1. The maximum electron temperature achieved by the bulk of the soft X-ray emitting plasma is about $20 \times 10^6$ K (e.g., Doschek et al. 1980; Feldman et al. 1980).

2. Continuous energy input well into the decay phase is required to explain the temperature-time behavior for many flares (e.g., Doschek et al. 1980; Feldman et al. 1980).

3. The electron density of the plasma at $20 \times 10^6$ K is high, $>5 \times 10^{10}$ cm$^{-3}$ (e.g., Cheng and Widing 1975; Feldman, Doschek, and Kreplin 1980).

4. The range of emission measures of the thermal soft X-ray emitting plasma observed from Skylab and the later instrumentation is $10^{45}$-$10^{50}$ cm$^{-3}$ (e.g., Doschek et al. 1980; McKenzie and Landecker 1981).

5. The spatial extent of the soft X-ray emitting plasma, as inferred from Skylab, P78-1, and SMM observations, is typically $<1'$; regardless of X-ray classification (e.g., Cheng and Widing 1975; Landecker and McKenzie 1980; Gabriel et al. 1981).

The numerical simulation tool used in our work is the NRL Dynamic Flux Tube Model (Boris et al. 1980), developed at NRL under the NRL Solar Theory Program. A brief description of the model is given in § II, and a more extensive description is given by Oran, Mariska, and Boris (1982) and Mariska et al. (1982).

Section III presents results from our simulations with comparisons to observation, and § IV contains a discussion and comparison with previous calculations.

II. THE NUMERICAL MODEL

a) Model Equations

The numerical model solves the one-dimensional two-fluid conservation equations for mass, momentum, and energy in the form

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial z} = 0 \quad (1)$$

$$\rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} \right) + \frac{\partial P}{\partial z} = \rho g \quad (2)$$

$$\frac{\partial}{\partial t} \left( \frac{P}{\gamma - 1} \right) + \frac{\partial}{\partial z} \left[ \frac{1}{2} \rho v^2 + \frac{P}{\gamma - 1} \right] v - \kappa_e \frac{\partial T_e}{\partial z} - \kappa_i \frac{\partial T_i}{\partial z} = \rho \gamma g - L \quad (3)$$

$$\frac{\partial}{\partial t} \left( \frac{P_e}{\gamma - 1} \right) + \frac{\partial}{\partial z} \left( \frac{\rho P_e}{\gamma - 1} - \kappa_e \frac{\partial T_e}{\partial z} \right) = -L + \gamma_{eq}(T_e - T_i) - P_e \frac{\partial v}{\partial z}, \quad (4)$$

where

$$P = P_e + P_i \quad (5)$$

with

$$P_e = N_e k T_e \quad \text{and} \quad P_i = N_i k T_i \quad (6)$$

$$N_e = \frac{\rho Z}{\mu m_p (1 + Z)} \quad (7)$$

and

$$N_i = N_e / Z. \quad (8)$$

Also,

$$\kappa_e \equiv 1.1 \times 10^{-6} T_e^{5/2}, \quad (9)$$

$$\kappa_i = \kappa_e / 25, \quad (10)$$

$$\gamma_{eq} = 1.4 \times 10^{-17} N_e^2 T_e^{-3/2}, \quad (11)$$

and

$$L = N_e N_p \Phi(T). \quad (12)$$

In the above equations, $\rho$ and $v$ are the fluid mass density and velocity; $P$ is the total pressure; $\gamma$ is the ratio of specific heats ($=5/3$ for this problem); $g$ is the gravitational acceleration at the solar surface; $\gamma$ is the gravitational acceleration at the solar surface; subscripts $e$ and $i$ refer to electron and ion quantities, respectively; $\kappa$, $k$, $\mu$, and $m_p$ are the thermal conductivity coefficient, Boltzmann's constant, the mean mass per particle, and the proton mass; and $Z$ is the average charge of the ions in the system. The plasma is assumed to be fully ionized and consists of hydrogen and a helium number density of 63% of the hydrogen number density (Ross and Aller 1976). The $\Phi(T)$ function in the radiative loss term is from Raymond (1979) and is basically the same as that described by Rosner, Tucker, and Vaiana (1978).

The convective derivatives in equations (1)-(4) are...
solved by the Flux-Corrected Transport method (e.g., Boris and Book 1976 and references therein). The nonconvective terms in equations (3) and (4) are solved implicitly. Convection, thermal conduction, and radiative losses are coupled by time step splitting techniques as described in Oran and Boris (1981).

The flux tube model described above also contains an atomic physics model for calculating the time-dependent behavior of individual ion densities at each location. We subsequently obtain spectroscopic parameters such as spectral line intensities which then are integrated over the entire height of the atmosphere. Such integrals represent the intensities that would be observed by an instrument with a field of view that includes the entire flux tube. In the calculations presented below, ionization equilibrium is assumed because ionization and recombination times are much shorter than hydrodynamic time scales at the high densities of flare plasmas. Previous calculations using nonequilibrium ionization packages support this simplification (Mariska, et al. 1981). The specific oxygen and iron lines which have been chosen for study are based on those available from Skylab, P78-1, and SMM observations. The abundances of the elements we shall consider are set equal to their solar values relative to hydrogen (Ross and Aller 1976).

The spectral lines studied can be divided into two broad categories: "transition region" lines emitted by the ions O iii, O iv, O v, and O vi at the lower temperatures, and "coronal" lines emitted by ions such as O vii, O viii, Fe xxii, Fe xxiv, and Fe xxv at the high temperatures. The transition region lines fall in the XUV and UV between about 500 and 1700 Å. The lines of O vii and O viii fall in the X-ray region around 20 Å. The iron lines, which fall near 1.85 Å, result from ls-2p type transitions formed by inner shell electron impact excitation and dielectronic recombination at temperatures above 10^{7} K. The sources of the oxygen atomic data are given in Mariska et al. (1981). The data needed for the Fe ion calculations and line excitations were obtained from Jacobs et al. (1977), Gabriel (1972), Bhalla, Gabriel, and Presnyakov (1975), Bely-Dubau, Gabriel, and Volonté (1979a, b), and Doschek, Feldman, and Cowan (1981).

b) The Initial Conditions

In this paper we address the cooling of a flux tube after it is heated to a temperature >10^{7} K, and a quasistatic equilibrium configuration has been established where radiative losses are balanced by energy transport due to thermal conduction. To establish these initial conditions it is necessary to provide a nonzero conductive flux $F_c$ at the top of the computational region since we are ignoring the heating term. The equilibrium model is calculated by solving the equation

$$dF_c/az = N_eN_H\Phi(T_e),$$

where

$$F_c = 1.1 \times 10^{-6}T_e^{5/2}(dT_e/az),$$

and the equation of hydrostatic equilibrium. The equilibrium configuration chosen for investigating the cooling of the plasma has a temperature at the top of the tube $T_e = 17.6 \times 10^6$ K and a density $N_e = 5.6 \times 10^{11}$ cm$^{-3}$. The temperature at the top of the chromosphere was taken to be 10$^4$ K. The length L of the transition region and corona in the tube is $2.5 \times 10^3$ km. These values were chosen because they are the same as those adopted by Antiochos and Krall (1979) for their numerical simulation, and therefore a comparison of our results with their results is facilitated. Rather arbitrarily, we have chosen $3.0 \times 10^7$ cm as the radius of the flux tube. The adopted temperature and density are quite typical for flares regardless of X-ray classification.

Below the transition region, there is an idealized optically thin chromosphere, maintained by conduction from above and below. The purpose of this chromosphere is to couple the higher temperature plasma to the lower atmosphere and to provide a heat sink as well as a source of mass if mass flow into the flux tube occurs. This chromosphere is about 10$^3$ km thick, and therefore the total length of the flux tube considered is 3600 km. Below we shall see that the most interesting results of the calculation are not critically dependent on the structure of the chromospheric region. The main effect of the chromosphere is to provide a source of mass to the flux tube and to act as a piston during the cooling, which results in a slow upward movement of the chromosphere into the flux tube.

The 3.6 $\times$ $10^7$ km length of our computational region is divided into 100 cells, of variable size, $\Delta z$. In the transition region of the flux tube, i.e., the region where the temperature gradient is very large between about 2 $\times$ $10^6$ K and 10$^9$ K, $\Delta z = 5$ km. This is the smallest value of $\Delta z$ used anywhere in the tube for a complete calculation and is still too large to satisfactorily resolve the transition region for intensity calculations of spectral lines arising there. However, the hydrodynamic results obtained with this cell size are similar to those obtained with a cell size twice as small, and therefore the resolution appears to be adequate for deducing the hydrodynamic motions that occur during cooling.

Equations (13) and (14) were solved by adopting different values of $F_c$ at the base of the tube and then integrating upward. This procedure was followed until the calculated temperature at the top of the tube reached about 17.6 $\times$ $10^6$ K. The resulting distribution of $T_e$ and $N_e$ with height $z$ provided the starting point for the numerical simulation. This distribution was stabilized numerically by keeping the temperature at the top of the tube fixed and allowing the program to run for many time steps (about 15 s). Only insignificant velocities were generated in the system during this time, and the resulting model atmosphere is very similar to that obtained by solving equations (13) and (14). The temperature and density distribution of this initial atmosphere are shown in Figures 1 and 2. Our calculations of flare cooling are initiated by establishing the equilibrium atmosphere shown in these figures and then setting $F_c = 0$ at the top of the system.
III. COMPUTATIONAL AND OBSERVATIONAL RESULTS

a) Hydrodynamic Results

Figure 1 shows the electron temperature as a function of height for the initial model and for a number of times after the onset of cooling. Corresponding displays for mass density, pressure, and velocity are shown in Figures 2, 3, and 4. Initially, equilibrium in the system is maintained by balancing the radiative losses of the system with energy input at the top of the loop. Energy is transported within the system by conduction, and almost all of the radiative losses occur in the chromosphere-corona transition region. When the energy input is suddenly turned off, the coronal part of the system continues to lose energy by conduction, which is then radiated away in the chromosphere, but there is no longer a compensating energy gain from the top of the system. This results in a decrease in coronal temperature and pressure and in the eventual collapse of the coronal plasma onto the chromosphere. The drop in coronal pressure also produces an upper expansion of the chromosphere, due to the resulting pressure gradient created in the system. The upward chromospheric velocities are much smaller than the downward directed coronal velocities, because of the much greater mass density of the chromosphere. Finally, the upward
expansion of the chromosphere results in a drop in the chromospheric temperature in regions near the chromosphere-transition interface, due to the work done by the chromosphere in expanding upward into the tube. Below we consider this discussion more quantitatively.

During the initial cooling phase, i.e., about the first 20 s of cooling, which we shall call the conduction phase, the maximum temperature decreases from $17.6 \times 10^6$ K to $10^7$ K. In this period, the region at temperatures greater than $10^6$ K cools by thermal conduction of energy down the flux tube. This energy is then lost to the system as it is radiated away in the transition region and upper chromosphere. Figure 2 shows that there is little change in the mass density at each location in the system during the conduction phase. The X-ray emission from the most highly ionized ions (e.g., Fe xxv) formed in the plasma occurs during this phase. Such ions produce negligible X-ray emission at temperatures less than $10^7$ K. For later times, radiative cooling becomes increasingly more important as conduction weakens.

The velocities produced in the system are small during the conduction phase. At the earliest times (< 70 s) there are primarily downflows on the order of 15 km s$^{-1}$. Later (after 70 s; Fig. 4), when the temperature in the low density region begins to fall rapidly and the condensation and collapse rate of this low density plasma increase, the velocities increase to values near 60 km s$^{-1}$. These velocity profiles are shown in Figure 4.

The picture that emerges after 70 s is as follows. The system can be divided into two regions: a dense plasma at chromospheric temperatures that slowly rises upward at small velocities on the order of 20 km s$^{-1}$ and less, and a low-density, much hotter plasma above the transition region that is accelerating onto the advancing high density region, with relatively high velocities. The upward moving transition region can be regarded as a moving wall. The downflow is greatest at the point immediately above the wall, because the gas at this point is radiating so rapidly that it has little pressure to support it and the overlying plasma. At positions farther up in the tube, the weight of overlying layers diminishes, and the acceleration is consequently less. The maximum downward speed is about 60 km s$^{-1}$ at 97 s.

An interesting result is the local temperature maximum at about 2140 km seen in the temperature distribution after 109 s (see Fig. 1). At 109 s, the total energy of the plasma immediately adjacent and on the high density side of the “wall” is 7 times the total energy of the corresponding low density plasma nearly adjacent to the wall, and in both cases this energy is about equally divided between internal and kinetic energy. However,
for the location immediately adjacent to the wall on the low-density side (2140 km at 109 s), the internal energy is about 6 times greater than the kinetic energy, and the total energy lies between the total energies of the cells on either side of it. The large temperature at this location (2 × 10^5 K) is due to the impact of the downfalloing plasma on the wall and the subsequent conversion of the kinetic energy of the cold dense plasma into internal energy.

The detailed behavior of the chromosphere is also interesting. As mentioned, the upward expansion of the chromosphere results in a temperature drop over part of the region due to the work done in the expansion on the downfalloing coronal gas. Although the expansion velocity eventually reaches ≈ 20 km s^{-1} adjacent to the transition region, the expansion velocities over most of the chromosphere are considerably less, on the order of 5 km s^{-1}. The temperature drops from about 10^4 K at the base of the chromosphere (where the temperature is maintained at 10^4 K as a boundary condition) to values as low as 4 × 10^3 K late in the cooling, when the coronal plasma downflow velocity is around 60 km s^{-1}. The temperature drop from 10^4 K to about 4 × 10^3 K is not precipitous, but occurs gradually from the base of the chromosphere to the transition region. A more quantitative discussion of the temperature behavior is not appropriate since we have in fact chosen a simplified physics necessary for a realistic evaluation of the consequences of the cooling of a flare flux tube.

b) Spectroscopic Results and Comparison to Observation

The hydrodynamics discussed in the previous section is investigated observationally using spectroscopic techniques. We may anticipate the spectroscopic consequences of the cooling of the flux tube by considering Figure 1. The initial model reaches a peak temperature of about 17 × 10^6 K, and much of the tube is at temperatures above 10 × 10^6 K. Therefore, these portions of the tube give rise to X-ray emission lines of highly ionized iron. Figure 1 shows that the tube cools quickly to temperatures < 10 × 10^6 K, and consequently the iron line emission would decrease rapidly. However, X-ray emission lines of cooler ions such as O iv and O v in would remain strong during and beyond this cooling phase, as they are formed at temperatures between 2 × 10^6 K and 10^7 K. The iron line fluxes monotonically decrease during the cooling, while we expect the X-ray fluxes of lines of cooler ions to remain constant or perhaps even increase for a period of time during cooling. This is qualitatively what is observed in the simulation. For example, eventually the peak temperature of the system becomes about 3 × 10^6 K. At this time the entire tube (excluding the chromosphere) would appear bright in X-ray lines of O vi.

The above discussion can be made more quantitative. The specific spectral lines we consider are the Fe xxiv resonance line (1s^2 2p^2 P_{3/2} - 1s2p 2P_{1/2}) at 1.8560 Å (line w, see Gabriel 1972); the Fe xxv line at 1.8660 Å due to the transition, 1s^2 2p^2 P_{3/2} - 1s2p^2 D_{5/2} (line j); the Fe xxv line at 1.8851 Å due to the transition, 1s^2 2p^2 P_{3/2} - 1s2p^2 D_{5/2} (line i); the resonance (1s^2 1S_0 - 1s2p 1P_{1/2}), inter-combination (1s^2 1S_0 - 1s2p 3P_{1/2}), and forbidden (1s^2 1S_0 - 1s2s 3S_1) lines of O iv at 21.60, 21.80, and 22.10 Å, respectively; and the O v ii Lyz lines (1s^2 2p^2 P_{3/2} - 1s2p 2P_{1/2}) at 18.97 Å. We also consider the emission due to 2p → 2s type transitions in lines of ions such as O iv, formed at typical lower transition region temperatures (~ 10^6 K). In all the calculations, ionization equilibrium is assumed. For each line, we calculate the total flux emitted by all 100 cells and then double the quantity since the 100 cells actually represent only one-half of the flux tube. Thus, the calculated flux in a particular line is the flux that would be measured by a hypothetical spectrometer that viewed the entire flux tube. For each line, this flux is given by,

\[ F(\text{photons cm}^{-2} \text{s}^{-1} \text{area}^{-1}) = \frac{1}{2\pi R^2} \int \text{d}z, \]  

where the area unit on the left-hand side of equation (15) refers to the cross section area of the flux tube, and \( R \) is the Earth-Sun distance. As mentioned, the radius of the tube is arbitrarily chosen to be 3.0 × 10^7 cm. The quantity \( \epsilon \) in equation (15) is the emission rate (photons cm^{-3} s^{-1}) and the integral extends over the entire flux tube. All spectral lines are assumed to be effectively optically thin.

The calculation of \( \epsilon \) for the iron lines is described in Doschek, Feldman, and Cowan (1981). The intensity ratio of the Fe xxiv line to the Fe xxv line is an electron temperature diagnostic. The Fe xxiv line is formed by dielectronic recombination of Fe xxv, and the Fe xxv line is formed by electron impact excitation from 1s^2 1S_0. These excitation mechanisms have different dependences on temperature, which results in a temperature-dependent line ratio. The theory is described in detail in Gabriel (1972) or in Doschek, Feldman, and Cowan (1981).

The calculation of \( \epsilon \) for the O vii lines is described in Gabriel and Jordan (1972), and the O vii atomic data used are from Pradhan, Norcross, and Hummer (1981). The ratio of the O vi forbidden line to the inter-combination line is electron density sensitive because the 1s2s 3S_1 level is quite metastable. At sufficiently high densities the 1s2p 3P_{1/2} level can be depopulated by collisional excitation into the 1s2s 3P levels, as well as by radiative decay to 1s^2 1S_0. The theory for this density diagnostic was developed by Gabriel and Jordan (1969, 1972); the reader is referred to the 1972 paper for details. The emission rate for the O viii lines is calculated using the Gaunt factor approximation (van Regemorter 1962), assuming a Gaunt factor of 0.2 and an oscillator strength of 0.42. The atomic data needed for all the emission rate calculations are obtained from the sources cited at the end of § IIa.

Spectral lines emitted from highly ionized iron only emit strongly during the first few seconds of cooling. Results calculated from the numerical simulation are © American Astronomical Society • Provided by the NASA Astrophysics Data System
The intensity ratio of the Fe xxiv line to the Fe xxv line can be used to derive an electron temperature, using the results in Table 1. This is the temperature that would be derived by a hypothetical experimenter analyzing the flux results from the simulation as if they were experimental data, and also assuming an isothermal plasma. This temperature is shown for the initial model (0 s) and later at 3.8 s into the cooling phase. At both times it is slightly less than the maximum temperatures in the tube of 17.6 \times 10^6 K and 14.3 \times 10^6 K. This difference is due to small contributions to the total iron line intensities from plasma at lower temperatures down to 12 \times 10^6 K. The temperature inferred from the iron lines j/w ratio is fairly typical of small flares.

The j/w temperature of course agrees with observational results since it is, in effect, an initial condition of the system, and we have chosen a temperature at 0 s that agrees with observation. More interesting is the intensity of the dielectronic satellite line of Fe xxii at 1.8851 Å, called γ in Doschek, Feldman, and Cowan (1981). The ratio of this line to line w is 0.17 at 0 s and 0.53 at 3.8 s, which is in good agreement with observation. Thus, the relative line intensities of the computed iron line spectra resemble the observed relative line intensities. As the system cools, the ratio of line γ to line w and similar line ratios increase, as expected. However, the flux in these lines decreases rapidly due to the decreasing temperature and consequent recombination of the highly ionized ions.

Also shown in Table 1 are two emission measures, N_e^2ΔV, where ΔV represents the volume within the tube in which the Fe xxv line is formed. This quantity can be calculated by a hypothetical experimenter from F(Fe xxv), multiplied by the flux tube area, and the temperature given in Table 1. This is the emission measure defined by the subscript “o”, and it is calculated in the manner described by Doschek et al. (1980). The emission measure can also be calculated directly from the numerical simulation (subscript s), by defining ΔV as the volume from which 70% of the Fe xxv emission arises and assuming a flux tube radius of 3 \times 10^7 cm. The emission measure calculated from the simulation at both 0 s and 3.8 s is less than the emission measure that would be inferred by a hypothetical observer analyzing the flux results in Table 1 as if they were data, by a factor of 2. This results from the fact that the observer uses the temperatures shown in Table 1. While they are only slightly less than the maximum temperatures of the flux tube, the emission rate for the Fe xxv line is a rapidly increasing function of temperature at these temperatures, and choosing a slightly lower temperature results in a substantial increase of emission measure. Both the “observed” and calculated emission measures are much smaller than typical values deduced from most real observations. Thus, our model flare corresponds to a very small event. Finally, we also give in Table 1 the length ΔL in the flux tube over which 70% of the Fe xxv emission occurs. This quantity is computed from the simulation, and an observer would need to know both a density and flux tube area to compute it.

Although the relative intensities of different iron lines agree with available data initially (0 s), the flux ratio (in photons) of the Fe xxv resonance line to the O vii resonance line at 21.60 Å is 1.46 \times 10^{-3}. For many flares near the time of maximum flux in the Fe xxv line, the observed ratio is much larger, by factors of 2–10. This difficulty in reproducing with static models the ratios of observed line intensities over large temperature ranges has been noted by Antiochos (1980). Inclusion of a heating term may help resolve this discrepancy but is beyond the scope of the present paper.

Turning now to the plasma at quiet coronal temperatures, there are the three interesting lines of O viii near 21 Å: the resonance line, the intercombination line, and the forbidden line (see McKenzie et al. 1980). Since the ratio of the forbidden to intercombination line is sensitive, and O viii is formed at temperatures near \approx 2 \times 10^6 K, this ratio provides an observational measure of N_e for T_e \approx 2 \times 10^6 K. That is, a hypothetical experimenter testing the computed O viii fluxes as data would derive a density at any instant during the cooling, from the forbidden to intercombination line intensity ratio. The computed energy flux in the resonance line allows an observational emission measure (N_e^2ΔV) to be determined, defined in the same sense as the density, if a temperature for the emission of O viii is also assumed. This temperature is usually determined by experimenters.

### Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>t = 0 s</th>
<th>t = 3.8 s</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(Fe xxv)</td>
<td>3.23(-14)</td>
<td>9.84(-15)</td>
</tr>
<tr>
<td>F(Fe xxv)</td>
<td>3.32(-14)</td>
<td>1.56(-14)</td>
</tr>
<tr>
<td>F(Fe xxv)</td>
<td>5.60(-15)</td>
<td>5.23(-15)</td>
</tr>
<tr>
<td>[F(Fe xxv)/F(Fe xxv)]_γ</td>
<td>0.66</td>
<td>0.93</td>
</tr>
<tr>
<td>T_e(K)</td>
<td>15.8(±6)</td>
<td>13.6(±6)</td>
</tr>
<tr>
<td>(N_e^2ΔV)[cm^{-4}]</td>
<td>4.0(±47)</td>
<td>3.8(±47)</td>
</tr>
<tr>
<td>(N_e^2ΔV)[cm^{-4}]</td>
<td>1.8(±47)</td>
<td>2.0(±47)</td>
</tr>
<tr>
<td>ΔL[Å]</td>
<td>1.9(±3)</td>
<td>1.9(±3)</td>
</tr>
</tbody>
</table>

* Spectral line fluxes defined by eq. (15).
* The ratio that would be measured by a hypothetical spectrometer.
* The Fe xxv line is blended by satellite lines of Fe xxiv that fall at almost the exact wavelength of the Fe xxv line. See Bely-Dubau, Gabriel, and Volonté 1979b for a detailed discussion.
* The temperature that would be deduced from the F(Fe xxv)/F(Fe xxv) ratio. At 0 s, the maximum temperature in the flux tube is 17.6 \times 10^6 K, and at 3.8 s it is 14.3 \times 10^6 K.
* The parameter ΔV is the volume over which the line is emitted. The volume emission measure N_e^2ΔV is an observational parameter deduced from F(Fe xxv), the area of the flux tube, and the calculated emission rate for the line. See Doschek et al. 1980, p. 732.

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as about $2 \times 10^6$ K (Doschek et al. 1981). Once $(N_e^2 \Delta V)$, and $N_e$ are derived, both $N_e \Delta V$ and $\Delta V$ can be derived as well, remembering that these quantities are defined in an observational sense, as discussed for the iron lines, and apply here only to the plasma at temperatures of $\approx 2 \times 10^6$ K.

The flux in the O vii resonance line, computed from the model and summed over the entire system, the electron density obtained from the ratio of the forbidden and intercombination lines, the temperature at the top of the tube, and a length parameter $\Delta L$ for O vii emission, are shown as functions of time in Figure 5. The length represents the distance in the flux tube over which substantial O vii emission occurs, as determined from the density sensitive line ratio. This length multiplied by the flux tube area is the approximate volume over which O vii emission occurs and is the one-dimensional analog of the volume $\Delta V$ discussed above that would be deduced by a hypothetical experimenter. The length is determined by assuming that the temperature of formation of O vii is $2.3 \times 10^6$ K at all times. This value is the average value obtained by forming the ratio of the sum of the forbidden and intercombination lines to the resonance line, which is somewhat temperature sensitive (see Gabriel and Jordan 1972). We again emphasize that lengths obtained in this way are the lengths that would be deduced from hypothetical soft X-ray spectral observations assuming the flux tube area is known.

The length can also be computed directly from the numerical simulation. The actual average temperature of formation of O vii in the system varies from $2 \times 10^6$ K to $2.6 \times 10^6$ K at various times during the cooling. If the length is defined as the length at any instant from which 70% of the total emission in the O vii line arises, a somewhat different result is obtained, compared to the length deduced using observational procedures. This length is also shown in Figure 5.

We have calculated various quantities from both the oxygen and iron line fluxes directly from the numerical simulation and indirectly by treating the computed fluxes as hypothetical data in order to establish a link between purely theoretical quantities and the usual parameters calculated and published by experimenters. Differences in the results help to evaluate the accuracy of the parameters derived from real data.

The time behaviors in Figure 5 of the O vii flux, the electron density deduced from O vii line ratios, and the length parameters are explained by noting that the O vii ion is formed initially in a thin zone at the top of the transition region. As the system cools, this region expands, and the density drops. The combined effect is such that the volume emission measure for O vii remains about constant, and therefore the flux in the resonance line remains about constant. Eventually most of the system is at a temperature near $2 \times 10^6$ K, and, as mentioned, the entire loop would appear bright in O vii radiation. After about 80 s when the temperature is below $10^6$ K, the Boltzmann factor in the excitation rate drops rapidly so that the flux in the O vii lines therefore falls. Between 0 s and 75 s, the length increases by a factor of 200. Large downflows in O vii would not be observed, since these motions are not appreciable up to 75 s. Note that the density determined from the O vii lines is always high, i.e., $>3 \times 10^{11}$ cm$^{-3}$.

Also shown in Figure 5 is the computed flux in the O vii Lyα line at 18.97 Å. The intensity ratio of this line to the O vii line is in fair agreement with observation (Doschek et al. 1981). The O vii line is formed over a much larger temperature region than the O vi line because O vii is a hydrogenic ion.

So far we have only considered spectral diagnostics for the plasma at $T > 10^6$ K. We can also examine the observational consequences of our calculations on the transition region by considering spectral lines of O iv and O v, e.g., lines of O iv at 554 Å or near 1400 Å, and lines of O v at 629 Å or 1218 Å. The long-wavelength lines are intercombination lines and are therefore...
optically thin (see Mariska et al. 1981 for atomic physics details). These lines are formed between $10^5$ K and $2 \times 10^5$ K, and flare observations were obtained from Skylab. Because of the coarse resolution of the model ($\Delta z > 5.0$ km) at the locations of these temperatures, accurate intensities are difficult to extract. However, the following results are essentially independent of the precise resolution used. Initially the O iv and O v lines are formed in a very high density region near $10^{13}$ cm$^{-3}$. The density at which most of the line radiation arises falls as the loop cools, and by $80$ s it is near $10^{12}$ cm$^{-3}$. Up to this time the lines are formed in the high density region that is slowly expanding into the flux tube. For times greater than $100$ s, $N_e \approx 10^{11}$ cm$^{-3}$, and the lines are then formed in the lower density plasma collapsing at velocities of about $40$ km s$^{-1}$. Thus, our calculations show relatively low density downflows toward the end of the cooling phase, and upflows at the onset of cooling. Steady downflows are in fact observed (Cheng 1978) near the footpoints of flaring flux tubes, but most upflow observations appear to be surge related (e.g., Doschek, Feldman, and Mason 1979). The high densities predicted by the model for transition region temperatures are consistent with available observations (e.g., Cheng 1978).

We may summarize the above discussion with the statement that although there is some agreement between calculations and observations, complete consistency between observations and numerical calculations cannot be expected until at least a heating function is included in the model. As noted in § I, continuous heating is a property of many flares. However, the construction of a detailed dynamic model of solar flare flux tubes is best undertaken by isolating and understanding in a stepwise manner the physics involved in a single tube. The numerical calculations discussed in § III represent a reasonable beginning in the development of a more complete model, in which the cooling of plasma is isolated from a heating process that is only ill-defined by available observations.

IV. DISCUSSION AND CONCLUSIONS

a) The Dependence of Cooling on Initial Conditions

In this section we show how the initial cooling of the system depends on the initial conditions assumed for the flux tube. The motivation for this analysis is the tremendous range in observed X-ray output from flares. We show that the ratio of thermal conduction energy loss time scales relative to radiative energy loss time scales depends strongly on the emission measure and electron density of the hot plasma. Some of the results described in § III therefore apply to a restricted class of flares.

The fluid velocities during the initial cooling phase described in § III (the conduction phase) are small. In this case the energy equation, for the hot coronal plasma where radiative energy losses are negligible, is given by

$$\frac{\partial}{\partial t} \left( \frac{P_e}{\gamma - 1} \right) = \frac{\partial}{\partial z} \left( \kappa_e \frac{\partial T_e}{\partial z} \right),$$

which can be solved by separation of variables (Antiochos and Sturrock 1976). The solution for the temperature at the top of the system is

$$T_e (L, t) = T_m (1 + t/\tau)^{-2/5},$$

where

$$\tau = \frac{10^{10} T_m^{3/7} N_e}{P_o}.$$  

In equations (16) and (17), $T_m$ is the maximum temperature of the system, and $P_o$ is the pressure at height $L$ at $t = 0$.

It is useful to express the solution for the temperature $T_e (L, t)$ in terms of the emission measure which is accessible to observation. Accordingly, we express $T_e (L, t)$ in terms of $T_m$, the total loop length $2L$, and the volume emission measure $N_e \Delta V (\equiv E_v)$ of the hot plasma, where $\Delta V$ is defined as the volume of plasma that contributes significantly to X-ray emission from Fe xxv. Measurements of $T_m$, $2L$, and $E_v$ are available from the observations discussed in § I. Expressing $T_e$ in terms of only $E_v$ and $L$ requires knowledge of the flux tube radius. There are no observations of the ratio of loop radius $R$ to loop length $2L$ for flares. We assume a reasonable value of $L/5$ for $R$.

In terms of $E_v$, equation (16) becomes,

$$T_e (L, t) = T_m [1 + 1.73 \times 10^9 T_m^{3/2} (E_v L)^{1/2}]^{-2/5},$$

with the auxiliary relationship

$$N_e (L) = 2 (E_v / L)^{1/2}.$$  

Rather than determining $L$ directly from observation, it is more instructive to determine $L$ from a chosen fixed value of $N_e$, with the requirement that the values of $L$ be consistent with observation. We adopt a typical density of $3 \times 10^{13}$ cm$^{-3}$.

Table 2 shows the effect of varying $E_v$ on the conduction cooling time $t_c$, while keeping $T_m$ and $N_e$ fixed. The conduction cooling time is defined as the time necessary for the loop to cool from $18 \times 10^5$ K to $10 \times 10^5$ K. Note that the lengths $L$ of all the representative loops considered are consistent with observation, i.e., they are all on the order of 1' or less. The conduction time varies by very large amounts depending on the loop length. The point is that since the range of parameters given in Table 2 has been observed, all of the cooling times shown are possible, depending on the emission measure $E_v$ and the other parameters.

**TABLE 2**

<table>
<thead>
<tr>
<th>$E_v$ (cm$^{-3}$)</th>
<th>$2L$ (\degree)</th>
<th>$t_c$ (s)</th>
<th>$t_1$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{48}$</td>
<td>9.8</td>
<td>26</td>
<td>550</td>
</tr>
<tr>
<td>$10^{49}$</td>
<td>21.0</td>
<td>120</td>
<td>550</td>
</tr>
<tr>
<td>$10^{50}$</td>
<td>45.0</td>
<td>550</td>
<td>550</td>
</tr>
</tbody>
</table>

* $T_m = 18 \times 10^5$ K; $N_e = 3 \times 10^{11}$; 1" = 725 km.
The times \( t_c \) shown in Table 2 can be increased by varying the geometry of the flux tube as discussed by Antiochos and Sturrock (1976). In our case the cross-sectional area is constant with height, and our \( t_c \) is the same as \( \tau_p \) given in the aforementioned paper. Antiochos and Sturrock (1976) have also shown that \( t_c \) increases when the area increases with height in the tube. Thus we consider the values of \( t_c \) given in Table 2 as lower limits.

The radiative cooling time \( t_r \) of the plasma near \( 18 \times 10^6 \) K can be estimated from the expression,

\[
t_r = 1.04 \times 10^{-7} (T_M - T_i(L, t)) L^{1/2}/E_0^{1/2},
\]

which is obtained by approximating \( \Phi(T) = \Phi(T_M) = 2 \times 10^{-23} \text{ cm}^2 \text{ s}^{-1} \). Table 2 shows that our conclusion that conduction dominates the early cooling phase depends on our choice of a small emission measure, or short loop length. However, if we had instead chosen longer loops which correspond to much larger emission measures in the range \( 10^{10} < E_0 < 10^{16} \) \text{ cm}^{-3}, then radiative and conduction cooling would have been comparable, and equation (15) would not have been valid.

Note that in Table 2, for \( 10^{49} < E_0 < 10^{50} \), the minimum cooling time is 2 minutes. However, a flare has been observed by P78-I for which \( E_0 \approx 5 \times 10^{49} \) \text{ cm}^{-3}, and nearly all the Fe xxv emission disappears in 20 s. In order to obtain a cooling time as short as the above calculations, the density must be higher. Consider the case where \( N_e = 10^{13} \) \text{ cm}^{-3}, and \( E_0 = 5 \times 10^{49} \) \text{ cm}^{-3}. Then \( t_c = 100 \) s, \( t_r = 16 \) s, and radiative cooling of the hot plasma dominates, i.e., there is no initial conduction phase. This particular flare may, therefore, have a higher density, much larger than most densities reported previously. A full discussion of this flare is in preparation (Feldman, Doschek, and Kreplin 1982).

We conclude from the above discussion that the cooling time is strongly dependent on the particular values of \( E_0 \) and \( N_e \) (or \( L \)) chosen. The dominance of conduction or radiative energy loss during the initial cooling phase is also a result of this choice. Thus, our calculations described in § III apply to flares with small emission measures. However, the physics of the cooling below \( 10^7 \) K is probably applicable to all flares.

b) Comparison to Previous Calculations

Antiochos and Krall (1979) have numerically simulated a cooling flare flux tube. As mentioned above, we have adopted their loop height \( L \) and peak temperature and density as initial conditions for our calculations. However, their initial model was somewhat different from ours since they assumed a constant heat flux at the beginning of the calculation. A major difference between their calculation and ours is that they did not attempt to resolve the transition region, but instead treated it as a discontinuity with fixed lower boundary. Quantities on either side of the boundary are related through conservation conditions, but the location of the interface is not allowed to vary. The temperature at the base of their model was higher than ours, i.e., \( 3 \times 10^4 \) K, and was fixed at this value throughout their calculation. They considered a family of models characterized by different variable cross-sectional areas \( A(z) \). The times \( t_c \) were determined by assuming different dipole magnetic field configurations. However, they found that variations in the functional form of \( A(z) \) had a negligible effect on their results.

The results obtained by Antiochos and Krall (1979) are qualitatively similar to ours, despite some quite different approximations used in the two calculations. The times needed to cool their loop to different peak temperatures agree quite well with our times. This is due to the fact that some of the parameters we use are identical, i.e., \( T_r, N_e, \) and \( L \), in our initial models. At times greater than 160 s, Antiochos and Krall (1979) observe large downflow velocities on the order of 75 km s\(^{-1}\), which is in qualitative agreement with our results. The implication of this agreement is that much of the hydrodynamic behavior of the system is not heavily dependent on initial conditions and boundary conditions that are unknown from observations.

Recently, Nagai (1980) has numerically simulated a flare flux tube including both a heating and cooling stage. He begins with a loop at a typical quiet coronal density and temperature \( (10^7 \text{ cm}^{-3}, 1.5 \times 10^6 \text{ K}) \). Heat is then deposited over a period of time and in a nonspatially uniform manner along the length of the loop. Different functional forms for the heating were investigated, and two different loop lengths were considered. Only in Nagai's (1980) Case A is cooling without energy input considered specifically. In this case the heating is turned off at a particular time, and the loop begins to cool. However, at this time mass is rising into the flux tube due to the prior heating, and therefore cooling begins with an initially dynamic condition, rather than with the static condition we have chosen. Nagai's (1980) numerical calculation underresolves the transition zone with a grid size of 10 km. However, as mentioned above, we have found from numerical simulations that in the absence of strong heating this difference, i.e., 10 km zones instead of 5 km or 2.5 km, alters the hydrodynamics only moderately. The actual width of the initial model transition region is in fact considerably less than 2.5 km.

There are several other differences between Nagai's (1980) calculation and ours, some major and some minor. He adopts a detailed chromospheric model (HSRA model, Gingerich et al. 1971); he alters the radiative loss function to account for opacity at low temperatures; and the functional dependence of his thermal conductivity coefficient on temperature is slightly different from ours at chromospheric temperatures. In spite of these differences between his calculation and ours, the qualitative behavior of the plasma during cooling deduced by Nagai (1980) is about the same as we have described. The transition zone in his calculations rises into the flux tube as in our model, and the downflow velocities are similar. Even the local rise in temperature that occurs late in the cooling at 109 s in our model...
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(discussed in § III) occurs at a relatively similar time in the Nagai (1980) model.

Finally, we note that the scenario of heating and cooling adopted by Nagai (1980) cannot, in its present form, explain the observational data satisfactorily. The reason is that the maximum density achieved by the plasma at $T_e \approx 18 \times 10^6$ K is only $10^{10}$ cm$^{-3}$. This density is too low (Cheng and Widing 1975; Feldman, Doschek, and Kreplin 1980). The density is determined in part by the choice of a quiet coronal loop as the loop initially heated. This loop has a density ($\approx 10^9$ cm$^{-3}$) which is too low (Cheng and Widing 1975; Feldman, Doschek, and Kreplin 1980). The model might be considerably improved by adopting a high density ($\approx 10^{11}$ cm$^{-3}$) transition region ($T_e \approx 10^5$ K) loop as the plasma to be heated.

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