I. INTRODUCTION

The linear properties of condensational instabilities in solar and astrophysical plasmas are simple to understand in principle. The nonlinear behavior, which describes the way in which the instability approaches saturation and eventually establishes a new dynamic equilibrium, is basically unexplored.

This paper considers some of these dynamic nonlinear aspects for a plasma medium such as the solar corona which is optically thin in the principal components of its radiation field. The volumetric energy loss rate may be written as \( L \sim \rho^2 \phi(T) \), where \( \rho \) is the local density and \( \phi(T) \) is a positive function of temperature. If the pressure is constant and there is a steady heat source, \( S \), some regions of the plasma cool slowly and others are heated according to the balance of \( L \) and \( S \). As an element of the plasma cools, the density increases so that pressure balance is maintained with the hotter surroundings. When \( L \) increases faster with \( \rho \) than it decreases with the lower temperature, there is a strong tendency for material to condense: this is called the condensational instability. Now consider a volume of plasma which is hotter than its surroundings. As it expands, the energy addition rate increases its dominance over the radiation cooling process. This runaway temperature effect, called the thermal instability, is equivalent to the condensational instability with a temperature perturbation of the opposite sign. These types of instabilities are exponential for small perturbations and are generally damped by thermal conduction for perturbations with a short enough scale length.

A substantial amount of research has been done to understand the linear condensational (thermal) instability in fluids at astrophysical temperatures and pressures. Perhaps the most complete analysis is that of Field (1965), who added substantially to the prior work of Parker (1953), Zanstra (1955a, b), and Weymann (1960). Field's work clarified the stability criteria and extended the linear analyses to include effects other than radiation loss such as magnetic fields, thermal conduction, rotations, and density stratification. Recently, Antiochos (1979) has extended this work to consider the stability of a stratified fluid for which the heating source term has a temperature dependence.

In this paper, we review briefly the linear stability analysis for a homogeneous fluid including radiation, thermal conductivity, a magnetic field, and a constant heat source. This analysis is then applied to standard semiempirical models of the solar transition region and corona. The effects of increasing density and magnetic field, as well as variations in the wavelength of the perturbations, are examined in order to provide quantitative information on the local stability of the solar transition region and corona. Finally, these results are used to choose initial conditions for detailed time-dependent simulations of the condensation instability using the Naval Research Laboratory (NRL) Dynamic Flux Tube Model. The results from the simulations are compared to theory in the linear regime and are also used to show the nonlinear evolution and saturation of the instability.

Our linear analysis is similar in many respects to that of Field (1965). We believe, however, that it is useful to discuss it in some detail. By discussing the specific case of a plasma under solar conditions, we can provide a physical feeling for its stability properties that is not present in Field's work. While the semiempirical model that we analyze is, in some sense, a mean atmosphere, it does describe a tremendous amount of observational data and is consistent with the equations for a static
equilibrium atmosphere. It is probably the best current estimate of the physical conditions in the outer layers of the atmosphere of the quiet Sun.

It is important to do the numerical simulations for several reasons. First, by comparing both calculations, the theory and the simulations provide a quantitative check on each other. The meaning of approximations in numerical model. Understanding and resolving discrepancies gives confidence in the answers and indicates where to expect inaccuracies. Since the computational model will also be used to describe more realistic nonuniform environments, this sort of quantitative calibration is invaluable. Finally, the numerical simulations provide insights into the end state of this important instability which have important consequences for transition region structure and dynamics, coronal transients, and bifurcation in the state of the interstellar medium.

One important result of the nonlinear analyses presented below is the demonstration that the transition region is a dynamically stable structure which is the response of the plasma to a condensational instability. By dynamically stable, we mean that the dynamically counterbalanced effects of conduction and radiation are so strong that rapid but small readjustments of the structure of the transition region track changes in external conditions, heat input, or other perturbations. Thus, studies of the stability of empirically and analytically determined solar atmospheres with transition regions as initial equilibrium states are in fact evaluations of the consistency of these atmospheres. When such postulated atmospheres are shown to be unstable, the instability is a result of some inconsistence or the assumed form of the heating function. Then the resulting readjustment of the transition region is very fast due to the lack of balance in the heating, loss (radiation), and source terms. This effect is more properly considered a lack of equilibrium rather than an instability, and the resulting change in the model structure is usually thought of as a quasi-static readjustment.

II. LINEAR ANALYSIS

The basic equations we linearize are those for conservation of mass, momentum, energy, and magnetic flux:

\[ \frac{\partial \rho}{\partial t} = -\mathbf{V} \cdot \mathbf{v}, \tag{1} \]

\[ \rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla P - \frac{\mathbf{v} \cdot \mathbf{B}^2}{8\pi} + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi}, \tag{2} \]

\[ \frac{1}{\gamma - 1} \frac{\partial P}{\partial t} + \frac{1}{\gamma - 1} \mathbf{v} \cdot \nabla P + \frac{\gamma}{\gamma - 1} \mathbf{v} \cdot \nabla \mathbf{v} + \gamma L + S + \nabla \cdot \mathbf{V} T, \tag{3} \]

\[ \frac{d}{dt} + \mathbf{B} \cdot \nabla \mathbf{v} - (\mathbf{B} \cdot \nabla) \mathbf{v} = 0, \tag{4} \]

with

\[ P = N_k T = \frac{\rho k_B}{\mu m_H} T, \tag{5} \]

where \( \rho, \mathbf{v}, P, B, \gamma, \) and \( T \) are the density, velocity, pressure, magnetic field, ratio of specific heats, and temperature respectively. The \( \kappa, k_B, \mu, \) and \( m_H \) are the thermal conductivity coefficient, Boltzmann's constant, the mean mass per particle, and the proton mass. The quantities \( S \) and \( L \) are the heat source and the radiation loss functions respectively. We assume that \( S \) is constant and assume \( L \) for an optically thin plasma is defined as

\[ L = N_e N_H \phi(T) \equiv c \rho^2 \phi(T), \tag{6} \]

where \( N_e \) and \( N_H \) are the electron and hydrogen number densities and \( \phi(T) \) has been provided by Raymond (1979). For the work presented in this paper, \( c = 0.7171/m_H^2 \). This is based on the constants given in Table 1. Note that, depending on the values used for the quantities in Table 1 consistent with those typically used for a solar plasma, \( c \) can be in the range from 0.5/m_H^2 to 0.75/m_H^2. Thus, a change in \( c \) could substantially influence the decay and growth rates calculated below.

The constant equilibrium values of the basic variables are \( \rho_0, T_0, P_0, B_0, \) and \( v_0 = 0 \). We consider perturbations of the form

\[ e^{\gamma t} e^{ik \cdot \mathbf{r}}. \tag{7} \]

To simplify the problem, we assume that the perturbation can only be either along the magnetic field direction, \( \hat{z} \), or along a perpendicular direction, \( \hat{x} \). The effect of either of these assumptions is to reduce the dispersion relation from a fifth order to a cubic polynomial in the growth rate. This cubic is simpler to study and yet contains the physics of principal interest.

Combining equations (1)–(7), we obtain the dispersion relation

\[ \Gamma^3 + a_2 \Gamma^2 + a_1 \Gamma + a_0 = 0. \tag{8} \]

For \( k \) parallel to \( B_0 \),

\[ a_2 = (\gamma - 1) \frac{T_0}{\frac{P_0}{P_0}} \left( k_B k^2 + c \rho_0^2 \frac{\partial \phi}{\partial T} \right), \tag{9a} \]

\[ a_1 = k_B^2 c_2 k_2^2 = k_B \frac{\gamma P_0}{P_0}, \tag{10a} \]

\[ a_0 = k_B^2 c_2 (\gamma - 1) \frac{T_0}{\gamma P_0} \left[ c \rho_0^2 \left( \frac{\partial \phi}{\partial T} + 2 \frac{\phi}{T} \right) + k_B k^2 \right], \tag{11a} \];
where \( c_s = (\gamma P_0 / \rho_0)^{1/2} \) is the speed of sound. For \( k \) perpendicular to \( B_0 \),
\[
a_2 = (y - 1) \left( \frac{T_0}{P_0} \right) \left( \kappa_1 k_1^2 + c p_o \frac{\partial \phi}{\partial T} \right),
\]
\[
a_1 = k_1^2 v_M^2 = k_1^2 \left( \frac{B_0^2}{4 \pi \rho_0} \right) + \left( \frac{\gamma P_0}{\rho_0} \right),
\]
\[
a_0 = k_1^2 c_s^2 (y - 1) \left( \frac{T_0}{\rho_0} \right)
\]
\[
\left[ \frac{B_0^2}{4 \pi \rho_0} \left( \kappa_1 k_1^2 + c p_o \frac{\partial \phi}{\partial T} \right) + c p_o \left( \frac{\partial \phi}{\partial T} - \frac{2 \phi}{T_0} \right) + \kappa_1 k_1^2 \right],
\]
where \( v_M \) is the magnetoacoustic velocity, \( v_M = v_A + c_s^2 \).

At least one of the three roots of equation (8) must be real, and this corresponds to the condensational mode. The other two roots may be complex conjugates of each other and correspond to damped magnetoacoustic waves if \( B_0 \neq 0 \) or damped sound waves if \( B_0 = 0 \). If the density and hence the radiation rate are large enough, these oscillatory roots may grow instead of damp.

The \( k_\parallel \) and \( k_\perp \) cases are two extremes. When \( k \) is parallel to \( B_0 \) or has a substantial component at an angle to \( B_0 \), there is a large stabilizing effect due to the parallel conductivity term. When \( k \) is perpendicular to \( B_0 \), the thermal conductivity term is much smaller, resulting in very much smaller characteristic values of the wavelength for which the system may be unstable. These effects become apparent from the figures shown in the next section. Although some analysis is done on the \( k_\perp \) case, in this paper we are primarily interested in \( k_\parallel \) since this is the case studied in the numerical simulations presented below.

There are several approximations to the solution of equation (8) which are of interest. When the wavenumber \( k_\parallel \) is small enough (large enough wavelength perturbation), the dispersion relation becomes \( \Gamma \approx -a_0 \). This occurs for the condensational mode when \( k_\parallel c_s^2 \ll \Gamma^2 \). Then, instability results only when \( \partial \phi / \partial T \) is negative and large enough to overcome the damping due to thermal conduction. When \( k_\parallel^2 c_s^2 \gg \Gamma^2 \), the dispersion relation becomes \( \Gamma = -a_0 / a_1 \). In this case, the condensation instability may occur even though \( \partial \phi / \partial T \) is positive. This arises because the plasma radiation rate is influenced as much or more by density variations as by local temperature variations. At \( k_\parallel = 0 \) (infinite wavelength), the density perturbations must be zero. Finally, we note that, for an important range of parameters, the plasma density is low enough so that the basic character of the solution to equation (8) is essentially the same as for the case of \( \phi(T) = 0 \). These specific limits of the solution will be discussed below with reference to a particular test case.

\[ a) \text{ Analysis of Models of the Transition Region and Corona} \]

We are primarily interested in the structure and stability of the region of the solar atmosphere with a temperature above about 30,000 K. From 30,000 K to 447,000 K, we use the average quiet-Sun model of Vernazza, Avrett, and Loeser (1981). This is a semiempirical model atmosphere that satisfies a large number of observational constraints. We have extended the atmosphere above 447,000 K by constructing an energy balance model which balances radiation losses with conduction from the overlying corona. The lower boundary conditions for the coronal model are those at 447,000 K in the semiempirical model. Figure 1 shows the temperature structure used as a specific case on which to perform the linear stability analysis.

In principle, we could analyze the response of the empirical model to any wavelength of perturbation. The physical meaning one could attach to finding a positive growth rate as one of the three roots of equation (8), however, varies with the wavelength. For wavelengths comparable to or longer than the density or temperature scale height given by the empirical model, a linear instability may be an artifact of, for example, inaccuracies in the semiempirical model, the improper treatment of the one-dimensional variable-coefficient eigenvalue problem, the particular choices of models for heating and radiation functions, or a combination of all these. Further, for perturbations with lengths on the order of the temperature or density scale height or longer, local stability criteria such as we developed above are at best approximate—there is no clear distinction between linear instability and lack of equilibrium.

On the other hand, we expect very short wavelength perturbations to be stabilized physically by the strong smoothing effects of plasma thermal conduction along magnetic field lines. There should be a range of scale

![Fig. 1.—Electron temperature and characteristic wavenumber as a function of height above \( R_{\odot} \) for a typical, quiet transition region–corona model atmosphere. The heavy dots indicate the special conditions examined in this study.](image-url)
lengths between the thermal conduction damping scales and the characteristic system scale lengths where local stability criteria are valid and the instability growth rates maximize when the equilibrium is unstable. We wish to focus on a characteristic perturbation wavenumber in this regime. This wavenumber \( k_c \) must vary with height because the temperature and density profile scale lengths vary rapidly with height. Parallel to the magnetic field, \( k_c \) is determined by the density or temperature scale height of the system, whichever is larger, i.e.,

\[
k_c = 10.0 \times \max (k_\rho, k_T).
\]

In equation (12), \( \max (k_\rho, k_T) \) indicates the maximum of two quantities \( k_\rho \) and \( k_T \), defined to be

\[
k_\rho = \frac{1}{\rho} \frac{\partial \rho}{\partial z} \quad \text{and} \quad k_T = \frac{1}{T} \frac{\partial T}{\partial z}.
\]

Figure 1 shows that \( k_c \) ranges from \( 10^{-10} \) to \( 10^{-5} \) for the altitude range considered. The factor of 10 in equation (12) emphasizes wavelengths short enough that our local treatment of stability suffices. This corresponds to wavelengths from \( 6 \times 10^{10} \) to \( 6 \times 10^5 \) cm.

b) Perturbations Parallel to \( B_0 \)

By solving equations (8) and (9a)-(11a), we find that this model atmosphere at the standard wavelengths is inherently stable. Figure 2 shows \( \Gamma \) as a function of altitude where \( k_c \) for each altitude is taken from Figure 1. We have defined

\[
\Gamma_i = \gamma_i + i\omega_i \quad \text{for } i = 1, 2, 3,
\]

where \( \gamma^{-1} \) is the growth or decay rate and \( 2\pi \omega^{-1} \) is the oscillation period of the perturbation. The figure shows two roots. The first one, \( \Gamma_1 = \gamma_1 \), is the condensational mode and the other, \( \Gamma_2 = \gamma_2 + i\omega_2 \), is a sound wave or magnetoacoustic wave. For the third root, \( \Gamma_3 = \Gamma_2^* \). In general, we have found that the roots corresponding to sound waves retain their damped oscillatory character while the condensational mode may either damp or grow.

Each altitude has a \( k_c \) value for which the system may go unstable. The marginal stability curve shown in Figure 3 indicates the roughly order-of-magnitude disparity between the wavenumber \( k_c \) chosen for characteristic disturbances and the wavenumbers \( k_{\text{mar}} \) which would cause instability. The disparity decreases at lower altitudes and temperatures where the gradients become very steep. However, we see that the static, unperturbed, models of the transition zone appear to be stable within the framework of this analysis. As we show below in §IV, the stability of the transition region to linear condensational modes is expected once we see that a transition region is the final structure toward which a condensational mode evolves. The shape and characteristic scale lengths in a transition region are determined by a balance between electron thermal conduction and the radiation loss term. The same effects are unbalanced at the beginning of condensational mode growth. The state of the plasma then changes to correct this imbalance by creating sharp temperature gradients which allow an increase in
thermal conduction and a localized volume over which radiation loss is strong.

Since the electron thermal conduction determines both the transition region structure and the instability cut off for the condensational mode, it is not surprising that \( k_c \) and \( k_{\text{mar}} \) are close at low temperatures. The factor of 10 used to define \( k_c \) in equation (12) is reflected quite graphically in Figure 3, where the marginal instability wavenumber lies basically one order of magnitude longer in wavelength than the characteristic scale length chosen. In this kind of dynamically balanced system, the equilibrium state would be different if any local element of the plasma were unstable. In fact, plasma and other fluids seek such a state of marginal instability in many contexts as a natural consequence of competing physical effects with short time scales (Manheimer and Boris 1978).

Such nonlocal stability analyses as have been performed (Antiochos 1979; An, McClymont, and Canfield 1980) need to make specific assumptions about heating functions, boundary conditions, geometry, etc., in order to get a time-independent equilibrium set of conditions to perturb. Instabilities which have been observed in such nonlocal analyses seem derivable from specific choices of the auxiliary conditions rather than an intrinsic instability of the plasma. The fact that the semiempirical models are observed as the average long-time behavior of the solar atmosphere argues against their being very unstable to major rearrangements.

Since local conditions on the Sun vary greatly, it is reasonable to investigate deviations from average behavior which could bring up intrinsically unstable conditions. For example, when the overall density is increased by a factor of 28, shown in Figure 4 and in the upper marginal stability curve in Figure 3, a sizeable region of instability develops.

We now investigate the stability of the particular point on Figure 1 marked with the solid circles. This corresponds to \( T^* = 7.293 \times 10^5 \) K and \( k_f^* = 6.84 \times 10^{-9} \) cm\(^{-1}\). The contours in Figure 4 show the behavior of the unstable condensational root of equation (13) as a function of both \( k_{\parallel} \) and density, which is represented as a ratio of an enhanced density to the density of the standard case. We know that the region of instability increases in size as \( k_{\parallel} \) and \( \rho \) become larger, i.e., as the thermal conduction is reduced and radiation rates are increased. Figure 5 shows the roots as a function of \( k_{\parallel} \) for the case \( \rho/\rho_0 = 28 \). As noted earlier in conjunction with Figure 4, a substantial region of instability is encountered.

Figure 5 points to some of the interesting limits of equation (8) which were discussed above. First let us consider the infinite wavelength perturbation corresponding to \( k_{\parallel} = 0 \). Then,

\[
\Gamma = -a_2 = \gamma_1 ,
\]

(15)

and, for the case we have been examining, \( a_2 \) is negative since \( \partial \phi/\partial T \) is negative. Thus, the root is real and corresponds to a growing condensation instability. From the figure, we see that \( \gamma_1 \) is relatively constant at low \( k_{\parallel} \) values, and \( \gamma_2 \) and \( \omega_2 \) appear to be going to zero.

At shorter wavelengths and therefore larger values of \( k_{\parallel} \), another linear solution regime dominates the condensation mode:

\[
\Gamma = -a_0/a_1 ,
\]

(16)

Thus \( a_0 > 0 \) ensures that the system is stable. Equation (16) gives the value of \( \gamma_1 \) at \( k_{\parallel}^* \) to within a few percent. It is
interesting to look at the term $a_0$ in some detail for this case,

$$a_0 = \frac{cp_0}{(\gamma - 1)k_\parallel} \left( T_0 \frac{\partial \phi}{\partial T} - 2\phi \right) + \frac{T_0}{\rho_0} \kappa_{\perp} k_\parallel^2,$$

since its sign determines the stability of the system. As Field (1965) indicated, the $2\phi$ term, resulting from the derivative of $L$ with respect to $\rho$, is destabilizing. Thus, there may be regimes for which $(T_0 \frac{\partial \phi}{\partial T})$ is positive, yet the system is unstable. The third term in $a_0$ represents the thermal conductivity and is a stabilizing term. For standard, quiet-Sun transition region and chromospheric models, this term dominates.

c) Perturbations Perpendicular to $B_0$

Figure 6 shows the values of $k_\perp$ needed to obtain a marginally stable solution to equations (9b)-(11b). For the same height in the atmosphere, these values of $k_\perp$ are much larger than the $k_\parallel$ values investigated above. This shift of scales occurs because similar values of $k_\perp^2 \kappa_{\perp}$ are needed to balance the other terms in equation (11b). Thus, the first term in $a_0$ represents the thermal conductivity and is a stabilizing term. For standard, quiet-Sun transition region and chromospheric models, this term dominates.

The conclusions presented above have potential consequences for the interpretation of solar observations. The condensation instability with a width-to-length ratio of $\sim (k_\perp/k_\parallel)^{1/2}$ should result in extremely fine filamentary structures. At this ratio of scale lengths, cooling or heating across field lines is roughly as important as it is along field lines. Thus, if the classical thermal coefficients remain valid, we may be able to use the measured ratio of these scales to infer a local value of $B_0$ in the corona. Such an analysis would require two-dimensional simulations that are analogous to the one-dimensional simulations presented below.

The appropriateness of using $\kappa_{\perp}$ and $\kappa_{\parallel}$ in the form given above must also be considered. First, $\kappa_{\parallel}$ may be increased $B_0$ also has a stabilizing effect.

Figure 7 shows the disparity between the parallel and perpendicular conductivities, $\kappa_{\parallel}$ and $\kappa_{\perp}$, for reasonable estimates of the magnetic field. Since the ratio of $\kappa_{\perp}/\kappa_{\parallel}$ is so small, we believe it is important to consider what implications this has for the physics of the transition region and corona. We know that when $k$ deviates even by small angles from $B_0$, the parallel conductivity dominates. Thus, there is really only a small regime where this perpendicular instability is meaningful: typically $k_\perp/k_\parallel > 10^4$ when $k_\perp/k_\parallel \sim 10^{-8}$.

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The appropriateness of using $\kappa_{\perp}$ and $\kappa_{\parallel}$ in the form given above must also be considered. First, $\kappa_{\parallel}$ may be
reduced from the classical (Spitzer) value by local turbulence or flux limitations in low-density regions (Colombant and Manheimer 1980, references therein; Manheimer, Max, and Thomson 1978; Max, Manheimer, and Thomson 1978). This latter effect may be induced by beaming or return current instabilities which tend to scatter electrons faster than they are scattered collisionally. Under quiet-Sun conditions these effects are probably not important.

The fact that perpendicular conduction may be anomalously large is more important. There are a number of effects, all studied to a greater or lesser degree by the magnetic fusion (tokamak) community, which tend to enhance cross field heat transport. Some of the mechanisms involve plasma microinstabilities, others concentrate on details of complex magnetic curvature, such as the neoclassical transport and cross field fluting as stressed field lines try to straighten (Hazeltine 1976; Manheimer 1977; Manheimer and Boris 1978; Manheimer and Cook 1979).

Consider a condensationally unstable plasma where $\beta$, the ratio of plasma pressure to magnetic pressure, is a few percent. As the plasma cools, there is little motion transverse to $B_0$ since local compressions of the field compensate for the reduced plasma pressure. If a cross field structure has a characteristic radius $R$, then $\Delta R/R \sim \beta/4$ for pressure balance displacements. The ratio of parallel to perpendicular heat transport in the tilted field line region will generally be in the range

$$\left( \frac{k_{\parallel} \Delta R}{2\pi} \right)^{0.5} \lesssim \frac{\kappa_{\perp}}{\kappa_{\parallel}} \lesssim \left( \frac{k_{\parallel} \Delta R}{2\pi} \right)^{1.0}. \quad (21)$$

Using the 0.75 power in equation (21) and assuming $k_{\parallel} \sim R^{-1}$ gives

$$R/\lambda \approx (\beta/8\pi)^{3/5}. \quad (22)$$

The 0.50 power gives

$$R/\lambda \approx (\beta/8\pi)^{1/3}. \quad (23)$$

These scale length ratios will generally be much larger than predicted by classical collisional theory alone because of the much larger transverse heat transport. Thus, we conclude that considerable additional analysis of this problem will be necessary before perpendicular versus parallel scales in condensational filamentation becomes useful as a diagnostic of the local magnetic field.

III. THE NUMERICAL MODEL

We have examined the nonlinear evolution of the condensational instability using the NRL Dynamic Flux Tube Model (Boris et al. 1980). For the problem addressed in this paper, this numerical model solves the set of one-dimensional two-fluid conservation equations for mass, momentum, and energy in the form

$$\rho \left( \frac{\partial v}{\partial t} + v \frac{\partial \rho}{\partial z} \right) + \frac{\partial P}{\partial z} = \rho g, \quad (25)$$

$$\frac{\partial}{\partial t} \left( \rho v^2 + \frac{P}{\gamma - 1} \right) + \frac{\partial}{\partial z} \left[ \left( \rho v^2 + \frac{\gamma P}{\gamma - 1} \right) v - \kappa_{\parallel} \frac{\partial T_e}{\partial z} - \kappa_{\perp} \frac{\partial T_i}{\partial z} \right]$$

$$= \rho v g - L + S, \quad (26)$$

$$\frac{\partial}{\partial t} \left( \frac{P_e}{\gamma - 1} \right) + \frac{\partial}{\partial z} \left( \frac{\nu P_e}{\gamma - 1} - \kappa_{e} \frac{\partial T_e}{\partial z} \right)$$

$$= -L + S + \gamma_{eq}(T_i - T_e) - P_e \frac{\partial v}{\partial z}, \quad (27)$$

where

$$P = P_e + P_i$$

$$P_e = N_e k_B T_e \quad \text{and} \quad P_i = N_i k_B T_i,$$

$$N_e = \frac{\rho Z}{\mu m_1 (1 + Z)}, \quad (30)$$

and

$$N_i = N_e / Z. \quad (31)$$

Also

$$\kappa_e = 1.1 \times 10^{-6} T_e^{5/2}, \quad (32)$$

$$\kappa_i = \kappa_e / 25, \quad (33)$$

and

$$\gamma_{eq} = 1.4 \times 10^{-17} N_e^2 T_e^{-3/2}. \quad (34)$$

In the single fluid limit, these equations reduce to equations (1)-(5) which have been analyzed theoretically above. Total fluid variables have been written without subscripts, such as, $P$, $v$, etc. The subscripts $i$ and $e$ designate ion and electron variables respectively. The $g$ is the solar surface gravity, $Z$ is the average charge of the ions in the system, the $N$'s refer to number densities and $\kappa$'s to the thermal conductivity coefficients. The radiation loss function $L$ has been defined in equation (6), and the heating source function $S$ is problem dependent. Other aspects of the numerical model, such as the atomic rate equations, the magnetic field terms, and the radial momentum equations, are not used here and will be discussed in future papers.

The convective derivatives in equations (24)-(27) are solved by the Flux-Corrected Transport method (see, for example, Boris and Book 1976, and references therein). The nonconvective terms in equations (26) and (27) are solved fully implicitly. All of the processes are coupled by time-step splitting and grid variation techniques as described in Oran and Boris (1981).

For the purpose of the simulations presented below, we have made an effort to ensure that the basic constants and the governing equations used are the same as in the linear model.
analysis. Thus, the parameters given in Table 1 and an identical radiation loss function are used in both theory and computation. Furthermore, the gravity terms have been set to zero, and we have forced $T_\varepsilon = T_\varepsilon^*$ for the detailed comparisons with the linear analysis to recover the single fluid plasma approximation.

The strength of the numerical simulation approach lies in the fact that a single specific initial value problem is being solved in both its linear and nonlinear phases. The connection between linear growth of an instability and the resulting dynamically balanced states which evolve is made clear. Since only one case at a time is solved, however, the general parametric behavior of the system and the underlying physical principles must be inferred from only a few data points.

**IV. THE NONLINEAR EVOLUTION OF THE CONDENSATIONAL INSTABILITY**

For the detailed numerical computations, we are considering now a rigid cylindrical flux tube with nontransmissive symmetry boundary conditions at its ends. The initial conditions are set to match those discussed in § IIb on perturbations parallel to $B_0$,

$$T_\varepsilon = T_\varepsilon^* = 7.293 \times 10^5 \text{ K}$$

and

$$\rho = \rho^* = 3.23 \times 10^{-14} \text{ g cm}^{-3}.$$  

A sinusoidal velocity perturbation $v_0(\varepsilon) = v_0 \sin (k_\varepsilon^* \varepsilon)$ triggers the instability, where

$$k_\varepsilon^* = 6.84 \times 10^{-9} \text{ cm}^{-1} = 2\pi/\lambda^*.$$  

The system length is set equal to $1.5 \lambda^*$ with zero velocity along $B_0$ at the ends of the tube. At each time step, an amount of energy $S$ (ergs cm$^{-3}$ s$^{-1}$) is deposited in each cell such that the initially uniform temperature would be maintained. The value of $S$ is based on the initial radiation loss rate and is maintained at that value throughout the calculation.

Figure 8 shows the velocity at the center of the system as a function of time. As we have seen from the linear analysis, three wave modes are present. The oscillatory sound waves are imposed on a growing exponential mode. From a least-squares analysis of the data in this figure, $\gamma_1$, $\gamma_2$, and $\omega_2$ may be determined. It is easiest to find $\omega_2$ and $\gamma_2$, which correspond to a period of 69.1–69.3 seconds and a damping rate of 263–264 seconds compared to the linear prediction of 69.2 and 264 seconds. It is more difficult to find $\gamma_1$, the growth rate of the instability. Our best estimate of this number from the data is an exponentiation time of 142–147 seconds for the condensational mode, as compared to 146 seconds derived from the theory.

We have in fact observed that this growth rate is extremely sensitive to the input and accuracy of the code. In order to evaluate this quantity from the data generated by the simulation, we have developed a least-squares Newton-Raphson iteration technique to evaluate $\gamma_1$, $\gamma_2$, and $\omega_2$ simultaneously. The general insensitivity of $\gamma_2$ and $\omega_2$ to small variations in density and the radiation function is consistent with what we see in Figure 5. The $\gamma_1$ curve is steeper, and we expect, for example, even more sensitivity at $\rho/\rho_0 = 20$ than $\rho/\rho_0 = 28$.

Figure 9 shows graphs of temperature as a function of position for several times in the simulation. Initially the temperature is constant; the perturbation is in the form of a sinusoidal velocity deviation about $v = 0$. As time passes, we note structure developing in the temperature as energy is transferred from the hotter regions to the colder, denser regions where it radiates away quickly. Finally, we see the formation of a structure which resembles a transition region, and a redistribution of temperature and density throughout the system in which the pressure has remained essentially constant. The system is now bifurcated with one region having a high temperature and low density and the other having a low temperature and high density. Since the heating function $S$ is constant in time and uniform in space, more heat is deposited in the large, high-temperature region than in the low-temperature region. Radiative loss occurs predominantly in the cold, high-density regions, thus the deposited energy must be transported by thermal conduction down the gradient just as it is from the corona to the chromosphere through the transition region. Figure 10 shows the detailed structure of the end-state temperature profile from the simulation described above.

The cool, condensed region, which we call the "condensation," is a stable structure that is a couple of orders of magnitude denser than its surroundings and is much like a chromosphere. On each side of it is a coronal region at constant pressure and with a constant energy input per unit volume. Thus, the regions to the left and right of the condensation shown in Figure 9 have a different amount of energy being deposited simply due to the difference in their sizes. Neither of the hot regions is able to radiate the energy being deposited. Thus, the structures continue to evolve seeking to attain a...
In the final state of the calculation, there are two transition regions, one at each wall, and they just sit there in an equilibrium. The high-temperature state has $T_e \approx 2 \times 10^6$ K and $\rho \approx 8.5 \times 10^{-15}$ g cm$^{-3}$, and the low-temperature state has $T_e \approx 18,000$ K and $\rho \approx 8.5 \times 10^{-13}$ g cm$^{-3}$. When a linear stability analysis is performed for these end states, we find that they are in fact stable. The extra factor of 28 in initial density which was added over the standard coronal values has condensed out leaving a corona-like hot plasma connected to a cooler, denser chromosphere-like region. It should be noted that the temperatures of the condensations are low enough that effects due to recombination and radiative transfer may become important. Thus the assumptions of a fully ionized plasma and an optically thin radiation loss rate used in the model equations begin to break down.

We have also performed calculations in which the system size was initialized to be $2.5 \lambda^*$ and $0.5 \lambda^*$. In the former case, there are more initial oscillations in temperature at early times. However, the final state looks exactly the same as the $1.5 \lambda^*$ case. This is also the final state of the $0.5 \lambda^*$ case. There is thus a demonstrated tendency for the lowest mode to dominate once the nonlinear characteristics of the system begin to dominate.

We observe that the nonlinear evolution of the condensational instability is a bifurcated system. The dense, cool condensation then moves subject to forces in the system. In our case, the force is a differential heat input; a gravitational force could act similarly. If heating is uniform and there is no gravity, as in these calculations, small displacements of the uniformly spaced dense condensations from the equilibrium position cause a destabilizing force because of uneven heat distribution. Two condensations, which are slightly closer together than the equilibrium situation, are accelerated toward each other in a secondary coalescence instability. In the case of Figure 9, the condensation about one-third of the way across the system is closer to its image in the left wall than it is to the condensation against the right wall. The situation is shown schematically in the upper right corner of Figure 11. The position of the center of this condensation is plotted versus time, and an equilibrium position at $L/3$
is indicated in the figure. The plot begins at 1000 seconds into the calculation of the condensation instability shown in Figure 8. The effective mass of the condensation in these calculations is $2.35 \times 10^{-5}$ g cm$^{-2}$. Figure 12 shows the displacement of this condensation from the equilibrium position plotted as a function of time. The displacement is increasing exponentially with a characteristic time of about 282 seconds.

It is clear from these calculations that it is possible for the condensation to sit stably in a hot plasma without evaporating. The bifurcated system is unstable to the progressive coalescence of condensations but dynamically stable to their existence. This was tested directly in a number of simulations. First, a condensation of large mass was put in a hot plasma whose temperature and density were determined by the simulation shown in Figure 9. The condensation was found to be stable and moved to the wall, as shown in Figure 13. Different, linearly unstable, initial conditions were tested, and, in all cases, a condensation was formed and its motion accelerated. When the two sides of the condensation were initialized at different temperatures and densities (pressure remaining constant), the lower temperature side became hotter until radiation balance was attained, and then the condensation was accelerated.

This coalescence of condensations is a manifestation on stellar scales of the ablative acceleration phenomenon which occurs in laser fusion (Orens 1980; McMahon et al. 1980; Felber 1977; Max, McKee, and Mead 1979, 1980). In this case, an incident laser beam is focused on a small, solid pellet of fusionable material. The energy deposited by the beam causes a high-temperature, dense plasma to be formed at the pellet's surface which then blows away. In fact, almost all of the incident energy is converted to outflowing kinetic energy. The reaction force to this rapid blow off of hot plasma accelerates the payload cold material. In the regime of solar parameters, the longer time scale permits radiation to balance conduction, thus reducing ablation of the cold material. This reduction is proportional to the differential heating and is better described on the Sun as evaporation.

When condensations accelerate and coalesce, only a small fraction of the incident energy goes into acceler-
tion. Most of the deposited energy is still being radiated. The equations governing this acceleration are

\[ \frac{dz}{dt} = V, \]

\[ M \frac{dV}{dt} = \Lambda \frac{de}{dt} \left( \frac{3z - L}{2} \right), \]

where \( z(t) \) is the instantaneous location of the condensation and \( V(t) \) is its velocity. The term \( \frac{de}{dt} \) is the energy deposition (ergs cm\(^{-3}\) s\(^{-1}\)) and \( \Lambda \), which has units of an inverse velocity, is a measure of the force generated by differential energy deposition.

Figure 14 shows the displacement versus time for two more numerical experiments with condensations. Again, the heavier condensation is treated by initializing an isobaric bifurcated state with the hot material at the final temperature and pressure of the previous condensation instability calculations. Only a single condensation was initialized, so the equilibrium position is at the system center. The upper curve was calculated for a condensation far from equilibrium and the lower curve for one near equilibrium. The predicted exponentiation rate,

\[ \alpha = \pm \sqrt{\frac{3\Lambda \frac{de}{dt}}{2M \frac{dV}{dt}}}, \]

depends on the condensation mass, overall energy deposition rate, and the coefficient of proportionality, \( \Lambda \). From the measured values of \( \alpha \) and \( M \) and the input energy deposition rate \( S = \frac{de}{dt} \), \( \Lambda \) can be determined qualitatively.

Craig and McClymont (1981) have recently investigated the formation of an active region coronal loop. Their conclusion that the loop will relax to a quasi-stationary equilibrium is in agreement with our calculations which show that the nonlinear evolution of the condensational instability leads to a dynamically stable structure. There are, however, areas where our calculations differ from theirs. Craig and McClymont are attempting to model an entire coronal loop, whereas we focus on a localized region of plasma with a scale comparable to the most unstable wavelength of the linear spectrum. They do not consider the linear theory or the linear condensational modes at all but rather disturb the system with a large (nonlinear) perturbation which provides both the necessary heating to reach the radiatively unstable regime and the spatial perturbation leading to a bifurcation of the plasma. Therefore, they are unable to compare with the linear theory. It is not clear to us that, on the Sun, it is possible to force the coronal plasma into such an unstable condition. Our results, which start with somewhat more idealized conditions, suggest that dynamic readjustment of the nonuniform loop plasma would prevent it from attaining such a truly unstable situation in the first place.

Finally, the Craig and McClymont (1981) formulation of the equations for the problem is somewhat different from ours. Their formulation is not energy conserving, so the accurate treatment of any shocks which might develop is impossible. They also use a heating source term which is assumed to be constant per unit mass, while we use one that is constant per unit volume. When the density changes in their model, the heating function follows it. This means that more energy is deposited in high-density regions, where radiation is stronger. The effects of radiation are therefore stronger, and the role of thermal conductivity is correspondingly weakened. Even the linear stability is changed.

V. CONCLUSION

In summary, we have used both linear analysis and nonlinear time-dependent numerical simulation to investigate the stability of plasma at temperatures and densities typical of solar transition region and coronal plasmas. We have found that, in the linear regime, the detailed simulations produce growth rates, decay rates, and frequencies which are in excellent agreement with the prediction of the linear analysis. In the nonlinear regime, the numerical solutions show a bifurcation of the plasma into a cooler, dense "condensation" surrounded by a hotter, tenuous corona. The condensation is then accelerated in one direction if it is "pushed" in any way, i.e., if there is differential heating on one side or if there are gravitational forces. A number of different scenarios were used to initialize the numerical model. However, if the linear analysis predicted instability, the qualitative results were always the same: a condensation was formed and then accelerated. Further sets of calculations in which the initial conditions consisted of condensations of various masses in a hot, tenuous plasma always showed results consistent with the above description. The condensation retained its integrity without evaporating.

The bifurcated condensed state which results naturally as the end product of the condensational instability is characterized by a dynamic balance between energy deposition, thermal conduction, and radiation. The transition region structure is the end result of condensation,
not the beginning point. The purpose of testing an empirical atmosphere for local condensational instability is not to determine if the structure has to be dynamic rather than static but rather to determine if the laminar profiles inferred are a superposition of distinct filaments or if they are consistent with a single atmosphere.

The results presented above also lead to interesting speculation concerning spicules, bifurcations in the interstellar plasma, and the recent observations of mass ejections in the corona. Spicules could start out as a product of a condensation. The heating or magnetic acceleration would cause the observed upward motion (Beckers 1972). Hildner et al. (1980) have observed coronal mass ejections, which appear to be cold, dense material moving through hotter, tenuous material. Finally, the condensational instability could cause the observed H I condensations in the interstellar medium (e.g., Dalgarno and McCray 1972).

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