The differences in the phase of the velocity oscillations between a pair of chromospheric Ca II lines was measured using the Vacuum Tower Telescope at the Sacramento Peak Observatory. The observed phase differences indicate that the acoustic modes are trapped or evanescent, rather than propagating, in the chromosphere. We find systematic distinctions in the phase delays between quiet network and cell interior regions for both intensity and velocity oscillations in photospheric and chromospheric lines. The theory of linear perturbations in an isothermal atmosphere is invoked to interpret these differences. From this analysis we find that one or more of the following explanations is possible: (1) the radiative damping is more effective in the network than in the cell interior; (2) the network features exclude oscillations of large horizontal wavenumber; or (3) the scale height of the chromosphere is larger in the network than in the cell interior.

Subject headings: Sun: atmosphere — Sun: atmospheric motions — Sun: chromosphere — Sun: sunspots

I. INTRODUCTION

The first paper in this series (Lites and Chipman 1979, hereafter Paper I) reported on observations which clearly demonstrated the vertical propagation characteristics of the solar “5 minute” oscillations. In this paper we report on continued analysis of those data, as well as on new data obtained by one of us (O. R. W.). This new experiment was designed to measure the phase delays of the oscillations between a pair of the Ca II infrared triplet lines, both of which are thought to be formed in the chromosphere. The observations, data reduction, and theoretical analysis reported in this paper address two questions concerning the solar oscillations: (1) Can the propagation characteristics of the oscillations in the chromosphere be determined from ground-based observations of optically thick chromospheric lines? and (2) What distinctions may be made in the phase delays of the oscillations between the supergranular network and the cell interior?

The phase delay of the solar 5 minute oscillations between two heights in the chromosphere gives a good indication of the phase velocity of disturbances. The waves are evanescent if the phase velocity is highly supersonic; and if the phase velocity is near the sound speed, one may be fairly confident that the waves are propagating energy as traveling acoustic disturbances. Most of the studies addressing this question with ground-based observations have employed the Ca II resonance lines and infrared triplet lines, and nearly all these studies report very high phase velocities in the chromosphere (Mein 1971; Mein and Mein 1976; Cram 1978; Mein and Schmieder 1980). Indeed, the phase velocity of acoustic waves with frequencies above the acoustic cutoff frequency, noted in Paper I for levels in the photosphere and temperature minimum region, appears to increase as one looks at higher levels in the chromosphere (Schmieder 1979). Spacecraft measurements of the phase delays between oscillations in the optically thick Ca II and Mg II resonance lines also indicate very small phase delays, and hence little propagation of acoustic energy (Artzner 1979). It has been suggested that these small observed phase delays among chromospheric lines may result from reflection at the chromosphere-corona transition region (Mein and Mein 1976; Provost and Mein 1979).

Radiative transfer effects in optically thick absorption lines can confuse the interpretation of the velocity and intensity fluctuations in the atmosphere. Much of the uncertainty in the phase of the oscillations can be removed through measurement of Doppler shifts in
effectively thin chromospheric emission lines rather than in the optically thick and often self-reversed absorption lines employed in the studies mentioned above. The few investigations made with OSO 8 that specifically address this problem give conflicting results. Chipman (1977, 1978) found only small differences in the phase of the intensity oscillations between the ultraviolet continuum formed near the temperature minimum and ultraviolet chromospheric emission lines. This result is in agreement with the evanescent character of the oscillations near periods of 3 to 5 minutes, as was verified by ground-based observations (Paper I). Chipman (1978) also found possible evidence for downward propagation of disturbances from the transition region to the temperature minimum. Contradictory results arise from two other analyses of the Si II and C IV OSO 8 data (White and Athay 1979; Athay and White 1979). These analyses of an extensive data base led to the conclusion that waves are propagating vertically in the chromosphere with phase velocities nearly equal to the sound speed throughout most of the 2.5–10 mHz band.

Faced with the conflict between ground based and OSO 8 measurements, we have undertaken an observational program to measure the phase delays between a pair of the chromospheric Ca II infrared triplet lines with increased precision using the improved ground-based observational technique described in Paper I. This decision was motivated by the great improvement of the phase spectra presented in Paper I over the results of the previous analyses. The pair of lines chosen are well suited to the study, since they are members of the same multiplet in the Ca II ion. They have nearly a factor of 9 difference in line opacity and hence should be separated by about 300 km in height of formation in the chromosphere. Their wavelengths also occur near the peak of the response of the photodiodes which make them advantageous lines for photoelectric observations.

In the other aspect of this study, we have analyzed the data from Paper I along with the new dataset obtained for the study described above to search for differences in the phase of the oscillations between cell interior and network structures. Very few studies have addressed this aspect of the solar oscillations. Mein and Mein (1976) separated their analysis of phase delays among chromospheric lines into cell center, network, and regions intermediate between these two extremes. The distinctions between these seem inconclusive, and it is possible that they did not have the observational accuracy to detect the small phase differences that we report in this paper. In an earlier paper, Mein (1971) noted a larger phase lag between velocity oscillations in a photospheric Fe I line and in the Ca II 8498 Å line for brighter regions in the Ca II line. The amount of the increase is about 9° throughout the evanescent range: 2 < v < 5 mHz. Our approach is to form separate phase diagrams, obtained in a similar manner as those in Paper I (for cell center and network features). We form a difference between the cell center and network diagrams to investigate the distinctions in the phase lags between these features. We then use a first-order theory to interpret the differences in the solar atmosphere and the wave propagation characteristics that could give rise to the observed phases.

## II. Observations

We present data obtained with the vacuum tower telescope, echelle spectrograph, and linear diode array on 1977 December 14 (dataset 1—one of the two datasets described in Paper I) and 1978 October 29 (dataset 2). The observational setup and experiment execution are described in Paper I, and the setup for dataset 2 is basically the same. Table 1 presents the observational parameters for dataset 2, which was obtained at disk center. The area scanned did not include regions of solar activity as determined by Ca II K spectroheliograms taken on that date. Dataset 2 differs from dataset 1 by the following changes: (1) the substitution of the second chromospheric line, Ca II 8542 Å, for the temperature minimum line Mg I 5173 Å; (2) narrower exit slits for the Ca II lines in order to limit the breadth of the line contribution functions; (3) exit slits placed somewhat closer to the center of the Ca II lines in order to sample higher in the chromosphere; and (4) a more rapid cadence of the areal scan: each point was sampled every 30 s. The area scanned was 128° × 128° with a resolution of 2°. A series of 256 scans obtained between 1527 and 1735 UT was analyzed. Stable seeing persisted with the image degradation estimated at 1–2° throughout most of the observing period. Some periods of ~3° seeing occurred during the last 20 minutes of the observing period.

### III. Data Reduction

We follow a data reduction procedure similar to that outlined in Paper I. We noted in Paper I that a differential offset of the diodes from line to line caused a degradation of the velocity–velocity (V−V) phase delays involving the Mg I line. Before forming the time series of the data used in the present work, we made a cross-correlation analysis of the images in the lines and then corrected the time series for relative shifts in the diodes.

The digital images of the area scanned by the diodes recording the Ca II lines clearly show the calcium network. As a criterion for location of the network and cell center features, we define the upper one-third of the 8498 intensity range as network and the lower one-third of the range as cell interior. For dataset 1, network represents 9.3% of the image area and cell interior represents 36.1%. The corresponding percentages for dataset 2 are 9.6% and 39.2%, respectively. We have...
IV. PHASE DELAYS OF THE CHROMOSPHERIC OSCILLATIONS

Figure 1 presents the Ca II 8498 Å – 8542 Å V–V phase diagram obtained from Doppler shifts derived from dataset 2. Throughout the entire range of 2–10 mHz, the solar oscillations appear to be in phase. If waves were propagating upward between the heights of formation of the two lines (separated by about 300 km—Paper I), one would expect the phase difference to increase at frequencies above 5 mHz, yet we even find a slight decrease in phase between the frequencies 2–6 mHz. Our study confirms previous ground-based measurements and OSO 8 measurements using optically thick lines: the chromospheric oscillations appear to have highly supersonic phase velocities. The phase spectrum for Mg I–Ca II 8498 Å shown in Figure 4 is the improved version of Paper I, Figure 6, where we have corrected for the differential offset of the diodes. In Figure 4 we can measure the phase delays; hence, we can use the data from this figure to check the mechanical energy flux derived in Paper I for the height of formation of the Ca II 8498 Å line. We obtain a mechanical flux of $2.35 \times 10^4$ ergs cm$^{-2}$ s$^{-1}$, using the phase diagram in Figure 4, as compared to $6.8 \times 10^4$ ergs cm$^{-2}$ s$^{-1}$ obtained from the Fe I–Ca II V–V phase diagram in Paper I. In this computation we have used both the same power spectrum for the Ca II line and the same density at the level of formation of Ca II as were used in Paper I. Our computation of mechanical flux given here differs from the previous calculation only in

Fig. 1.—Distribution function of phase differences between velocities measured in the lines $\lambda 8498$ and $\lambda 8542$ of Ca II as a function of frequency in mHz from dataset 2. The phase difference is a result of the subtraction of the $\lambda 8542$ phase of the velocity fluctuations from the corresponding $\lambda 8498$ phase. This diagram is called the $\lambda 8498 - \lambda 8542$ V–V phase diagram in the text. The darker regions of the diagram indicate higher concentration of the product of the oscillatory power present in both of the lines, and the phase distribution is normalized to unity at each frequency. The phase diagrams presented here are produced in a manner identical to those of Paper I except that the frequency scale is displayed in mHz rather than rad s$^{-1}$. 

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The Fe i 5576 – Mg i 5173 $\nu - \nu'$ phase diagram is presented on the left, and the corresponding “difference phase diagram” (cell center diagram minus network diagram) is on the right. Therefore, in the diagram on the right, lighter shading indicates a higher concentration of power at the value of that phase and frequency in the network than in the cell interior, and darker shadings correspond to a higher concentration of power in the cell than in the network. The diagrams are produced from dataset 1.
Fig. 3—Phase diagram and corresponding difference phase diagram from dataset 1 for Fe i 15376–Ca ii 3498 V. See Figs. 1 and 2 for an explanation.
Fig. 4.—Phase diagram and corresponding phase diagram from dataset 1 for Mg I 5173 – Ca II 8542. $\nu - \nu'$. See Figs. 1 and 2 for an explanation.
Fig. 5.—Phase diagram and corresponding difference phase diagram from dataset 1 for Fe I λ5576 $V-I$. In this figure the phase of the intensity fluctuations was subtracted from the phase of the velocity fluctuations. See Figs. 1 and 2 for an explanation.
Fig. 6.—Phase diagram and corresponding difference phase diagram from dataset 1 for Mg $\lambda$ 5173 $V-I$. See Figs. 1 and 2 for an explanation.
Fig. 7.—Phase diagram and corresponding difference phase diagram from dataset 2 for Fe I $\lambda 5576$ – Ca II $\lambda 8542$ $V - V$. See Figs. 1 and 2 for an explanation.
Fig. 8. — Phase diagram and corresponding difference phase diagram from dataset 2 for Ca II 8498 Å V − I. See Figs. 1 and 2 for an explanation.
Fig. 9.—Phase diagram and corresponding difference phase diagram from dataset 2 for Ca ii 28542 $V - I$. See Figs. 1 and 2 for an explanation.
that we use the difference in the height of formation of the Mg I and Ca II lines given in Paper I, and in that we use the phase velocities derived from Figure 4. Errors in our estimates of height of formation cannot possibly explain the threefold difference in the mechanical flux computations. Evidently, waves above the photospheric acoustic cutoff frequency (~5 mHz), and hence also above the chromospheric cutoff frequency, may propagate energy in the photosphere, but our observed phase relationships indicate that the oscillations do not propagate much energy in the chromosphere at any frequency. Therefore, our results are consistent with Schmieder’s (1979) conclusion.

A search for any sign of phase coherence of high frequency acoustic waves above 10 mHz was another objective of the observational study that resulted in dataset 2. The increased cadence of the observations in dataset 2 extended our frequency coverage beyond 16 mHz. A comparison of the phase spectrum in Figure 3, obtained from dataset 1, to that of Figure 7 from dataset 2, shows that the power in the observed signature of the solar oscillations at frequencies higher than 10 mHz is too low to measure phases accurately.

V. THE CHROMOSPHERIC VELOCITY—INTENSITY PHASE SPECTRA

In Paper I we discussed the difficulties encountered in trying to understand the phase relationship between the velocity and intensity (V—I) oscillations within a chromospheric line. For evanescent adiabatic oscillations, we would expect the phase lag on our diagrams to be —π/2, and for adiabatic acoustic waves propagating upward, we expect the magnitude of the phase lag to diminish at higher frequencies. It was shown in Paper I that the Ca II 28498 and Mg I 125173 lines departed from this expected behavior. In Figures 8 and 9 we present the V—I phase diagrams for both of the Ca II lines. The concentration of phases near 0 radians at frequencies above 5.5 mHz is less than that given by Mein (1971). The diode located in the 28542 array. We therefore concentrate our attention to the frequency range 0–10 mHz in these diagrams. It is quite surprising that the phase diagrams for these two lines, obtained simultaneously in as nearly identical fashion as possible, show such different appearance. The phase diagram for the more opaque line, 28542, behaves as one would expect for evanescent oscillations in the range 2–9 mHz, while for the 28498 line, the phase difference decreases from ~90° to ~135° over the same frequency range. Since the V—I phase diagram for this pair of lines shows very little phase difference, one must conclude that the distinction here lies in the measured intensities, or our measurement of the intensities in the two lines. This empirical fact strongly points to the need to understand fully the interaction of the wave disturbances with the medium, including the effects of radiative transfer, in order to properly interpret the chromospheric V—I phase spectra.

VI. THE OBSERVED DISTINCTIONS BETWEEN THE PHASE DIAGRAMS OF CELL INTERIOR AND NETWORK FEATURES

The right-hand images in Figures 2 through 9 present the difference of the phase diagrams (cell center minus network) corresponding to the average (cell + network) phase diagram on the left. In these representations, light is negative and dark is positive, so light areas on the right-hand image correspond to higher concentrations of oscillatory power in the network than in the cell at the indicated phase and frequency. The frequency resolution of the “difference phase diagrams” on the right is much coarser than the standard phase diagrams, since we have limited the length of the data string in the cell-network study to 32 measurements.

We note the following features of the V—V difference phase diagrams:

1. At frequencies between 2.5 and 5 mHz in the V—V difference phase diagrams of Figures 2, 3, 4, and 7, we note that the network phase differences lead the cell phase differences. The V—V difference phase diagram for 28498—28542 is not given here because it shows very little structure, yet there is some indication that it has a similar behavior at these frequencies. Table 2 compares the centroids of the phase distribution for cell and network in the frequency band centered near 4 mHz. We note that the shift in the phase distributions between the cell center and network features is at most 4°, a value much smaller than that given by Mein (1971).

2. We point out some indication in the data in Figures 2 and 7 of the network phase delay falling short of the cell phase delay at frequencies 5.5 < v < 7.5 mHz. This falls within the frequency range where the atmosphere admits propagation of acoustic waves. This trend is not nearly so evident in Figures 3 and 4, but it must be remembered that we are looking for very small effects at frequencies where the solar oscillatory power is very low.

3. There does seem to be some systematic cell-network effect at the very lowest frequencies of the λ5576—285173 V—V difference phase diagram in Figure 2. The phase relationship shown in Figure 2 is the best defined of all

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that we present and also reflects conditions at the lowest levels in the atmosphere. The strong vertical magnetic field structure in the network may alter obliquely propagating internal gravity waves, and hence change the phase characteristics at low frequencies. We discuss this in more detail below.

The \( V-I \) difference phase diagrams (Figs. 5, 6, 8, and 9) also indicate that the phase delays in the cell and network features are different. Table 2 also summarizes the cell-network differences near \( v = 4 \) mHz.

4. At the lowest levels of the atmosphere sampled, the difference phase diagram for \( \lambda 5576 \) \( V-I \) (Fig. 5) indicates that, when compared to the cell phases, the network oscillations in the range \( (2 \leq v \leq 5 \) mHz) lie somewhat closer to the \(-\pi/2\) phase delay expected for adiabatic evanescent oscillations.

5. The \( V-I \) phase diagrams for Mg I \( \lambda 5173 \), and Ca II \( \lambda 8498 \) in Figures 6 and 8 indicate that the velocity fluctuations in the evanescent frequency range \( (2 \leq v \leq 5 \) mHz) lag the intensity fluctuations by more than \( \pi/2 \). In the corresponding difference phase diagrams, we note that the network again attempts to pull the phases back toward \(-\pi/2\), but this time the direction of the change in phase is opposite to that in the photospheric \( \lambda 5576 \) Fe I line. This effect may not be simply related to increased adiabaticity in the chromospheric network because the phases overshoot \(-\pi/2\) in the \( \lambda 8498 \) line, and they even increase away from \(-\pi/2\) in the \( \lambda 8542 \) line as one goes from cell to network (Table 2).

Figures 6, 8, and 9 point to cell-network distinctions in the \( V-I \) phase diagrams at low and high frequencies. Here, as in the \( V-V \) diagrams at the same frequencies, the distinctions may not be solar in origin. The cell interior sample is about 4 times larger than the network sample, so the statistical noise in the cell phases will be only one-half that in the network. Systematic errors in the measurement process, such as image motion due to seeing and guiding, may affect the cell and network signals in different ways. The former error will generally cause either a correlation or an anticorrelation of measured velocity with intensity in bright features associated with steady flows. Since the chromospheric network is associated with down flows, one would expect the measurement process to introduce some tendency for brightenings to be associated with redshifts in the Mg I and Ca II \( V-I \) phase diagrams. This effect would cause the network phases to be preferentially shifted towards \( \pm \pi \). We point out just such an effect at high frequencies \((v > 10 \) mHz) in Figures 6, 8, and 9, and we also note that the cell-network differences for these lines in the evanescent range of frequencies is in the opposite sense of this systematic error, lending credence to our interpretation of the observed phase differences as solar in origin.

VII. IMPLICATIONS OF THE CELL-NETWORK PHASE DIFFERENTIAL

The phase diagrams in Figures 2-9 show small but consistent differences in the height dependence of the phase of the oscillations between cell interior and the network. This distinction in the phase between cell and network provides an opportunity for better understanding of the difference in atmospheric structure between the cell and network and also for an empirical understanding of the ways in which the network features modify the behavior of the solar nonradial oscillations. We adopt a first-order approach to the analysis of the phase of the oscillations by invoking the theory of linear perturbations in an isothermal atmosphere. This theory may fail to predict accurately both the amplitude of the oscillations and the amount of differential phase delay between a typical cell interior and network structures. It does, however, have a limited capability for predicting the sign of the local differential phase delay due to changes both in the atmosphere and in the propagation characteristics. We recognize another shortcoming of this analysis: the phase of the oscillation can be greatly changed by reflection and trapping due to nonisothermal effects. We feel these results are nevertheless interesting by themselves, and they point to an analysis now underway in which we relax the isothermal assumption. We avoid many of the complications of the theory introduced by radiative transfer effects by limiting our application of the linear isothermal hydrodynamics to interpretation of the phase difference between Doppler shifts sampled at two heights in the atmosphere and also by using a Newtonian cooling approximation in the wave equations.

We adopt the results of Souffrin (1966, 1972) for radiatively damped acoustic modes modified by gravity in a stratified isothermal atmosphere. Schmieder (1976) has also applied this type of analysis toward the interpretation of some aspects of phase delays in the solar atmosphere. This theory admits an analytical expression for the fluctuations in the vertical component of velocity \( v_z \) as a function of height \( z \):

\[
\Phi = \rho^{1/2} v_z \propto e^{i(p + iq)x} e^{i(\omega t + k \cdot x)} .
\]

Here, \( \rho \) is the density given for a static isothermal atmosphere, \( \omega = 2\pi v \) is the angular frequency of the oscillation in rad s\(^{-1}\), \( k \) is the horizontal wave vector for oscillations in the plane perpendicular to the vertical, and \( x \) is the displacement vector in that plane. In this sign convention, as well as in the data analysis, \( \omega \) is positive and phase increases with time. The magnitude of \( k \), which we denote as \( k \), is then the horizontal wavenumber. In this representation, \( p \) and \( q \) are real quantities given by

\[
(p + iq)^2 = a + ib ,
\]

where the real quantities \( a \) and \( b \) occur in the wave equation \( (d^2\Phi/dz^2) = (a + ib)\Phi \) that results from solution of the linearized hydrodynamic equations. Here,

\[
a = \frac{\omega^2 - \Omega^2}{c^2} + k^2 \left( \frac{N^2}{\omega^2} - 1 \right) - \frac{1}{1 + \omega^2 \sigma^2} \cdot \frac{N^2}{\sigma^2} \left( k^2 - \frac{\omega^2}{g^2} \right) .
\]
In (3) we have used the notations:

- **Acoustic cutoff frequency:**
  \[ \Omega = \frac{c}{2H}, \]

- **Brunt Väisälä frequency:**
  \[ N = \left( \frac{\gamma - 1}{\gamma} \right)^{1/2}, \]

where \( \gamma, \, \beta, \, \tau_R, \, \text{and} \, H \) are the ratio of specific heats, the local acceleration of gravity, the radiative relaxation time, and the scale height, respectively.

Equation (1) shows that the height dependence of the phase of the oscillation is given by \( \phi = \frac{p}{c} \), so we solve (2) for \( p \) in terms of the known quantities \( a \) and \( b \):

\[ p = \pm \left[ \frac{1}{2} \left( a + \left( a^2 + b^2 \right)^{1/2} \right) \right]^{1/2}. \]

We adopt the negative branch in (5), since we wish to consider upward propagating acoustic disturbances. In this case the gradient in \( \phi \) with respect to \( z \), \( d\phi/dz \), is always negative and is given by \( p \).

If we consider the atmosphere locally isothermal, we can determine if a local change in \( \tau_R \) or \( k \) steepens or flattens the phase gradient. We compute the partial derivatives \((\partial/\partial \tau_R)(dp/dk)\) and \((\partial/\partial k)(dp/dz)\) for the local conditions in a model solar atmosphere, and the algebraic sign of these derivatives at various heights indicates the direction of the effect on the phase of the velocity fluctuations. These derivatives are given in the analytic forms (gradients with respect to \( H \) will be useful later):

\[
\frac{\partial}{\partial (\tau_R, \, k, \, H)} \left( \frac{dp}{dz} \right) = \frac{\partial p}{\partial \tau_R} \left( \frac{d\phi}{dz} \right) \frac{1}{\theta(\tau_R, \, k, \, H)} \left[ 1 + \frac{a}{b^2 + b^2} \right]^{1/2},
\]

where

\[
\frac{\partial a}{\partial \tau_R} = \frac{2N^2(\tau_R^2)}{(1 + \omega^2)^2} \left( \frac{k^2 - \omega^2}{\beta^2} \right),
\]

\[
\frac{\partial b}{\partial \tau_R} = \frac{1 - \omega^2 \tau_R^2}{(1 + \omega^2)^2} \left( \frac{k^2 - \omega^2}{\beta^2} \right),
\]

\[
\frac{\partial a}{\partial k} = 2k \frac{N^2(\tau_R^2 - \omega^2 \tau_R^2 - 1)}{(1 + \omega^2)^2},
\]

\[
\frac{\partial b}{\partial k} = 2k \frac{N^2 \tau_R^2}{(1 + \omega^2)^2}.\]

In order to compute these derivatives we need a representation of the local conditions that includes an estimate of the radiative relaxation time as a function of height in the solar photosphere and chromosphere. We adopt the representation of the atmosphere given by Giovanelli (1978) which includes some new estimates of radiative loss rates in optically thick lines in the chromosphere. Because this representation has a rather sparse distribution of points with height, we have interpolated the values using cubic spline fits to the data. The interpolations were performed on the natural logarithm of the density (derived from the total hydrogen density). We have adopted constant values of \( g = 2.74 \times 10^4 \text{ cm s}^{-2} \) and \( \gamma = \frac{5}{3} \).

Figure 10 gives the height variation of the quantity \( \tau_R (\partial p/\partial \tau_R) \) for a horizontal wavenumber corresponding to a wavelength of \( 10^6 \text{ km} \). The curves represent this derivative at various frequencies throughout the evanescent and propagating acoustic gravity wave regimes. It is clear that at frequencies where we detect a cell-network differentiation in the phase delays (\( \omega \geq 0.02 \text{ s}^{-1}, \, v \approx 3 \text{ mHz} \)) this derivative is positive at all significant heights. Therefore, a decrease in the local value of \( \tau_R \) will cause a decrease in \( p \), which is already negative, resulting in a larger positive difference between the phases at two levels in the atmosphere.

If the radiative damping is the primary influence for the cell-network phase differential, then the data tells us that the radiative relaxation time is smaller in the network than in the cell at equivalent heights of formation of the lines. For \( \omega \geq 0.02 \) there is very little change in the curves of Figure 10 over a large range of horizontal wavelengths (5-50 x \( 10^3 \text{ km} \)), but the height versus \((\partial p/\partial \tau_R)\) relationship is very sensitive to the horizontal wavelength at small wavelengths and frequencies. This behavior is shown in Figure 11 for a frequency \( \omega = 0.01 \text{ s}^{-1} \) and for horizontal wavelengths of 5000 and \( 10^4 \text{ km} \). Such a behavior is to be expected, since internal gravity wave modes become important at the lower frequencies (Mihalas 1979).

The analogous derivative \( k (\partial p/\partial \tau_R) \) is presented in Figure 12 for the angular frequencies \( \omega = 0.02, 0.03, \) and \( 0.04 \text{ s}^{-1} \). These curves are shown for a horizontal wavelength of \( 2 \times 10^4 \text{ km} \). Here again, the derivative is positive at all heights and remains so for the decade range in horizontal wavelengths 5-50 x \( 10^3 \text{ km} \). Following the same line of reasoning given above for the radiative relaxation time, these curves would indicate that the network features restrict the wave modes to large horizontal wavelengths, or to small horizontal wavenumbers \( k \).

Figure 11 shows the behavior of the computed \( k (\partial p/\partial \tau_R) \) versus height at \( \omega = 0.01 \text{ s}^{-1} \) for horizontal wavelengths of \( 10^4 \) and \( 2 \times 10^4 \text{ km} \). A changeover occurs in the sign of the derivative from positive to negative as the horizontal wavenumber increases. There is some hint of this kind of
behavior in the sharpest of the $V-V$ phase relationships (Fig. 2), where the cell-network difference phase spectrum shows the possibility of reversal at the lowest frequencies. The magnitude of this effect would depend sensitively upon the power present in the gravity waves and upon the extent to which waves of large horizontal wavenumber are excluded from the network features, two parameters of the wave motions that are to some extent mutually exclusive, since gravity wave modes tend to have high horizontal wavenumbers.

A somewhat different approach must be adopted to investigate the differential effects in the phase upon changing the scale height. Whereas local changes in $\tau_R$ and $k$ can, in principle, be effected at a given height in the atmosphere without changing the heights of formation of the line profiles, local changes in the scale height cannot. If, as in the isothermal case, the atmosphere has a constant scale height, then that scale height defines a unique height $z_1 = z_1(H)$ where the line is formed. We assume that this height occurs at a fixed column mass per unit area $m_1$ above $z_1$, defined by

$$m_1 = \int_{z_1}^{\infty} \rho(z)dz,$$  \hspace{1cm} (8)

Fig. 10.—The quantity $\tau_R(\partial p/\partial z)$ is plotted as a function of height in the model atmosphere for four values of the angular frequency $\omega$ given in rad s$^{-1}$. The curves are shown for a specified value of the horizontal wavenumber corresponding to a wavelength of $10^4$ km.

Fig. 11.—The quantities $\tau_R(\partial p/\partial z)$ and $k(\partial p/\partial k)$ are plotted vs. height in the model atmosphere for a frequency of $\omega = 0.01$ rad s$^{-1}$ for the horizontal wavenumbers shown.
where \( \rho(z) \) is given by the isothermal atmosphere condition
\[
\rho(z) = \rho_0 e^{-(z-z_0)/H}. \quad (9)
\]
Here, \( \rho_0 \) is the density at some reference height \( z_0 \) in the atmosphere. Integrating (8) we find:
\[
m_1 = \rho_0 H e^{-(z_1-z_0)/H}, \quad (10)
\]
from which we derive \( z_1(H) \):
\[
z_1(H) = z_0 - H \ln \left( \frac{m_1}{\rho_0 H} \right). \quad (11)
\]
Thus,
\[
\frac{\partial z_1}{\partial H} = 1 + \frac{z_1 - z_0}{H}. \quad (12)
\]
The phase of the oscillation at \( z_1 \) is given by \( \phi = \rho z_1 \), so we may calculate the change in this phase with respect to the scale height:
\[
\frac{\partial \phi(z_1)}{\partial H} = z_1 \frac{\partial \rho}{\partial H} + \rho \frac{\partial z_1}{\partial H}. \quad (13)
\]
Using equation (12) this becomes:
\[
\frac{\partial \phi(z_1)}{\partial H} = z_1 \frac{\partial \rho}{\partial H} + \rho \left( \frac{z_1 - z_0}{H} + 1 \right), \quad (14)
\]
where \( (\partial \rho/\partial H) \) is given by equations 6 and 7. We assume that both the cell and network structures have identical physical conditions at the level of unit optical depth in the continuum at 5000 Å, and this roughly corresponds to the height \( z_0 = 0 \) in the model atmosphere of Giovanelli (1978). A measure of the sign of the differential phase delay between two levels of the atmosphere \( z_1 \) and \( z_2 \) would be given by
\[
\delta = \frac{d\phi(z_1)}{dH} - \frac{d\phi(z_2)}{dH}. \quad (14)
\]
When \( \delta \) is positive, the effect of increasing the scale height will be to increase the phase difference between the two heights in the atmosphere. To carry out this study we ensure that the model atmosphere reflects the isothermal condition by fixing \( H = 100 \) km. This is necessary, since we must actually compute differences between two heights of the atmosphere, not just local gradients. We select heights in the atmosphere from Paper I roughly the same as the heights of formation of the Fe I 5576 Å, Mg I 5173 Å, and Ca II 8498 Å lines: 155 km, 418 km, and 936 km, respectively. In Figure 13 we present the results in the form of curves of \( \delta \) versus \( \nu \) for all three combinations of \( z_1 \) and \( z_2 \). Throughout most of the evanescent region—2 \( \leq \nu \leq 5 \) mHz—\( \delta \) is positive for phase lags involving the chromospheric line. The observed phases would then imply that \( H \) is larger in the chromospheric network than in the interior of the supergranular cells, further implying that the network structures are somewhat compressed. Between 3.5 \( \leq \nu \leq 5.5 \) mHz, the curves in Figure 13 undergo a large positive excursion. This excursion occurs just above the acoustic cutoff frequency and is caused by the dominance of the first term on the right-hand side of equation 9 within this frequency range. Figure 13 shows relatively small phase gradients and even negative values near \( \nu = 3 \) mHz for the phase difference between the photosphere and temperature minimum.
From our analysis of the Ca II $\lambda 8498 - \lambda 8542$ $V - V$ phase lags we find no evidence for vertically propagating acoustic disturbances in the chromosphere. The chromospheric contribution functions calculated for these lines in Paper I indicate a minimum height differential of 300 km over the bandpass listed in Table 1, and so phase delays for acoustic waves should be comparable to those shown in Figure 2 for the $\lambda 5576 - \lambda 5173$ $V - V$ diagram. Since we do not find such an easily detectable signature in the phase spectrum, we lend further observational support to the theoretical prediction (Kahn 1961, 1962; and as discussed recently by Provost and Mein 1979) that acoustic waves may be trapped in the chromosphere by the rapid rise in temperature, that leads to a rapid rise of the sound velocity with height in the chromosphere-corona transition region. The $\lambda 8542$ $V - I$ phase spectrum taken by itself would also indicate that the waves are standing rather than propagating through the chromosphere.

One must not overlook the possibilities of other chromospheric effects that might influence the waves and their velocity signatures in spectral lines. These include radiative transfer effects resulting from velocity fluctuations which may alter the line profile shapes and thus change the observed phases of the velocities. The observed differences in the $\lambda 8498$ and $\lambda 8542$ $V - I$ phase diagrams (Figs. 8 and 9) point to the possibility of significant differences in the radiative transport between these two lines. Height dependences of the radiative relaxation time in the chromosphere caused, for example, by hydrogen ionization or resonance line emission may also influence the $V - I$ phase differences. We note that, according to the contribution functions of Paper I, Table 3A, the bandpass used for each of these lines samples quite different regions of the atmosphere. The sampling we have used in dataset 2 for the $\lambda 8498$ line profile covers the temperature minimum and low chromosphere (330–830 km), whereas the $\lambda 8542$ line profile is sampled well into the middle chromosphere (830–1200 km). The $\lambda 8498$ $V - I$ phase diagram does resemble the $V - I$ phase diagram for the temperature minimum line of Mg I (Fig. 6). The shape of the $V - I$ phase diagram may therefore result from conditions in various regions in the atmosphere rather than being purely a radiative transport effect. Resolution of this question awaits a more complete treatment of non-LTE line formation in media containing propagating waves. Other height dependent properties of the solar atmosphere that might influence the observed phases of both the velocity and intensity oscillations include chromospheric inhomogeneities and magnetic fields. The phase delays of the oscillations measured in the Si II line profiles using OSO 8 data (White and Athay 1979) remain in conflict with this study and with other ground-based observations. This discrepancy may only be resolved by future high resolution ultraviolet measurements of effectively thin chromospheric emission lines.

Our analysis of the cell-network phase spectra shows systematic changes in the phase delays of the velocities measured at two different heights of the atmosphere, and systematic changes in the phase difference between the velocity and intensity signals measured in one spectral line. Our first-order analysis of the $V - V$ phase spectra indicates that the network (1) undergoes a decrease in the characteristic radiative damping time $\tau_R$ at heights at and above the temperature minimum, (2) causes a decrease in the mean horizontal wavenumber $k$ of the waves, or (3) has a larger chromospheric scale height $H$ than the cell interior. Of course, the actual situation may be a combination of these effects, along with effects that do not appear in a linear isothermal analysis.
One could argue physically for the case of all three possibilities listed above. If the network emission is indeed confined to very narrow magnetic flux tubes, then the lateral transport of radiation will introduce an effective sink of photons for these structures. The lines and continua responsible for the radiative relaxation time will radiate energy away from the network structure at a greater rate than will the more homogeneous cell interior regions and hence reduce the characteristic radiative relaxation time. The difference in the atmospheric structure between cell and network structures can also influence the radiative relaxation time at the levels where the lines are formed, but the direction and magnitude of that effect may only be properly determined from very detailed radiative transfer computations of the type carried out by Vernazza, Avrett, and Loeser (1980).

Vertical magnetic flux tubes would allow only those wave modes with nearly vertical wave vectors (corresponding to external $p$-mode oscillations with small horizontal wavenumbers $k$) to propagate; hence, we would expect the network features to respond to waves smaller in mean $k$ than are present in the cell interior. This "guiding" of magnetoacoustic waves in flux tubes has been discussed by Scheuer and Thomas (1981) for chromospheric waves and Hollweg and Roberts (1981). This difference would lead to some of the observed features of the $V - V$ difference phase diagrams as outlined in the previous section. This line of reasoning would also lead us to expect some reduction in the power of the oscillations in the network as compared to the cell interior due to the selection of modes with small $k$ by the network. Table 3 presents the ratios of the mean power spectra in the network to that of the cell interior for dataset 1. This table shows that the power in the velocity excursions is indeed less in the network than in the cell at the temperature minimum and above (Mg I $\lambda$5173 and Ca II $\lambda$8498) where the magnetic pressure becomes important relative to the gas pressure. Note also that Table 3 shows the intensity fluctuations in all the lines to be comparable for the cell and network, with the exception of the chromospheric Ca II line. The intensity level in the Ca II line is much greater in the network line than in the cell at the temperature minimum, so a fractional change in the intensity due to a passing acoustic disturbance would cause a larger peak in the intensity power spectrum in the network than it would in the cell interior. The cell-network differences in the power spectra support the viewpoint that we detect a reduction in the mean value of $k$ in the network, but they are by no means conclusive evidence. The difference in the atmospheric structure between the network and the cell interior could influence the power spectra as well.

Finally, our theoretical arguments presented in the previous section indicate that an increase in the scale height $H$ would produce many of the features of the difference phase diagrams at chromospheric heights, but not for phase lags between photospheric lines. The fact that the photosphere-temperature minimum difference phase diagram (Fig. 2) clearly shows higher phase lags in the network than cell interior would indicate that changes in the scale height are not responsible for this observed effect below the temperature minimum. Semiempirical models of the solar atmosphere predict a compression of the upper chromosphere in the network due to the increased coronal pressure (Vernazza, Avrett, and Loeser 1980). This leads to an increase in the scale height in the upper chromospheric layers. These models also predict higher temperatures in the network, which would also indicate that $H$ increases as one moves from cell to network. The semiempirical models do not incorporate the effect of magnetic fields in the network on the hydrostatic pressure balance. If we let $P_i$, $P_0$, and $P_B$ be the gas pressure internal to the flux tube (network), the gas pressure in the field free region external to the flux tube (cell), and the magnetic pressure, respectively, then lateral pressure balance gives us

$$P_i + P_B = P_0 \ .$$

Using the definition of the scale height,

$$H = - \left( \frac{1}{\rho} \frac{dp}{dz} \right)^{-1},$$

and assuming the atmosphere is locally isothermal, we derive the ratios of the scale heights outside and inside of the flux tube by taking the derivative of (11) with respect to height $z$. We then use the ideal gas equation of state

$$P_{0,0} = P_{0,0} k_B T_{0,0},$$

where $k_B$ is the Boltzmann constant, to obtain

$$H_0 = \frac{\rho_0 T_0}{\rho_i T_i} + \frac{H_0 dP_B}{P_i dz}. \ \ (12)$$

If the magnetic pressure $P_B$ dominates over the gas pressure inside the flux tube, then the density inside, $\rho_i$, will be much smaller than the external density $\rho_0$. In this case the small fractional increase in temperature inside the flux tube will not counterbalance the density differential, and the first term on the right of equation 12 will lead to a reduction in $H_i$ with respect to $H_0$. One expects the magnetic field, hence the magnetic pressure, to decrease with increasing height, so the second term on the right in equation (12) will have the effect of increasing $H_i$ with respect to $H_0$. It is clear that the behavior of the magnetic

<table>
<thead>
<tr>
<th>Line</th>
<th>$\lambda$5576V</th>
<th>$\lambda$5173V</th>
<th>$\lambda$8498V</th>
<th>$\lambda$5576I</th>
<th>$\lambda$5173I</th>
<th>$\lambda$8498I</th>
<th>Continuum I</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{network}}$</td>
<td>0.99</td>
<td>0.86</td>
<td>0.69</td>
<td>0.98</td>
<td>0.96</td>
<td>1.33</td>
<td>0.96</td>
</tr>
</tbody>
</table>
field strength in the flux tubes that comprise the chromospheric network can influence the scale height, and it is conceivable that this effect could cause a reduction of the scale height in the network that would produce the phase differences opposite to that seen in the observations.

This discussion has shown that we can interpret the direction of some of the cell-network phase delay differences using a first-order hydrodynamic analysis. It is likely that the calculation of the wave modes in a nonisothermal model atmosphere will point to which, if any, of the possibilities discussed above are viable processes for producing the observed degree of the phase differences. One effect in the nonisothermal atmosphere suggested by Spruit (1980) would be a higher degree of tunneling of the waves through the temperature minimum region into the chromosphere because of the higher temperatures found in the network features. Any increase in the tunneling of the waves would result in more phase difference in the velocity fluctuations at two levels of the atmosphere, as we have observed. It is therefore important to follow up this study with computations of the wave hydrodynamics in nonisothermal model cell interior and network atmospheres.

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