ENERGY BALANCE IN SOLAR AND STELLAR CORONAE

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The methods by which observed EUV and X-ray fluxes may be used to derive models of stellar chromospheres and coronae are discussed. The importance of measuring the electron density through spectroscopic techniques is stressed.

Energy may be lost from each region of the atmosphere by radiation, conduction and mass motions. The radiation losses can be calculated from the emission measure distribution derived from line fluxes and the theoretical radiative power function, without knowledge of the temperature and density structure of the atmosphere.

The pressure is required in order to calculate the energy transferred by thermal conduction and mass motions. It is also needed if comparisons are to be made between the energy fluxes implied by observed non-thermal motions and those which could be carried by particular wave processes.

Some examples of the results obtained through applying the methods to particular stars are given. The broad conclusions which may be drawn so far are that main sequence stars have hot coronae with thermal conduction an important energy transfer process, whereas giants and supergiants may have either hot or cool coronae, depending mainly on the surface gravity and the presence of a strong stellar wind.

1. INTRODUCTION

The main aims of current work on stellar chromospheres and coronae are to understand the processes by which these atmospheres are heated, and to find how these processes depend on conditions in the sub-photospheric convective zones. In short - why do some stars have hot coronae but not others? Observations of the EUV and X-ray spectra of stars are now being carried out with these goals in mind.

The present discussion will be concerned with (a) the methods by which the terms of the energy balance equation may be investigated and (b) results available so far for a range of late-type stars. Many of the methods have been developed in the context of the solar atmosphere and are well-known to solar physicists, but they are included to give a systematic account, beginning with observation of emission line fluxes and profiles.

The term 'corona' will be interpreted to include any region where the emission lines are effectively thin and are excited predominantly by electron collisions or recombination. Avrett has discussed the more complex problems of stellar chromospheres in the previous lecture.

Section 2 gives the methods by which the temperature and density structure can be found. Since it is essential to measure the electron density or pressure a range of possible diagnostic techniques is discussed.

The processes by which energy is lost from the atmosphere, or transferred within it are the subject of Section 3. Mass loss is only briefly mentioned since it is discussed in other lectures later in this volume.

In Section 4 the methods are applied to a range of stars, including the Sun, to show how the relative importance of different processes changes with spectral type and luminosity class.

2. DERIVATION OF STRUCTURE FROM OBSERVED LINE FLUXES

(a) The Emission Measure Distribution

The methods used for finding the emission measure distribution, and relative abundances in the solar atmosphere were pioneered by Pottasch(1).

Each emission line flux observed at the earth, $F_\nu$, can be expressed as
\[ F_\phi = \frac{1}{2} \hbar c \int \frac{N_e^2}{A_{21}} dV/4\pi d^2\lambda. \] (1)

where $\hbar c/\lambda$ is the quantum energy, $N_2$ is the population density of the excited level, $A_{21}$ is the spontaneous transition probability, $d$ is the distance to the star, and the factor $\frac{1}{2}$ is an approximation to the fraction of photons emitted in the outwards direction.

It is assumed that the line is optically thin. Provided no other decay process competes with spontaneous decay in the line, equation (1) is also valid for lines which have optical depth, $\tau$, greater than unity. The opacity will affect the line profile but not the total flux.

In statistical equilibrium $N_2A_{21}$ can be expressed in terms of the excitation processes. In main-sequence stars the lines from ions are predominantly excited through electron collisions, but recombination (and radiative processes) may be important for lines of neutral atoms. In giant and supergiant stars recombination and other processes may be relatively more important.

Using collisional excitation as an illustration,

\[ N_2A_{21} = C_{12}N_eN_1 \] (2)

where $C_{12}$ is the collisional excitation rate, $N_e$ is the electron density, $N_1$ is the lower level (usually the ground state) population. Statistical equilibrium should be valid, except perhaps for a few metastable levels, although ionization equilibrium may not be achieved (see below). Wherever the lower level population itself depends on density some iteration using approximate values of $N_e$ may be necessary.

Expressing the collisional excitation rate in terms of an average collisional strength, $\Omega_{12}$, (2) gives

\[ C_{12} = 8.6 \times 10^{-6} \Omega_{12} \exp(-\bar{W}_{12}/kT_e)/T_e^{1/2} \omega_1 \] (3)

where $\omega_1$ is the statistical weight of the lower level and $W_{12}$ is the excitation energy.

$N_1$ is expressed in terms of other quantities which can be
calculated or measured,

$$N_1 = \frac{N_1}{N_{\text{ion}}} \cdot \frac{N_{\text{ion}}}{N_E} \cdot \frac{N_E}{N_H}$$  \hspace{1cm} (4)$$

where $N_1/N_{\text{ion}}$ must be estimated, but for ions without metastable levels will be $\approx 1.0$; $N_{\text{ion}}/N_E$ is the fractional population of the ion, which can be calculated simply if ionization equilibrium is appropriate, $N_E/N_H$ is the element abundance. If $T_e \geq 2 \times 10^4 K$, $N_H = 0.8 N_e$; below this temperature $N_e/N_H$ should be calculated as part of the iterative modelling procedure. For stars it cannot of course be automatically assumed that abundances are solar, but if these are adopted, abundance anomalies will be revealed in the resulting emission measure distribution.

Regarding ionization balance, Brown et al. (3) have pointed out that in the cool, low density chromospheres of giants and supergiants the times for ionization and recombination are long. In the presence of mass motions and temperature gradients ionization equilibrium may not be a valid approximation. Dupree et al. (4) have also discussed non-equilibrium in the context of the solar transition region.

Putting the temperature dependent parts of equations (3) and (4) as

$$g(T_e) = T_e^{-\frac{1}{2}} \exp(-W_{\text{ion}}/kT_e) \frac{N_{\text{ion}}}{N_E}$$  \hspace{1cm} (5)$$

and assuming, initially, that there is a spherically symmetric uniform atmosphere, gives for the line flux, at the star,

$$F_* = \frac{hc}{\lambda} \cdot 8.6 \times 10^{-6} \cdot \frac{\Omega_{12}}{\omega_{l}} \cdot \frac{N_E}{N_H} \int_{\Delta h} N_e^2 g(T) \, dh$$  \hspace{1cm} (6)$$

where $F_* = F_0 (2/\theta)^2$

and $\theta$ is the stellar angular diameter in radians. $\Delta h$ is the region over which a line is predominantly formed. The effects of a non-uniform surface distribution are discussed below.

Since $\int_{\Delta h} N_e^2 g(T) \, dh$ may vary rapidly with $T_e$, particularly in the chromosphere, the common procedure of replacing $g(T)$ by an average value and removing it from the integral should be examined. Several approaches are possible (eg. 1,5,6). Following
Jordan and Wilson (5), each $g(T)$ function is computed over a wide range of temperature, say that over which $g(T)$ has decreased by two orders of magnitude from its peak value; the percentage of the area contained by $\log T_m \pm 0.15$ is calculated and then the normalization factor required to approximate the region contained within $\log T_m \pm 0.15$ by a constant $g(T)$ is found. $T_m$ is the temperature of the peak of the $g(T)$ function. Then

$$\frac{\log T_2}{\log T_1} \int g(T) \, d \log T = G \cdot g(T_m) \quad (7)$$

and $G$ is the combined normalization constant. $T_1$ and $T_2$ are the temperatures well outside $\log T_m \pm 0.15$. It is important to adopt a constant temperature width rather than a constant fraction of $g(T_m)$ otherwise the different shapes of the $g(T)$ curves will lead to errors in relative abundances. Once the initial emission measure distribution is found, taking $G \cdot g(T_m)$ outside the integral, an iteration should be performed to take into account the variation of $\int N_e^2 \, dh$. Alternatively $\int N_e^2 \, dh$ can be plotted as a function of $T_e$, using the actual values of $g(T)$. These loci of the value of $\int N_e^2 \, dh$ sufficient to produce the observed line emission place strong constraints on the acceptable emission measure distribution since the final function must be single valued at each $T_e$. An example of the resulting distribution of $\int N_e^2 \, dh$ with temperature, for Procyon ($\alpha$ CMi, F5 IV-V) $\Delta T_e$ is shown in Figure 1, taken from Brown and Jordan (7). Such distributions form the starting point of further modelling.

(b) Methods of Determining the Density and Pressure

The emission measure may be written as

$$E_m = \int \frac{N_e^2 \, dh}{\Delta h} \quad (8)$$

and expressed in terms of $P_e = N_e T_e$ and $dh/dT$ such that

$$E_m = \int \frac{P_e^2}{T_e^2} \cdot \left( \frac{dh}{dT} \right) \cdot dT \quad (9)$$
The emission measure distribution for Procyon (7) using fluxes from IUE spectra obtained at high and low resolution. Some data from the Copernicus satellite are included (22). The points for O I refer to excitation by collisions (c), recombination (r) and via Ly β. The full line is from the model by Ayres et al. (61).
In order to proceed beyond this point assumptions must be made. There is a variety of these in the literature, and a full review will not be given. Since the energy input function is the unknown quantity which is sought assumptions concerning this or the relative magnitudes of the radiative and conductive fluxes will be deliberately avoided. (See below also). Without such assumptions it is essential to know $P_{\text{E}}$ in order to find the conductive flux and to compare specific energy input processes with energy losses deduced from the observations.

The high fluxes observable from solar EUV and X-ray lines allow both good spatial and spectral resolution to be achieved. Also ratios of lines involving weak transitions can be measured accurately. Thus methods of measuring $N_e$ through density sensitive line ratios can be widely applied in solar physics. Several reviews of these diagnostic techniques have been given recently (8,9,10). Unfortunately many of the methods cannot be applied to the spectra of late-type stars now being obtained with the International Ultraviolet Explorer satellite (IUE) because the lines involved are too weak to observe at the high resolution required. Some exceptions are discussed below.

Ideally lines from the same ion, having different dependences on $N_e$, and little dependence on $T_e$, should be used. Pairs of lines from the ions C III and Si III are in principle suitable(11,13), depending on the range of $N_e$, but it is difficult with IUE to observe both components of the line pairs. A line from one ion can be compared with a line from a different stage of ionization, but this is clearly not ideal. Doschek et al. (12) and Cook and Nicolas (13) have discussed methods involving ratios of C III to Si III and C III to Si IV, in the context of IUE observations. However, in stars earlier than around G 8 the strength of the continuum above $\sim 1800$ A makes it difficult, or impossible, to observe the C III and Si III lines at 1909 A and 1892 A. In giants and supergiants later than around K 2 these lines are not present, the features observed at low resolution being due to lines of S I (14). Thus the method seems to be limited to main sequence stars later than $\sim$ K 0 and perhaps a few giants around K 0. (Some binary systems may have exceptional spectra, eg. HR 1099, G5 V, (15)).

For bright objects where high resolution observations are possible the density sensitive O IV lines around 1400 A could be resolved from the stronger Si IV doublet, but their absolute intensity is usually rather low.

In the late type giants and supergiants, Brown et al. (3) have pointed out that line ratios within the C II multiplet at $\sim 2335$ A can be used to measure $N_e$ in the range $10^7 - 10^9$ cm$^{-3}$. The necessary transition probabilities have been calculated by
Dankwört and Trefftz (16) and Jackson (17) has calculated the total collision stength. For example, application of this method to β Gru (M2 II) gives $N_e \approx 10^8$ cm$^{-3}$. The long wave-length spectra of late-type stars can be observed at high resolution with moderate exposure times and the method seems potentially useful.

If even long exposures at high resolution do not show the density sensitive lines an alternative way of at least limiting $N_e$ can be used. This method, given below, derives from analyses of limb to disk line ratios in the solar atmosphere (18) and has been applied to Procyon (7, 19).

The optical depth at line centre, $\tau_0$, in a Doppler broadened line can be written in terms of the absorption coefficient, leading to

$$\tau_0 = 1.2 \times 10^{-14} \lambda(A) f_{12} M_i^{\frac{1}{2}} \int \frac{N_e}{N_H} \frac{N_{\text{ion}}}{N_{\text{H}}} \frac{N_{\text{H}}}{N_{\text{ion}}} T_i^{-\frac{1}{2}} dh \quad (10)$$

where $f_{12}$ is the oscillator strength, and $M_i$ the atomic weight of the atom or ion concerned. The ion temperature can be expressed in terms of observed FWHM of the line, $\Delta \lambda$, since

$$T_i^{\frac{1}{2}} = \Delta \lambda M_i^{\frac{1}{2}} / 7.1 \times 10^{-7} \lambda \quad (11)$$

Now the expression for $\tau_0$ and that for the flux (equation 6) contain many similar quantities, since $\Omega_{12}$ can be expressed in terms of $f_{12}$ and $g$ the gaunt factor, i.e.

$$\Omega_{12} / \omega_1 = 1.6 \times 10^{-2} f_{12} g \lambda(A) \quad (12)$$

Thus the ratio of the opacity to the surface flux can be written, approximately as

$$\tau_0 / F_* = 6.1 \times 10^{-6} \lambda^2 (A) T_e^{3/2} \exp \left( \frac{W_{12}}{kT_e} \right) / g P_e \Delta \lambda \quad (13)$$

Although in some cases (see below) $\tau_0$ can itself be measured through the ratios of lines from a common upper level (20), in main sequence stars only the O I triplet may be suitable. It is also possible to identify which lines have $\tau > 1$ through departures from the ratios of central intensities expected under optically thin conditions, for lines within a multiplet. The
The appearance of self-reversals in lines also indicates a high opacity. Then limits can be placed on $P_e$ from equation (13). When applied to Procyon this method places quite stringent limits on $P_e$ (7).

The opacity can be measured directly for many lines of neutral atoms and Fe II observed at high resolution in the spectra of late-type giants and supergiants. Although this can lead to $\int N_H \, dh$, which is valuable, the line formation processes are complex and it is not easy to combine the flux and opacity measurements (3).

A method of finding the minimum pressure has been developed (21,22). It is assumed that the hottest observed line is formed in an isothermal corona over a height such that

$$d \ln P_e / dh = -0.43 / \Delta h = -1 / H$$

where $H = 1.4 \times 10^8 T_{\text{max}} / g_*^2$ is the scale height. Then

$$P_{\text{min}} = 1.3 \times 10^{-4} \left( E_m g_* T_{\text{max}}^{1/2} \right)$$

is the minimum pressure. (For a collisionally excited line with flux $\propto N_e^2$ a value a factor $\sqrt{2}$ smaller would be more appropriate.)

In spectra obtained with IUE the hottest line observed varies between C II and N V, but emission measures obtained from X-ray observations can be used in the same way, and are indeed preferable.

Upper limits to the pressure in the corona can be found from models of stellar chromospheres made by obtaining an optimum fit to the fluxes and profiles of lines such as Ca II H and K and Mg h and k. The electron and gas pressure is calculated as part of the modelling procedure, and the total pressure at about 8000 K can be used to limit the electron pressure at higher temperatures. Some models for individual stars are mentioned in Section 4.

Even though the pressure may be measurable at one or two temperatures it is necessary to make assumptions concerning the pressure variation as a function of height. It is usual to assume either that $P_e = \text{const.}$, (below the corona) on the grounds that the atmosphere is much less extended than the local isothermal scale height, or that the pressure varies according to hydrostatic equilibrium. The latter assumption is easy to include in the formulation and is adopted below on the grounds that it is better than assuming a constant pressure. In the
presence of strong flows or turbulent motions, this will still
only be an approximation.

Just as the pressure associated with the random motions of
individual electrons is \( P_e = N_e k \ T_e = \frac{1}{2} \ m_e \ v_e^2 \), so random large
scale turbulent motions can be considered as exerting a pressure.
Thus the pressure associated with the turbulent motions is
\[
P_T = \frac{1}{2} \ \rho \ \overline{V^2_p} \tag{16}
\]
where \( \overline{V^2_p} \) is the mean square velocity amplitude.

The total pressure then becomes
\[
P_{\text{tot}} = P_g + \frac{1}{2} \ \rho \ \overline{V^2_p} \tag{17}
\]
where \( P_g = 1.8 \ P_e \) is the total gas pressure (for \( T_e \approx 2 \times 10^4 \ \text{K} \)).

The pressure associated with the non-thermal motions in
the solar atmosphere is not large and causes an increase to \( P_e \)
of only \( \approx 25\% \). To give an upper limit, if \( V_p = C_s \), the sound
velocity, the additional term is only \( \approx 80\% \) of \( P_g \). Thus unless
super-sonic turbulence is present the correction to \( P_e \) lies
within the uncertainty inherent in the diagnostic methods
discussed above.

In the absence of flows the equation of hydrostatic equili-
brium is
\[
d \frac{\rho_e}{dh} = -7.1 \times 10^{-9} \ \rho_e \ \frac{g_A}{T_e} \tag{18}
\]
Allowing for a flow over a constant cross-section \( A \) leads to
the additional term
\[
\frac{1}{A} \ \rho^2 u^2 \ \frac{d}{dh} (1/\rho A) \tag{19}
\]
where \( u \) is the local flow velocity.

Flows will be excluded from the analysis which follows.

(c) The Temperature Structure

If \( P_e \) is slowly varying then it is a good approximation to
remove it from under the integral in equation (9), which refers
only to the region of line formation, not the whole atmosphere.
The temperature gradient is also, initially, assumed to be constant over $\Delta h$. Then

$$\frac{dT_e}{dh} = P_e^2 / 1.4 E_m T_e$$ \hspace{1cm} (20)$$

where $\Delta h$ has been taken as the region where $\Delta \log T_e = \log T_m \pm 0.15 = 0.30$. Other authors prefer to remove $T_e^{5/2} dT_e/dh$ in regions where the conductive flux is almost constant. Then the factor $1/2$ in (20) is replaced by $3/2$. The assumption of $dT_e/dh$ constant over $\Delta h$ appears crude but it should be remembered that for the solar atmosphere $E_m$ is known at intervals less than 0.30 in $\log T_e$.

Equation (20) can be combined with equation (18) to give

$$P_e dP_e/dT_e = -7.1 \times 10^{-9} g* 1.4 E_m$$ \hspace{1cm} (21)$$

or

$$P_e^2 = P_T^2 + 2.0 \times 10^{-8} g* \int_{T_e}^{T} E_m dT$$ \hspace{1cm} (22)$$

where $T$ refers to the highest $T_e$ at which $E_m$ is known.

Thus if the pressure and $E_m$ are known at one temperature the variation of $P_e$ with $T_e$ with height can be found.

Figure 2 shows the temperature structure for Procyon for three boundary conditions on $P_o$, the pressure at $2 \times 10^5 K(7)$.

(d) Effects of Limiting Emitting Area

Suppose that the emission in the EUV lines comes solely from an area $A = X(T_e).4\pi R_e^2$, where $X(T_e) \leq 1$. In deriving the emission measure distribution all values must be increased by a factor $1/X(T_e)$. For the same pressure, assuming these are measured from line ratios, the temperatures gradient would be reduced by the same factor.

The estimate of the minimum pressure from equation (15) will be increased, but by a factor of $1/X(T_e)^{1/2}$. In this case combining $P_e$ with $E_m$ in equation (20) gives no change in $dT_e/dh$, at the top of the atmosphere. If $X(T_e)$ was constant with $T_e$ there would also be no change in $dT_e/dh$ lower in the atmosphere, but if $X(T_e)$ decreases with decreasing $T_e$, as is likely, there would be some increase in $dT_e/dh$ at the lower temperatures.

Multi-component models can be made in the same way. Since it is difficult to find the departures from a uniform stellar atmosphere it is pointed out that a comparison between $P_e$ measured from line ratios and $P_e$ measured from opacity arguments can in
Models of the temperature structure of the atmosphere of Procyon (7) for three values of the pressure at $2 \times 10^5$ K. At temperatures below 8000 K the model of Ayres et al. (61) is used.

principle reveal a non-uniform surface emission, since the latter method depends on the emitting area and will overestimate $P_e$ if this is assumed to be the whole stellar surface. In practice the accuracy obtainable at present may be insufficient.

3. THE ENERGY BALANCE

It is necessary to model the atmosphere in order to determine the net conductive flux through particular regions. In many stars a simple estimate of the net conductive flux shows that it is unimportant compared with radiation losses. In such cases the energy input may balance only the radiation losses, which do not depend on the model, only the emission measure distribution. However, in order to make a comparison between this empirical energy input and particular processes a model is required.

The energy balance equation can be written as
\[
\Delta F_m = \Delta F_R + \Delta F_C + \Delta F_k
\]  
(23)

where \( \Delta F_m \) is the energy flux required to balance \( \Delta F_R \) - the radiation losses, \( \Delta F_C \) the net conductive flux and, \( \Delta F_k \) the net energy flux from flows. \( F_k \) is given by

\[
F_k = \frac{1}{2} \rho u^3 + 5\rho \, k \, T_u - \rho g_x \, Ru
\]

(24)

These terms arising from the kinetic energy flux, the enthalpy flux and the rate of doing work against the gravitational field. (See references (23) and (24) for discussions of energy and momentum flux).

\( \Delta F_R \) and \( \Delta F_C \) will now be formulated in terms of the parameters used for modelling. The theory of stellar winds will be treated in later papers in this volume.

(a) The Radiation Loss

Using a temperature range \( \Delta \log T = \log T_m \pm 0.15 \), since this is the typical range of \( T_e \) over which the emission lines are formed the flux lost by radiation from such a layer is

\[
\Delta F_R(T_e) = \int 0.8 \, N^2 \, \rho_{e \text{ rad}}(T_e) \, d\, \text{erg cm}^{-2} \text{s}^{-1}
\]

(25)

(expressions involving \( \frac{dF}{dT} = 0.43 \, \frac{1}{0.30 \, T_e} \) can be used if preferred).

The radiative power loss function has been calculated by several authors (25, 26, 27). It is usual to make a series of straight line fits to the power loss function and there is no large discrepancy between that adopted by different authors, except where a decreasing gradient over the whole range from \( \sim 2 \times 10^6 \) to \( 10^7 \) is assumed. This should be avoided because the sign of the gradient changes at around \( 2 \times 10^5 \) K.

Over each region \( \Delta h \) the radiation flux can be written approximately as

\[
\Delta F_R(T_e) \approx 0.8 \, E_m \, P_{\text{rad}}
\]

(26)

Thus \( P_e \) is not required explicitly, only the emission measure distribution.

The calculations of Post et al. (27) are particularly useful for stellar work since the contribution from each element is given individually. The effects of different stellar abundances can then be taken into account if necessary. If the atmosphere is not static but instead ions are caused to pass rapidly across
a steep temperature gradient then enhancement of radiation may occur through the sensitivity of the collisional excitation rate to $T_e$. For example the emission from helium lines is particularly sensitive to such processes (28,29). The related change in ion populations, for example for the lithium-like ions, may also cause enhanced radiation (4).

As mentioned above the low density in the envelopes of giants and supergiants leads to long ionization and recombination times. The radiation losses under these circumstances bear closer examination.

(b) The Net Conductive Flux

The conductive flux is given by

$$F_c = -\kappa \frac{T_e^{5/2}}{T_e} \frac{dT}{dh}$$  \hspace{1cm} (27)

where $\kappa \approx 10^{-6}$.

For $dT/dh$ from equation (20)

$$F_c = -\kappa \frac{P_e^2}{T_e^{3/2}} \frac{T_e^{3/2}}{\sqrt{2}} \frac{e}{E_m}$$  \hspace{1cm} (28)

Thus in the region above $2 \times 10^5 \text{K}$, where in the Sun the emission measure gradient is $\approx 3/2$, an assumption of constant pressure obviously leads to $F_c = \text{const. (30)}$. It is not necessary to use $P_e = \text{const.}$ and the formulation below continues with $P_e$ varying according to hydrostatic equilibrium. From (34),

$$\frac{dF_c}{dT_e} = -\kappa \frac{dP_e^2}{2T_e} \frac{T_e^{3/2}}{E_m}$$ \hspace{1cm} (29)

d$p_e$ can be found from equation (21) in terms of $P_e, E_m$ and $g_*$, and after some algebra, the general expression for $\Delta F_c$ becomes

$$\Delta F_c = 9.9 \times 10^{-9} g_* \frac{T_e^{5/2}}{T_e} - 0.49 \frac{P_e^2}{E_m} \left( \frac{T_e^{3/2}}{E_m} - \frac{d \log E_m}{d \log T_e} \right)$$

$$- 9.9 \times 10^{-9} g_* \frac{T_e^{3/2}}{E_m} \left( \frac{T_e^{3/2}}{E_m} - \frac{d \log E_m}{d \log T_e} \right) \int E_m \frac{T_e^0}{E_m} dT$$  \hspace{1cm} (30)

where $P_e$ and $T_e$ refer to $T_e = 2 \times 10^5 \text{K}$.

This reduces to the form given by Jordan (31) if $E_m$ is replaced by a general power law $E_m^{0} = a T_e^{0}$.  

It is clear from equation (30) that $\Delta F_c$ does not depend only on the emission measure gradient, but also the absolute value of $E_m$ unless $d \log E_m/d \log T_e = 3/2$. Equation (30) then
reduces to the simple form (first term) given by Jordan (32), when conductive is a loss term. When \( d \log E_m / d \log T_e < 3/2 \) conduction becomes an energy deposition term.

At a given temperature the ratio of \( \Delta F_R / \Delta F_c \) always depends on \( E_m \), the local absolute value. The variation of \( \Delta F_R / \Delta F_c \) with \( T_e \) depends on the emission measure gradient and on \( \alpha \).

If the terms of equation (30) are examined using typical solar values or values from the analysis of Procyon it is found that once \( d \log E_m / d \log T_e < 0 \) (which is below \( \sim 2 \times 10^5K \)) then the second term dominates. For typical giants, where \( P_e, E_m \) and \( g_e \) are all lower, this may not be so, but the absolute value of \( \Delta F_c \) may be far smaller than \( \Delta F_R \).

Therefore, for near main sequence stars, below \( \sim 2 \times 10^5 K \), conduction deposits energy whilst radiation loses energy. In the sun it has been known for many years that the energy deposited by conduction at the base of the transition region is far larger than is radiated away in the layer immediately below. It has been suggested that this energy deposition will drive various types of motions until the energy is carried to a region from whence it can be radiated (31,33-36).

The condition for radiation to stably dispose of the conducted energy is \( \Delta F_R \geq \Delta F_c \)
i.e. \[
0.8 \frac{E_m^2}{P_{\text{rad}}} \geq 0.49K\frac{E_m^2}{P_0} T_e^{3/2} \left( \frac{3}{2} - \frac{d \log E_m}{d \log T_e} \right)
\]
where \( d \log E_m / d \log T_e < 3/2 \),
which is independent of gravity for near main-sequence stars.

Alternatively one can say that all energy conducted back from a hot corona at \( T_e > 2 \times 10^5 K \) must eventually be radiated away, but since the majority of the conducted energy is deposited immediately simply choosing a base temperature \( T_b \) such that

\[
F_c (at T_0) = F_R (T_b \text{ to } T_0)
\]
ignores the problem of the stability. To make correct use of this boundary condition the radiation losses in a dynamic atmosphere would need to be considered. Dynamic models will be required for stars which appear to have hot coronae and steep transition region temperature gradients.

The use of either \( \Delta F_R = \Delta F_c \) or \( F_c (at T_0) = F_R (T_b \text{ to } T_0) \) to determine \( P_o \) should be avoided. If conduction is large, \( P_o \) will be underestimated, but if conduction is small and \( \Delta F_R \) is balanced only by direct deposition of mechanical energy (as in Procyon) \( P_o \) will be overestimated.
The Region Above $2 \times 10^5$K

The formulation for $\Delta F_R$ and $\Delta F_C$ given above is general and can be applied provided the local emission measure is known. EUV observations, for example for the IUE satellite, allow the region up to $T_e \sim 2 \times 10^5$K to be studied. Lines formed at higher temperatures are observed above 1000 Å in the solar spectrum but they are either weak magnetic dipole transitions or transitions between excited states (e.g. O VII 1s2s $^3S$-1s2p $^3P$). X-ray observations can be used to investigate the region above $T_e \sim 10^6$K. In order to find the total radiation logs or the structure of the atmosphere between $2 \times 10^5$K and $10^6$K some interpolation must be adopted. Observations of fluxes in two different energy bands in the X-ray region can be used to determine an average temperature and the corresponding emission measure; this approach has been widely applied to solar observations. Some measurements of X-ray fluxes, or upper limits to these, are available from rocket flights, the ANS satellite, HEAO I satellite and Einstein observatory (37-42).

The principle of finding the volume emission measure from the X-ray fluxes is essentially the same as that used for individual lines, but the total contribution of the lines and continuum in the relevant wavelength range must be known as a function of $T_e$ (43,44). If fluxes from only one energy band are observed then $\int v N_e^2 dv$ cannot be found uniquely, but the range of suitable combinations of $\int v N_e^2 dv$ and $T_C$ can be investigated. The corresponding values of $\Delta F_R$ for the coronal region can then be established.

There is no agreed method of interpolating through the region between $2 \times 10^5$K and $10^6$K. Also controversy exists concerning suitable scaling laws for coronal parameters. Rosner et al. (45) have proposed a scaling law based on constant pressure and boundary conditions on $F_C$. Whilst this may adequately fit small dense closed loop structures in the solar atmosphere the assumption of constant pressure makes it unsuitable for application to open stellar coronae as a whole. Mullan (46,47) has predicted coronal parameters by extending Hearn's (48) minimum energy flux concept.

The present writers prefer to assume that the emission measure gradient above $T_e \sim 2 \times 10^5$K has a value of $\approx 3/2$, as found in the solar atmosphere. The justification is that there appears to be underlying cause in the stability of the atmosphere leading to this particular gradient (31). If the emission measure above $2 \times 10^5$K is written as

$$E_m = a T_e^{3/2}$$

(33)
then equation (22) for hydrostatic equilibrium becomes

\[ P_e^2 - P_o^2 = 8.0 \times 10^{-9} a g_\lambda (T_e^{5/2} - T_o^{5/2}). \] (34)

Defining \( T_c \), the 'coronal' temperature as that temperature at which \( P_e \) and \( dT/dh \to 0 \) gives

\[ T_c^{5/2} - T_o^{5/2} = 1.2 \times 10^8 \frac{P_o^2}{a g_\lambda} \] (35)

where \( T_o \) and \( P_o \) are the temperature and pressure at \( 2 \times 10^5 \)K. Thus if \( F_m \) and \( P_o \) are known at \( 2 \times 10^5 \)K, the coronal temperature can be predicted. This method has the advantages that neither constant pressure nor a particular form of the heating function are assumed, but the hypothesis that the emission measure gradient is constant from star to star is as yet unproven. Further X-ray observations are required before a satisfactory method of treating the coronal parts of the atmospheres can be established.

With the same formulation the radiation losses above \( T_o \) can be summed to give

\[ F_R(T_o) = 1.6 \times 10^{-16} a (T_c^{1/2} - T_o^{1/2}), \] (36)

where \( P_{rad} = 6.0 \times 10^{-17}/T_e \) has been used.

The conductive flux back at \( T_o \), from equations (28) and (35) becomes

\[ F_c(T_o) = 5.6 \times 10^{-15} g_\lambda (T_c^{5/2} - T_o^{5/2}) \] (37)

(d) Use of Line Widths

If high resolution observations are available then it is possible to use the widths of optically thin lines to examine the non-thermal motions present. The method below has been used by McWhirter et al. (26) and later authors. For Doppler broadening and a turbulent velocity (most probable velocity), \( \xi_o \), the full-width at half maximum (FWHM) \( \Delta\lambda \) is given by

\[ \frac{\Delta\lambda}{\lambda} = 7.1 \times 10^{-7} \left( \frac{T_e}{T_{line}} + \xi_o^2 \frac{M_i}{2k} \right)^{1/2} \] (38)

where \( M_i \) is the ion to proton mass ratio. The non-thermal energy density, \( E \), can be expressed in terms of the r.m.s. velocity, \( <v_T^2> \),

\[ E = \frac{1}{2} \rho <v_T^2> \] (39)

If this energy is related to the passage of waves through the
atmosphere, the energy flux, $\phi_m$, can be written as

$$\phi_m = 2 \, \text{EC}$$

where C is the appropriate propagation velocity. The variation of $\phi_m$ with temperature can then be compared with dissipation rate implied by the values of $\Delta F_m$ discussed in the previous section.

Given the uncertainties in present measurements of $P_e$ and line widths the comparisons may not lead to firm conclusions but in the region up to $2 \times 10^5 K$, where $\Delta F_m$ is determined mainly by $\Delta F_R$ useful constraints may emerge.

The obvious wave processes with which comparisons may be made are sound waves, Alfvén waves and shock waves. The propagation velocities for sound waves and Alfvén waves are, respectively,

$$C_S = (\gamma P/\rho)^{1/2} \approx 1.5 \times 10^4 \, T_e^{1/2} \, \text{cm s}^{-1}$$

and

$$C_A = B/(4\pi \rho)^{1/2} \approx 2 \times 10^{11} \, B \, T_e^{3/2} P_e^{1/2} \, \text{cm s}^{-1}.$$  

Following Kuperus (49) the flux carried by shock waves can be written as

$$\phi_m = 4 \rho \, C_S^3 \, (M^2 - 1)^{2/3} \, (\gamma+1)^2 \, M$$  

(40)

where $M$ is the Mach number.

This type of approach can be used to show that in the solar atmosphere at $T_e \sim 3 \times 10^4 K$ a pure acoustic flux cannot carry sufficient energy to heat the corona above. On the other hand fields of only 2 Gauss lead to Alfvén wave fluxes equal to the acoustic flux. Heating mechanisms will not be discussed further in this paper. Discussions of the rôle of acoustic heating in the low chromosphere may be found in the series of papers by Ulmschneider and colleagues (50). Rosner et al. (51) have revived interest in DC heating modes. For stars with magnetic fields and gas pressures not too dissimilar from the sun the succession of MHD wave processes proposed by Osterbrock (52) may well be relevant. In particular the viscous damping of short-period Alfvén waves (in the weak field limit) is difficult to rule out for either the solar atmosphere or that of Procyon (7, 32).

4. RESULTS

In the long term a detailed study of a wide range of stars will be required in order to investigate the dependence of coronal structure and heating on the stellar parameters. Meanwhile surveys of Mg II fluxes (53, 54) have shown distinct differences between
the gravity dependence of Mg II flux and the energy flux predicted from the deposition of acoustic energy. Moreover, the limited sample of high gravity stars does not show the predicted flux decrease with decreasing effective temperature. (The flux is \( \propto T_{\text{eff}}^{-4} \) rather than \( T_{\text{eff}}^{-8} \).) These trends can be seen from Figure 3, taken from ref. (53). The observed fluxes are compared with calculations of the acoustic flux generated (55) and that available to the chromosphere (56). These differences are also apparent in the survey of X-ray fluxes (42).

Some of the methods discussed in the previous sections have been applied previously to stars for which chromospheric models and IUE spectra are available (57). Since then the IUE fluxes have been found to require correction (58). The examples below are restricted to stars whose emission line fluxes are known to have been corrected.

(a) Near Main Sequence Stars

The Sun (G2V). Many of the methods discussed above have been developed from solar work. The value of solar observations lies in the high spatial and spectral resolution that can be achieved, for both strong and weak lines, over a very wide region of the spectrum. Stellar observations are limited to the integrated fluxes, for the stronger lines, in the region above the Lyman continuum and below \( \sim 100 \) Å. EUV fluxes should also be measurable for a few nearby stars. Thus one must rely on whatever is known about the spatial structure of the solar atmosphere, and about the rôle of the magnetic field in particular, to obtain guidance in modelling stellar coronae. However, the 'average' quiet sun model that would be obtained from integrated fluxes would give a reasonable idea of the relative importance of the various total energy loss and deposition terms. For the purpose of comparisons with other stars the parameters and energy terms are as follows. The electron pressure at \( 10^5 \) K and coronal temperature are \( 5.6 \times 10^{14} \) cm\(^{-3}\)K and \( 1.5 \times 10^{5} \)K respectively. The overall radiation losses amount to \( \sim 5 \times 10^6 \) erg cm\(^{-2}\) s\(^{-1}\) (23) with only \( \sim 4 \times 10^4 \) erg cm\(^{-2}\) s\(^{-1}\) above \( T_e \sim 2 \times 10^5 \)K. The main energy loss from the corona is by conduction back to the chromosphere from whence it is radiated, but not apparently in a static configuration (31,34) i.e. the condition for stable deposition of the conductive flux (equation 31) is not satisfied. Line profile measurements show the presence of non-thermal motions, but interpreted as due to the passage of acoustic waves the flux carried is insufficient to account for coronal heating (31,59).

Procyan (\( \alpha \) CMi, F5 IV-V). The F-stars are important in that models of the convective zone structure suggest that the acoustic flux generated is largest around the late F's, dropping rapidly for earlier type stars (60). Procyan is sufficiently close and
The ratio of Mg II flux to surface luminosity plotted against the effective temperature. The dashed lines show the theoretical acoustic flux generated (55) and the full lines show the flux calculated to be available to the chromosphere and corona (56). The Figure is from Linsky and Ayres (53).

bright to allow UV spectra to be obtained with IUE at both low and high resolution, and the methods described above can be applied (7,19). Also a model of the chromosphere has been made from Ca II and Mg II profiles and fluxes (61). The emission measure distribution has been shown in Figure 1. The electron pressure can be limited through $p_{\text{min}}$ and opacity arguments and the chosen model has $P_e \sim 1.2 \times 10^{14} \text{ cm}^{-3} \text{K}$ at $2 \times 10^5 \text{K}$. The electron pressure at $\sim 10^4 \text{K}$ is about a factor of three higher than that in the chromospheric model (61). Figure 2 shows the temperature structure which results from three pressure boundary conditions. The 'coronal' temperature in the final model is $\sim 3 \times 10^5 \text{K}$. Neither the pressure or temperature agree with the
PREDICTIONS OF HEATING BY THE SHOCK DISSIPATION OF ACOUSTIC FLUX (60, 62). IT IS INTERESTING TO NOTE THAT IN SPITE OF THE LOW CONDUCTIVE FLUX AT 2 × 10³ K, (≤ 3 × 10³ erg cm⁻² s⁻¹) THE ATMOSPHERE HAS A STEEP 'TRANSITION REGION'. ALSO THE LARGE SURFACE FLUX OF ≈ 14 TIMES THE SOLAR VALUE, AT C IV, DOES NOT IMPLY A CORRESPONDINGLY LARGER PRESSURE. LINE WIDTHS HAVE ALSO BEEN MEASURED AND ALTHOUGH THEY SHOW THE PRESENCE OF NON-HEAT ENERGY MOTIONS THE DATA ARE NOT CONSISTENT WITH THE DEPOSITION OF ACOUSTIC FLUX. THE Mg II FLUX FROM PROCYON IS ABOUT THREE TIMES THAT IN THE SUN. THE TOTAL RADIATION LOSS ABOVE 10⁴ K IS ABOUT A FACTOR OF FOUR GREATER THAN FROM THE SUN, ≈ 2 × 10⁶ erg cm⁻² s⁻¹. THUS MORE ENERGY MUST BE DEPOSITED, BUT THIS APPEARS TO BE LOST THROUGH RADIATION IN A LOWER PRESSURE, MORE EXTENDED CHROMOSPHERE THAN IN THE SUN. ONLY UPPER LIMITS TO THE X-RAY FLUX FROM PROCYON ARE AVAILABLE (38, 63) GIVING Lₓ < 10²⁸ erg s⁻¹. WITH Tₓ ≈ 3 × 10⁵ K THIS GIVES AN EMISSION MEASURE LESS THAN 3 × 10²⁶ cm⁻⁵, CONSISTENT WITH THE RESULTS SHOWN IN FIGURE 1.

ε ERI K2 V. ε ERI WAS OBSERVED WITH IUE IN THE INTERNATIONAL COLLABORATIVE PROGRAMME (15). A MODEL OF THE CHROMOSPHERE HAS BEEN MADE BY KELCH (64). A RECENT PAPER BY SIMON ET AL. (65) CORRECTS THE PREVIOUS IUE FLUXES AND CONCLUDES THAT THE ELECTRON PRESSURE MUST BE ≈ 3.6 × 10¹⁵ cm⁻³ K, HIGHER THAN IN THE PREVIOUS MODEL, WHICH GAVE 2.3 × 10¹⁴ cm⁻³ K. X-RAY LUMINOSITIES OF 3.8 × 10²⁸ erg s⁻¹ AND 2.0 × 10²⁸ s⁻¹ HAVE BEEN REPORTED FROM HEAO I (41) AND THE EINSTEIN OBSERVATORY (42). IF THE UV SURFACE FLUX RATIO IS COMBINED WITH THE X-RAY LUMINOSITY THROUGH THE 'CORONAL' RADIATION LOSS, (EQUATION 26) AND THE SCALING LAW GIVEN BY EQUATION (35), THEN THE VALUES OF THE TRANSITION REGION ELECTRON PRESSURE AND CORONAL TEMPERATURE WHICH RESULT ARE 5.2 × 10¹⁵ cm⁻³ K AND Tₓ = 3.5 × 10⁶ K. THE CONDUCTIVE FLUX BACK FROM THE CORONA (FROM EQUATION 37) IS THEN 4.6 × 10⁶ erg cm⁻² s⁻¹, GREATER THAN CAN BE RADIATED AWAY AT AROUND 10³ K (I.E. 1.0 × 10⁵ erg cm⁻² s⁻¹), BUT LESS THAN THE TOTAL WHICH IS LOST FROM THE CHROMOSPHERE. THE RADIATION LOSS ABOVE 2 × 10⁵ K, FROM EQUATION (36) IS ≈ 9 × 10⁸ erg cm⁻² s⁻¹. THE TOTAL RADIATION LOSSES WILL BE ABOUT TWICE THOSE FROM THE SUN, GIVEN THE LARGER SURFACE FLUX OF THE Mg II EMISSION AND UV EMISSION. THIS IS CONTRARY TO THE RESULTS EXPECTED FROM THE GENERATION OF ACOUSTIC ENERGY.

(b) GIANTS AND SUPERGIANTS

IUE OBSERVATIONS HAVE SHOWN THAT ALTHOUGH EVIDENCE FOR HOT (≈ 10⁵ K) MATERIAL IS PRESENT THROUGH C IV EMISSION IN GIANTS EARLIER THAN AROUND K 0, EVEN BY K2 THE LINES ARE BARELY STRONG ENOUGH TO DETECT (66, 67, 68). SUPERGIANTS SHOW A SIMILAR BEHAVIOUR, WITH C IV BEING VISIBLE IN THE G2 Ib STAR, α AQR, BUT NOT IN Λ VEIL (K5 Ib) (67, 68). LINSKY AND HAINICH (69) HAVE SUGGESTED THAT THERE IS A SHARP DIVISION IN THE H-R DIAGRAM BETWEEN STARS WHICH DO AND DO NOT SHOW C IV EMISSION, THE FORMER HAVING HOT
coronae like the sun. The position of their dividing line is similar to that proposed by Stencel and Mullan (70) for the region where there is evidence of mass outflow from the asymmetry of the Mg II lines. Other loci relating to evidence of outflow from Ca II asymmetries (72) and of mass-loss from circumstellar lines (73) have been proposed.

Although the methods discussed above are in principle applicable to stars showing predominantly emission from cool species much of the necessary work on the line excitation processes and derivation of ion and atom densities has not yet been carried out.

Two examples of late-type giants are now briefly discussed.

β Gem. KO III. β Gem shows emission from C IV and other moderately ionized species and is of interest since it lies close to the dividing line proposed by Linsky and Haisch (69). A chromospheric model has been calculated from the Ca II and Mg II fluxes and profiles (74). Some early observations were made from the Copernicus satellite (75) showing that both H Lyα and the Mg II lines have profiles with blue rather than red wing enhancements suggesting that any outflow is not an accelerating wind. The present authors have obtained a long exposure low resolution spectrum with the IUE satellite. The surface fluxes of lines formed between $2 \times 10^4 K$ and $3 \times 10^5 K$ are lower than from the sun by factors of up to 5.

The chromospheric model (74) gives $P_e \sim 1.3 \times 10^{-2}$ dynes cm$^{-2}$ at $\sim 8000 K$, giving an upper limit for $P_e$ of $4.7 \times 10^{-3} \text{ cm}^{-3} \text{K}$. The emission measure from the C IV line places a lower limit on $P_e$ of $7 \times 10^{12} \text{ cm}^{-3} \text{K}$. Using these pressures the coronal temperature is predicted to lie between $1.6 \times 10^6 K$ and $2 \times 10^5 K$, and the X-ray luminosity to be less than $5 \times 10^{28}$ erg s$^{-1}$. Linsky (private communication) informs us that β Gem is observed as an X-ray source by the Einstein observatory.

The conductive flux back at $2 \times 10^5 K$ must be $< 1.6 \times 10^4$ erg cm$^{-2}$ s$^{-1}$, compared with the local radiation loss of $\sim 4.3 \times 10^3$ erg cm$^{-2}$ s$^{-1}$. The radiation losses below $2 \times 10^5 K$ and the Mg II flux are lower than in the sun by about a factor of four.

Although the results are highly dependent on the pressure, which is not known to better than a factor of two, a coronal temperature similar to that of the sun is possible through the combination of lower pressure, lower emission measure and lower gravity (see equation 35). The energy requirements are, however, smaller.
$\alpha$ Tau K5 III. $\alpha$ Tau does not show C IV emission, and the surface fluxes of lines such as C II and Si II are lower than in $\beta$ Gem by around a factor of 15, i.e. $\sim 10^{-2}$ the solar values (14). The spectrum is composed mainly of lines from neutral atoms and singly charged ions, e.g. H Ly$\alpha$, C I, O I, S I, C II, Si II, Fe II. The strength of the O I resonance lines is likely to be due to pumping by H Ly$\beta$ as in $\alpha$ Boo (76). This behaviour is typical of other late-type giants observed with IUE (66,67,77). The O I lines appear to excite the nearby S I lines, resolved in high resolution spectra (14). Lines of C I and O I are present which owe their strength to radiation trapping in lines with a common upper level (3,14).

The line formation processes require further work but initial studies of opacity and density sensitive lines suggest $2 \times 10^{12} < \int N_\text{H} \, dh < 8 \times 10^{21} \text{ cm}^{-2}$, and $N_\text{e} \sim 10^8 \text{ cm}^{-3}$. The chromospheric model (74) gives $P_\text{e} \sim 6.1 \times 10^{12} \text{ cm}^{-3} \text{K}$ at $\sim 7200 \text{ K}$. Assuming the C II lines to be formed in an isothermal 'corona' at $10^4 \text{K}$ gives $P_\text{min} \sim 10^{12} \text{ cm}^{-3} \text{K}$. The radiation losses from Mg II are lower than in $\beta$ Gem by about a factor of five (54). High resolution observations of H Ly$\alpha$, Mg II and the O I lines (14,75) reveal strong red wing enhancements indicating an accelerating outflow. The S I lines are also asymmetric. Interpreted as a spread in velocities limits the relative outflow velocity to $< 10 \text{ km s}^{-1}$. The energy carried in a stellar wind then appears to be small compared with the total radiation losses.

The upper limit on the C IV flux can be used to limit the temperature gradient at $\sim 10^5 \text{K}$, and hence estimate the coronal temperature. If there is really no material above $\sim 10^4 \text{K}$ this is an artificial procedure and a spuriously high value of $T_\text{c}$ would result. With values of $P_\text{e}$ given above $T_\text{c}$ would lie between $7.4 \times 10^6 \text{K}$ and $1.6 \times 10^6 \text{K}$. Apart from arguments concerning the escape velocity, $\alpha$ Tau would then emit a substantial X-ray flux ($> 2 \times 10^{28} \text{ erg s}^{-1}$), whereas it is not reported as an X-ray source. A typical upper limit of $6 \times 10^{27} \text{ erg s}^{-1}$ for K giants in the Einstein survey (42) would limit the coronal temperature to $< 2 \times 10^5 \text{K}$. Thus the low temperature solution with a real absence of material above $\sim 2 \times 10^4 \text{K}$ seems appropriate.

The total energy requirement of $\alpha$ Tau appear to be at least an order of magnitude less than predicted by the deposition of acoustic flux.

It has become clear over the past few years that neither the broad properties or detailed structure of late-type chromospheres and coronae fit the early predictions based on the
generation and propagation of acoustic flux in the sub-photospheric convective zone. These theories must either be substanti-
al revised or the controlling factor sought elsewhere, for example in the combined efforts of the magnetic field and convective motions. As the energy requirements for the different types of stars became available from both UV and X-ray observations other correlations, for example between rotation and activity (78,79), can be investigated. Even in the cool giants it may be necessary to invoke magnetic modes (Alfvén waves) to provide sufficient momentum to drive the mass loss (80). The change in the structure of the coronae apparent from the decrease of C IV emission does seem to be closely related to the develop-
ment of an accelerating outflow as seen in the Mg II lines (72). However, the energy input required, as deduced from Mg II fluxes (see Figure 3 and ref. 54) or IUE spectra, decreases steadily, rather than suddenly, across the region around KO III (~4700 K). Thus the change in the structure appears to be related more closely to the large drop in surface gravity for stars evolved beyond KO III rather than to a sudden change in the total energy flux requirements. Cassinelli (81) has recently reviewed the current state of observations and theories relating to mass loss. Although the present paper has concentrated on methods of investigating the structure of late type coronae and their relative conductive fluxes, the momentum flux remains a crucial factor to be included in any satisfactory theory.
References
