THE OPTICAL CONTINUUM OF SOLAR
AND STELLAR FLARES

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(Received 8 May; in revised form 30 October, 1980)

Abstract. A further development of the Kostyuk–Pikelner’s model is presented. The response of the chromosphere heated by non-thermal electrons of the power-law energy spectrum has been studied on the basis of the numerical solution of the one-dimensional time-dependent equations of gravitational gas dynamics. The ionization and energy loss for the emissions in the Lyman and Balmer lines have been determined separately for the optically thin and thick Lα-line layers. Due to the initial heating, a higher-pressure region is formed. From this region, disturbances propagate upwards (a shock wave with a velocity of more than 1000 km s⁻¹) and downwards. A temperature jump propagates downwards, and a shock is formed in front of the thermal wave. During a period of several seconds after the beginning of this process, the temperature jump intensifies the downward shock wave and the large radiative loss gives rise to the high density jump (ρ₂/ρ₁ ~ 100). The numerical solution has been analyzed in detail for the case heating of the ionized and neutral plasma, and a value of this heating is close to the upper limit of the admissible values. In this case, the condensation located between the temperature jump and the shock wave front, may emit in the observed optical continuum.

In their essential features, the gas dynamic processes during the flares in red dwarf atmospheres are the same as those in the solar atmosphere. However, the high atmospheric densities, smaller height scale in red dwarf atmospheres, and greater energy of this processes in stellar flares, give rise, in practice, to the regular generation of optical continuum. The photometric parameters of a source with n ~ 10¹⁵ cm⁻³, T ~ 9000 K, and Δz ~ 10 km are in a good agreement with observations.

1. Introduction

The processes producing the optical continuum during flares on the Sun, and on UV-Ceti stars, are very important to study. Numerous papers have been devoted to elucidating the nature of white light solar flares (for example, Švestka, 1976; Machado and Rust, 1974; Brown, 1973). Most of these papers attribute the continuum to penetration of accelerated particles into the chromosphere. It was assumed that the highest-energy particles could penetrate as deep as the photosphere, while the low-energy particles are absorbed in the surface layers, i.e., in the upper chromosphere. Therefore, a white light flare model was extended in height. Besides that, the effects observed in dense layers could only be observed on the assumption of a large flux of the highest-energy particles which is at variance with the
hard X-ray burst observations. The models proposed earlier were also deficient in that they failed to explain the motions observed during flares.

The gasdynamic motions was first considered in terms of the problem of the response of the chromosphere to the heating by accelerated electrons (Kostyuk and Pikelner, 1974). Later on, a similar problem was treated by Somov et al. (1977, 1979), who considered mainly the processes in the high-temperature flare with \( T > 2 \times 10^4 \) K. We have extended these studies mainly to the low-temperature zones of flares taking into account that the plasma becomes optically thick in the \( \text{La} \) -line. As a result, we have modelled the gasdynamic process determining the entire set of events which we shall call, henceforth, the dynamic response* of the chromosphere to heating, in particular to the electron beam heating. The dynamic response discussed in Section 2 has been featured by a plasma condensation which appears in front of the temperature jump and moves slowly towards the photosphere. This condensation with \( n = 10^{14} - 10^{15} \) cm\(^{-3} \), \( T = 8000 \) K, and \( \Delta z = 10 - 30 \) km may well be the source of line emissions at the bright points of a low-temperature flares. The present work is an attempt to determine in which cases the optical continuum will be generated by our dynamic response model.

2. The Dynamic Response of the Solar Chromosphere to the Heating by the Accelerated Electrons

2.1. Gas-dynamics Equations

An electron acceleration event is assumed to occur sufficiently high in the atmosphere. At lower altitudes, a sufficiently strong magnetic field controls the motions of the accelerated electrons as well as the heat conduction and gasdynamic flows. The magnetic field in the low-temperature zones of the bright points of flares will be assumed to be vertical. As in the earlier works, the treatment will be limited to a one-dimensional approach to the gasdynamic problem (see Figure 1).

The appropriate gasdynamics equations in the Lagrange variable are of the form (Samarsky and Popov, 1975; Kostyuk and Pikelner, 1974, formulas (4)–(6))

\[
\begin{align*}
\frac{\partial z}{\partial t} & = v, \\
\frac{\partial}{\partial t} \left( \frac{1}{n} \right) & = -\frac{\partial v}{\partial \xi}, \\
\frac{\partial v}{\partial t} & = \frac{1}{m_H} \frac{\partial p}{\partial \xi} - g, \\
\frac{\partial e}{\partial t} & = \frac{p}{m_H} \frac{\partial v}{\partial \xi} + \frac{1}{m_H} \frac{\partial W}{\partial \xi} + P_e - \mathcal{L} + Q, \\
W & = \chi n \frac{\partial T}{\partial \xi},
\end{align*}
\]

* The term was proposed by S. I. Syrovatskii.
where \( z \), the Euler coordinate, is the height above the photospheric level with optical depth in the continuum near 5000 Å taken to be 1; \( t \) is the time; \( v \) is the velocity; \( p \) is the pressure; \( n \) is the density of the neutral and ionized hydrogen atoms; \( \varepsilon \) is the internal energy; \( g \) is the acceleration due to gravity; \( m_{\text{H}} \) is the hydrogen atom mass; \( \xi \) is the Lagrange variable (mass column density); \( d\xi = -n \, dz \), \( W \) is the thermal conduction flux; \( T \) is the temperature; \( \kappa \) is the coefficient of heat conduction; \( P_e \) is the power of the heating of the chromosphere by the accelerated electrons; \( \mathcal{L} \) is the radiative energy loss; and \( Q \) is the power of the sources which sustain the initial temperature distribution (wave heating). This power is assumed to equal the radiative loss calculated at the values of the parameters corresponding to the initial model.

The Equations (1) have been written for a one-fluid approximation assuming simultaneous heating of electrons and ions in the surface layer. In the case of initial heating of electrons, which then transfer their energy to ions (the two-temperature approximation with \( T_e > T_i \), see in Dyachenko and Imshennik (1963)), we get somewhat higher maximum temperatures \( T_e \) and, correspondingly, a higher thermal flux conducted into the chromosphere. This means that the heating flux resulting in the same chromospheric effects should be assumed for the single-temperature approximation to be somewhat higher compared with the two-temperature approximation. Since the general pattern of the processes developing in the chromosphere is independent of whether the electron-ion temperature gap is included, we have assumed that \( T_e = T_i \).
The solar plasma is assumed to be an ideal gas consisting of hydrogen only. The equation of state including the partial ionization is of the form

\[ \varepsilon = \frac{3}{2} \frac{k}{m_H} (1 + x) T + \frac{\chi_H}{m_H} x, \]

\[ p = k (1 + x) n T, \]

where \( x = n_e/n \) is the degree of ionization; \( n_e \) is the electron density; \( k \) is the Boltzmann constant; \( \chi_H = 13.6 \) eV is the potential of hydrogen ionization. It will be noted that, when (2) was applied at large \( \xi \), the value \( x \) was replaced by the analogous value \( x^* \) (see below).

2.2. The boundary conditions

The atmosphere was considered to be of finite extent in the Lagrangian coordinate \( \xi \), namely from \( \xi_{\text{min}} \) to \( \xi_{\text{max}} \). The lower boundary \( \xi_{\text{max}} = 2.3 \times 10^{24} \) cm\(^{-2} \) was chosen to lie sufficiently deep in the photosphere that we assume no motions, i.e., \( v(\xi_{\text{max}}, t) = 0 \). The upper boundary was located in the corona at \( z = 10^{10} \) cm\((\xi_{\text{min}} = 10^{15} \) cm\(^{-2} \). The values of \( \xi_{\text{min}} \) and \( \xi_{\text{max}} \) were so selected that the character of the solution in the chromosphere remained unaffected by them. It was assumed at the upper boundary that \( p(\xi_{\text{min}}, t) = 0 \), i.e., the so-called open model was adopted. The temperature boundary condition corresponded to the absence of the thermal conduction flux at the boundaries, \( W(\xi_{\text{max}}, t) = W(\xi_{\text{min}}, t) = 0 \).

2.3. The initial model of the solar atmosphere

To eliminate the motions at the initial moment \( t = 0 \) due to the pressure gradients, the initial model was assumed to be in hydrostatic equilibrium. The model was calculated from the equation \((1/m_H) (\partial p/\partial \xi) = g\) with the values of \( T(\xi, 0) \) and \( n \) specified at \( z = 500 \) km. The temperatures in the photosphere and lower chromosphere and \( n = 3 \times 10^{15} \) cm\(^{-3} \) at \( z = 500 \) km (the temperature minimum) were taken from Gingerich et al. (1971). Since the cold elements of the active chromosphere occupy most of its volume, the temperature was taken to be lower than or equal to \( 10^4 \) K up to \( z = 4000 \) km. The initial model is shown by the dashed lines in Figures 4 and 5. The small values of coronal densities in the initial hydrostatic model constitute a defect which, however, does not affect the character of the solution at the chromospheric levels.

2.4. The choice of the functions which enter the equations (1)

Our preliminary numerical solutions of Equations (1) with various assumptions concerning the radiative energy loss have made it possible to find the maximum range of uncertainty in the loss term, which does not significantly affect the results of the numerical calculations. We confirmed, first of all, that the radiative loss should be calculated simultaneously with the degree of ionization of the hydrogen plasma. If, following earlier work, the degree of the ionization is calculated according to Brown

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(1971, 1973), which is close to \( x^* \), then the radiative \( \text{La} \)-line loss proves to be about two orders of magnitude larger than real values. Such errors distort the entire process in the beginning of it. It has also proved possible to avoid the simultaneous solution of the equations of gasdynamics and radiative transfer for a many-level atom and, instead, to use an approximate method for calculating the energy loss.

The degree of ionization and the radiative energy loss at each moment have been calculated here separately for the layers which are optically thin and thick in \( \text{La} \)-line. The degree of ionization \( x = n_e/n \) in the upper optically thin layer was calculated in the same manner as for the solar corona. The degree of ionization \( x^* = n_e/n \) and the \( \text{Ha} \)-line loss in the lower layer with \( \tau_{\text{La}} > \tau_{\text{cr}} \) were calculated considering that the population of the second level of a hydrogen atom is determined by multiple scattering of \( \text{La} \)-quanta. At \( \tau_{\text{La}} > \tau_{\text{cr}} \), the solution for the two-level radiative transfer problem results in a Boltzmann distribution between the first and second levels of a hydrogen atom with the excitation temperature equal to the kinetic one.

Fig. 2. The degree of hydrogen ionization \( x = n_e/n \) and \( x^* \) in the \( \text{La} \) optically thin and thick layers respectively (see the expressions (A1) and (A2)).

(Ivanov, 1973). Figures 2 and 3 present values of \( x, x^* \), and the radiative loss at \( T < 4 \times 10^4 \, \text{K} \) for the case of the escape of all line photons from the medium. In addition, the total losses include the line emission of ions with \( 10^7 > T_i > 4 \times 10^4 \, \text{K} \) and the free-bound and free-free emissions from the hydrogen plasma. Some details of the calculations of the radiative loss may be found in the Appendix and in our work (Kosovichev et al., 1980).
The heating rate produced by the accelerated electrons due to Coulombic loss in partly-ionized plasma may be written (Kostyuk and Pikelner, 1974) as

\[ P_e/n = xP_1(\xi) + (1-x)\beta P_2(\xi), \]

where \( P_1(\xi) \) and \( P_2(\xi) = 0.3 P_1(\xi) \) (see expression (2) in Brown, 1973) are the ionization loss in the ionized and neutral media respectively. The factor \( 0 \leq \beta \leq 1 \) characterizes the portion of the energy of the total ionization loss which is directly converted to heat. The second term in (3) is, generally speaking, much smaller than the first term and, therefore, is usually neglected. The reason is that, if a neutral atom is excited by an accelerated electron, the excitation is removed when a photon is emitted. In this case, there is no heating of neutral gas and \( \beta \leq 0.1 \). The value \( \beta = 0.1 \) has been used in this section.

The function \( P_1(\xi) \) for an electron beam with energy \( E_1 \leq E \leq E_2 \) at \( \xi = \xi_{\text{min}} \) and

\[ \nu(E)N(E) \sim E^{-\gamma} \text{ cm}^{-2} \text{ s}^{-1} \text{ keV}^{-1}, \]

where \( N(E) \text{ cm}^{-3} \text{ keV}^{-1} \), the number of accelerated electrons has been taken to agree with Syrovatskii and Shmeleva (1972)); see the Appendix. It should be noted
that, according to the modern concepts (Vlahos and Papadopoulos, 1979), an isotropic beam agrees better with X-ray observations than an anisotropic beam. However, the calculations for the isotropic distribution of the velocities of non-thermal electrons gives but a small increase of the heating (the relevant discussion see in Somov et al., 1979).

The numerical characteristics of the heating flux will be selected considering that a hard X-ray burst, whose duration in energetic events is several minutes, may be divided into individual elementary bursts of durations ranging from several to ten seconds each (Van Beek et al., 1974, 1976). Accordingly, the actual lifetime of the non-thermal beam does not exceed 10 s in an elementary event. It seems questionable if the consecutive electron beams are injected at the same point on the Sun. Therefore, the effect of an electron beam on the chromosphere in a single elementary event of duration $t_h = 10$ s will be treated below (the energy of the beam incident onto the chromosphere $F_0 = \text{const at } 0 \leq t \leq t_h = 10$ s). It will be noted that the hard phase of a flare is composed of a number of such elementary events.

In the case of the Sun, the energy of accelerated electrons is chosen to be $F_0 \leq 3 \times 10^{11}$ erg cm$^{-2}$ s$^{-1}$. This choice is reasonable considering that the differential photon spectrum at $10 \leq h\nu \leq 100$ keV in the most studied large burst of November 5, 1970 was of the form $I = 2 \times 10^7 (h\nu)^{-\gamma}$ photon cm$^{-2}$ s$^{-1}$ keV$^{-1}$, where $\gamma = 4$ in the given case ($h\nu$ is expressed in keV). Such photon spectrum for the 'thick target' model corresponds to the energy injected in the layer (Korchak, 1976; see also formula (8) in Pikelner and Livshits, 1977) $\approx 6 \times 10^{29}$ erg s$^{-1}$.

Detection of a noticeable polarization of the X-ray emission from the above-mentioned flare requires that at least 10% of the power should be due to the emission of anisotropic beam. Since this energy is injected into the bright points of a $5 \times 5''$ total area ($S = 1.2 \times 10^{17}$ cm$^2$), we get the energy flux $F_0 \sim 5 \times 10^{11}$ erg cm$^{-2}$ s$^{-1}$. Our calculations (Kosovichev et al., 1980) were carried out for the fluxes $F_0 = 10^{10}$ and $10^{11}$ erg cm$^{-2}$ s$^{-1}$ at $\gamma = 3$ and 6. Since this soft X-ray burst was studied in the group of very small spots on November 5, 1970 ($\gamma = \varphi + 1 = 5$ in the thick-target model), we have assumed for a white flare that $F_0 = 3 \times 10^{11}$ erg cm$^{-2}$ s$^{-1}$ and $\gamma = 3$.

### 2.5. Numerical solution

Equations (1) were solved by the method of finite differences. Use was made of the completely conservative difference scheme (Samarsky and Popov, 1975; Kosovichev and Popov, 1979) in which the basic conservation laws of gasdynamics are satisfied in the difference grid in the presence of the gravitational field. The effect of the strong temperature dependence of the functions $\mathcal{L}$ and $x$ was eliminated using the Newton iteration method in each time step. The spatial grid was made denser in the region of pronounced variations in the calculated parameters. A time step in the initial stage of the process was $\sim 10^{-3}$ s, which was quite sufficient to test for the possible development of thermal instabilities in our calculations. An artificial viscosity was introduced when calculating the shock waves.
2.6. The dynamic response of the chromosphere to the heating by accelerated electrons

The numerical solution for the problem at $F_0 = 10^{11}$ erg cm$^{-2}$ s$^{-1}$ admits the following physical interpretation. During the initial stage $t = 0.5 - 1$ s, the chromosphere is heated down to $\xi_0 = 2 \times 10^{19}$ cm$^{-2}$ (the expression for $\xi_0$ is presented in the Appendix), i.e. down to altitudes of $\sim 2000$ km in the model applied. Motion are still practically absent and the density does not show any significant changes. Shock waves propagate upwards and downwards from the formed zone of high pressure (see Figure 1). The subsequent downward propagation of the disturbance is just the solution of the type of the second-kind temperature wave (TW II, Volosevich et al., 1963). The latter is characterized by a subsonic-velocity propagation of the thermal wave which drives a shock wave ahead. Figure 4 shows the propagation of the disturbance into the chromosphere at $F_0 = 10^{11}$ erg cm$^{-2}$ s$^{-1}$, $\gamma = 3$, $\beta = 0.1$ and the heating time $t_h = 10$ s. The abrupt change of the density at high-altitude side of condensation coincides with the temperature jump from $T \approx 8500$ K to $T \approx 2 \times 10^7$ K. It can be seen from Figure 4 that $n_{\text{max}} \leq 10^{15}$ cm$^{-3}$, $\Delta Z$ varies from one to several tens of km. The temperature behind the downward shock front is $1.5 \times 10^4$ K at time $t = 2.4$ s and 8500 K at time $t = 9.6$ s, whereas the mean temperature over the condensation is $\approx 8000$ K. Although, in accordance with the assumption made by Kostyuk (1976), the condensation may prove to be the source of the Balmer line emission (the H$\alpha$-line center optical depth of the condensation reaches $\sim 100$), the optical depth of the condensation in the continuum is small,

![Graph](image)

Fig. 4. The density profile of the condensation moving downwards. The time of heating by the accelerated electrons is 10 s.
$\Delta T_{5000} \sim 10^{-4}$. Hence, if only the ionized component is heated ($\beta = 0.1$), the gas under the temperature jump remains essentially cold and weakly-ionized and, therefore, cannot produce the continuum emission.

The thermal instability developing in the relaxation zone behind the shock front gives rise to formation of cold and dense structures within times $\sim 10^{-2}$ s. However, in the case of a powerful heating with $F_0 \geq 10^{11}$ erg cm$^{-2}$ s$^{-1}$, the generated fluctuations are smoothed within $\approx 2 \times 10^{-2}$ s in the course of finding of the equilibrium parameters behind the shock front (the specific analysis see in Kosovičev, 1979).

3. White Flare on the Sun

It was noted above that the optical depth of the downward propagating condensation in the continuum at $F_0 \leq 10^{11}$ erg cm$^{-2}$ s$^{-1}$ does not exceed $10^{-4}$. However, the conditions under which, according to the assumption made by Somov et al. (1979), a downward condensation may well prove to be the source of the optical continuum are probably realizable in the case of extremely large and very hard fluxes of the accelerated electrons in the presence of an additional heating of neutral gas due, for example, to absorption of an anomalously high flux of soft X-rays. The white-light continuum is extremely rare during solar flares and, at the same time, constitutes a characteristic feature of more powerful flares in the UV Ceti type stars. We consider first the possibility of white light emission to be generated at bright points in solar flares and, after that, in the flare stars.

In downward propagating condensations at high $F_0$, the density of neutral gas can exceed $n = 10^{15}$ cm$^{-3}$. In this case the probability of the occurrence of a second-type impact rather than of an emission of a quantum after excitation is $n_2 C_{21}/(A_{21} + n_2 C_{21})$, where $n_2$ is the population of the second level; $C_{21}$ is the de-excitation rate. Such probability is $\sim 10^{-2}$ in the condensation ($\leq 10^{-6}$ outside the disturbance), which is quite sufficient for the energy to be transferred into heat as a result of the second-type collisions during multiple scatterings.

Also, soft X-ray emission from powerful solar flares is characterized by $EM \geq 10^{50}$ cm$^{-3}$ and $T \geq 3 \times 10^{7}$ K. The absorption of such emission (Somov, 1975) in the case of incident flux $\phi_0$ (erg cm$^{-2}$ s$^{-1}$) results in the heating $P = 0.1 \phi_0 n(\xi)/\xi$ which gives $P \sim 10^{-7} \phi_0$ erg cm$^{-2}$ s$^{-1}$ at $\xi = 10^{21}$ cm$^{-2}$ and $n = 10^{15}$ cm$^{-3}$. Such heating at large values of $\phi_0 (\sim 10^9$ erg cm$^{-2}$ s$^{-1}$) begins to exceed the heating of neutral gas in the corresponding deep layers by the electron fluxes of moderate hardness ($\gamma > 3$). The absorption of $\lambda > 912$ Å radiation may also contribute to the heating of the deepest zones of the condensation. The absorption of soft X-rays and EUV emission and the second-type impacts have been included by formally setting $\beta = 1$ in (3) instead of $\beta = 0.1$ as in paper by Kosovičev et al. (1980).

The energy loss was calculated in the same way as described in Section 2 and in the Appendix. At large $F_0$, however, $\tau_{Ha}$ at line center in the condensation, is already close to $10^3$. In this case, the probability of Hα-quantum escape from the deep layers becomes less than unity. The probability of the Hα-photon escape from the level...
Fig. 5. The profiles of temperature, velocity, and density at various moments during a solar flare. The positive velocities correspond to the plasma moving away from the surface. The photosphere is to the left; $\xi = 10^{21}$ cm$^{-2}$ at $n = 10^{15}$ cm$^{-3}$ corresponds to $\Delta Z = 10^6$ cm. The dashed line shows the initial model.
\( \tau_{\text{Ha}} = 1000 \) has been taken to be 1/2 (using the same expressions as for the \( \text{L} \alpha \), see Ivanov (1973) and Kosovichev et al. (1980)), which actually means a somewhat overestimated loss from the deepest layers.

Here we have presented the results of the calculations for \( F_0 = 3 \times 10^{11} \text{ erg cm}^{-2} \text{ s}^{-1} \), \( \gamma = 3 \), \( \beta = 1 \), \( t_0 = 10 \text{ s} \). It can be seen from Figure 5 that, in the condensation, \( n_{\text{max}} \approx 10^{15} \text{ cm}^{-3} \) and the velocity of the downward motion of the gas \( v = 100 \text{ km s}^{-1} \). The temperature behind the shock front is \( \approx 10^5 \text{ K} \) \( (t = 0.8 \text{ s}) \), \( 1.4 \times 10^4 \text{ K} \) \( (t = 2.3 \text{ s}) \) and, after that, coincides practically with the overall temperature of the condensation which remains high \( (T \approx 8500–9000 \text{ K}) \). In this case, the degree of ionization \( \chi^* \) is \( 10^{-2}–10^{-1} \), thereby increasing the optical depth \( \Delta \tau_{5000} \) of the downward propagating condensation by several orders of magnitude. The derived distribution of physical parameters can be used to calculate

\[
\frac{d\tau_{5000}}{\kappa_{5000} \, dz} = a p_e \, d\xi,
\]

where \( a \) is the coefficient of continuous absorption per atom of neutral hydrogen and per unit electron pressure \( (p_e) \); \( a \) has been taken from Bode (1965). By integrating (4), for example, for \( t = 7 \text{ s} \), we get the optical depth of the condensation \( \Delta \tau_{5000} = 0.027 \). Figure 6 shows the excess of the white-light flare brightness \( I_{fl} \) over the photosphere \( I_{ph} \) at \( \lambda = 5130 \text{ Å} \) at the center of solar disc \( ((I_{fl} - I_{ph})/I_{ph}) \). Obviously, \( a \sim 100\% \) excess of the brightness is the upper boundary of the possible values in our

![Fig. 6. The light curve in \( \lambda = 5130 \text{ Å} \) for an elementary event on the Sun.](image-url)
model, while a decrease in the value of neutral gas heating (i.e., in $\beta$) gives rise to the corresponding decrease in the white-light flare contrast discussed here.

4. Flares on UV-Ceti Type Stars

The recent results obtained when studying the structures of red dwarf chromospheres, and in detecting the X-ray emission from the flares on red dwarfs, have made it possible to carry out the same calculations for these objects. Equations (1) were solved together with (2). The functions for heating (3) and cooling of the plasma were selected in the same way as in the dynamic response with due account of the remarks made in the beginning of Section 3. Because of the absence of information about the coronas of the flare stars, we shall somewhat modify the boundary conditions. Namely, the upper chromospheric boundary $\xi = \xi_{\text{min}}$ will be treated as a free surface (Somov et al., 1979). In this case the effect of the corona will be included by setting a constant external pressure $p(\xi_{\text{min}}, t) = p_0$. According to the preliminary analysis of the IUE observations (Linsky et al., 1978), the pressure at the coronal base $p_0$ is three times the solar value (i.e., $p_0 = 0.3$ dyn cm$^{-2}$, as for the $\varepsilon$ Eri (K2 V)), because amongst IUE-objects the spectral type of this star is the most similar to the spectral type of UV-Ceti stars), whence $\xi_{\text{min}} = p_0/(m_{\text{H} \text{g}}) = 3 \times 10^{18}$ cm$^{-2}$. The lower boundary $\xi = \xi_{\text{max}} = 5 \times 10^{23}$ cm$^{-2}$ have been selected to lie deep in the photosphere where we have assumed $v(\xi_{\text{max}}, t) = 0$. The thermal conduction flux at the lower and upper boundaries has been again assumed to be absent: $W(\xi_{\text{max}}, t) = W(\xi_{\text{min}}, t) = 0$.

The initial model shown with the dashed line in Figure 7 was selected in the following way. A level with $z = 1200$ km at which $n = 2 \times 10^{16}$ cm$^{-3}$, $T = 3000$ K was assumed to be the lower boundary in the atmosphere of a dM1--2 star (Mould, 1976) with $T_{\text{eff}} = 3250$ K and $\lg g = 4.75$. The temperature in the higher (chromospheric) levels was determined from the condition that $dT/d\lg \xi \sim 1500$ K for stars of this type (Kelch et al., 1979), whence we get $T = 7500$ K near the upper boundary as $\xi$ changes from $10^{22}$ cm$^{-2}$ to $10^{19}$ cm$^{-2}$. After that, the height dependence of the temperature was used to calculate the initial model on the basis of the hydrostatic equation.

Presented here are the results of calculations for $F_0 = 10^{12}$ erg cm$^{-2}$ s$^{-1}$, $\gamma = 3$, $\beta = 1$. These values were selected on the following grounds. We assume that, as in the Sun, a major portion of the flare energy is in non-thermal beams similar in magnitude with the energy of soft X-rays. In the large flare of the YZ CMi, the X-ray luminosity in the 1--7 keV energy range was $3.6 \times 10^{30}$ erg s$^{-1}$ (Heise et al., 1976). Assuming here and, henceforth, that the flare area is $\approx 0.01$ of the surface of the visible disc of the star ($S = 3 \times 10^{18}$ cm$^2$ as inferred from the optical observations; see, for example, Gershberg, 1978), we obtain that the energy flux heating the chromosphere is $F_0 = 10^{12}$ erg cm$^{-2}$ s$^{-1}$. The Fe xxv lines near 6.6 keV in the X-ray spectra of the flare are sufficiently intense (Kahn et al., 1979). If these lines are actually generated by the absorption of the accelerated electrons in a 'thick target', then the detection of the intense $\approx 6.6$ keV lines is indicative of a hard incident flux. Considering also that
the solar 6.6 keV line is detectable during the hard phase of flares with large 50–100 keV electron fluxes, we conclude that the spectrum of the electrons accelerated in red dwarfs is relatively flat ($\gamma = 3$).

The dynamic response of a stellar chromosphere is the same in its general features as on the Sun (see Figure 7). A downward motion of $a \geq 100$ km s$^{-1}$ velocity appears under the temperature jump behind the shock front. The coronal plasma with $T \sim 3 \times 10^7$ K is streaming outwards at a $\sim 1000$ km s$^{-1}$ velocity. On red dwarfs, the thickness of the downward condensation is $\sim 1$ km in the beginning of the process and increases to 10 km, i.e., an order below the relevant values for the Sun. At densities $n \geq 10^{11}$ cm$^{-3}$, the temperature in the condensation is maintained at 9000 K and, correspondingly, $x^* \approx 0.07$. The calculations of the optical continuum depth using the same procedure as in Section 3 gives $\Delta \tau_{4500} \sim 1$ (see Figure 8). This result is important since it shows that a condensation is formed and that this condensation is responsible for the flare continuum.

The relationship between the luminosity $L_{\text{fl}}$ of a flare at $\lambda = 4500$ Å and the star luminosity $L_*$ may be expressed as

$$\frac{L_{\text{fl}}}{L_*} = \frac{SB(T_{\text{fl}})}{\pi R_*^2 B(T_{\text{eff}})} [1 - \exp(-\Delta \tau_{4500})],$$

where $B(T)$ is the Planck function. For $S = 3 \times 10^{18}$ cm$^2$, $R_* = 0.3 R_\odot$, $T_{\text{fl}} = 8900$ K, and $T_{\text{eff}} = 3250$ K for the times from 3 to 8 s, we get $L_{\text{fl}}/L_* \approx 0.7$. Obviously, in the case of occurrence of such a flare on a star of smaller radius ($R_* \approx 0.1 R_\odot$), we get $L_{\text{fl}}/L_* \approx 6.3$, which corresponds to a large flare in the B filter.

The theoretical diagrams, calculated by Grinin and Sobolev (1977) for hydrogen plasma with $n = 10^{15} - 10^{17}$ cm$^{-3}$ and $T = 5000 - 20000$ K, have made it possible to estimate the photometrical parameters of the emission from such a condensation. At $n = 2 \times 10^{15}$ cm$^{-3}$, $T \approx 9000$ K, and a 10 km thickness, we get $U - B = -1$, $B - V = 0.5$ in a good agreement with the observed blue-white radiation of stellar flare (Gershberg, 1978). Again in accordance with Grinin and Sobolev (1977), and for the same parameters assuming an optically thin hydrogen plasma, we find that the Balmer jump is $\approx 0.8$. The fact that $\Delta \tau_{4500} \sim 1$ reduces this theoretical estimate, thereby making it closer to the observed values.

5. Discussion

We shall discuss only the processes in the low-temperature region of the bright points of flares. The gas condensation may be the source of the Balmer-line emission (see Figure 4). Although the major energy release from the heating agent takes place in the zone of the temperature jump, which is located immediately above the condensation, a slight heating of the inner zone of the condensation also takes place. At very high hydrogen densities the Balmer loss proves to be very effective. In this case, the energy balance in the condensation results in temperatures not above 8000 K. This
Fig. 7. The profiles of $T$, $v$, and $n$ for the process on a red dwarf. The positive velocities correspond to the ascent of gas. The dashed line shows the initial model.
Fig. 8. The optical thickness of the condensation in \( \lambda = 4500 \, \text{Å} \) on a red dwarf.

circumstance determines the relevant low values of \( x^* \) and \( \Delta \tau_{5000} \sim 10^{-4} \), so the computed optical continuum is negligible.

However, the above described results with \( F_0 = 10^{11} \, \text{erg cm}^{-2} \text{s}^{-1} \), mean that the H\( \alpha \)-center optical depth can reach \( \sim 100 \) in agreement with the observations of bright points. In the case of a somewhat more intense heating of the ionized and neutral components of plasma, \( \tau_{\text{H}\alpha} \) will exceed the critical value \( (\tau_{\text{H}\alpha} \sim 1000) \) at which the H\( \alpha \)-photon escape is hampered. The Balmer loss fails to compensate for the heating producing the temperature increase.\(^\dagger\) It is the increase in \( T \) by 500–1000 K, inside the condensation, that results in the generation of the observable optical continuum. It should be noted that the value \( T \approx 8500 \, \text{K} \) is in a good agreement with observations of white-light flares on the Sun and UV-Ceti stars (Machado and Rust, 1974; Mochnacki and Zirin, 1980).

Unfortunately, it has not been clear as yet in what form the energy is supplied from above to the bright-point zones (anisotropic or isotropic beams; accelerated particles, heat or radiative fluxes). This is why the numerical characteristics of the energy flux are determined in terms of simple model representations ('thick model', etc.). At the same time, our calculations have shown that the characteristic secondary processes can well be developed in the case of a sufficiently powerful heating of the chromosphere lasting for several seconds. In terms of the studied secondary processes, the generation of white continuum is facilitated by either \( F_0 > 3 \times 10^{11} \, \text{erg cm}^{-2} \text{s}^{-1} \) at \( \gamma \leq 3 \) or an intense source of soft X-rays with \( T_0 > 5 \times 10^7 \, \text{K} \) or a combined effect of two factors. Indeed, an excess of the white light continuum

\(^\dagger\) This conclusion does not follow from our calculations directly. We ignored the decrease of the probability of H\( \alpha \)-quantum escape from the medium. Thus, the Balmer loss was overestimated. We formally compensated the influence of this factor by the same increase of \( F_0 \).
brightness by more than 5% is possible not only in the case with $\beta = 1$ studied here but also for $\beta = 0.3$, $T_0 = 5 \times 10^7$ K, etc. The only factor of significance here is that the applied characteristics of the heating source are very close to the highest values inferred from the X-ray observations.

In the case of multiple successive occurrences of the elementary events discussed above, the shape of the light curve in the optical continuum (or of the curve of stellar flare brightness) should coincide with the form of the time profile of the heating flux. The profile resembles an X-ray burst at $h\nu \approx 5-10$ keV, and the durations of the white light and X-ray emissions may be different in extremely rare cases.

Thus, in the case of an ordinary low-temperature flare, the heating within the condensation is balanced by Balmer-line emission. Once the optical depth in white light becomes noticeable, the loss in the optical continuum becomes the dominating radiative loss. In this case the energy balance gives temperatures about $10^4$ K. During the initial stage of each of the elementary events, a condensation with temperatures of tens of a thousand degrees is formed to exist for a short time. However, our calculations have shown that, when averaged over the hard phase, the temperature should not exceed $10^4$ K. This fact, as well as the downward motion of the condensation at velocities of tens of km s$^{-1}$ in the initial stage of each elementary event, deserves serious experimental verification.

**Appendix**

Let us consider the hydrogen plasma kinetics as applied to sufficiently dense regions in the chromospheric layers of a flare. In a layer from which L$\alpha$-quanta may escape, the hydrogen ionization is calculated in the same way as in the case of the optically thin plasma:

$$n_1 n_e q_1 = n_1 n_p \alpha,$$

or

$$\frac{1-x}{x} = \frac{\alpha}{q_1},$$

(A.1)

where $x = n_e/n$, $q_1$ is the rate of ionization by electron impact of a hydrogen atom from the ground level in cm$^3$ s$^{-1}$; $\alpha$ is the rate of recombination to all levels of the hydrogen atom in cm$^3$ s$^{-1}$; $n_1$ is the population of the ground level; $n_e$ and $n_p$ are the electron and proton densities.

In the deeper chromospheric layers, the relative population of the second level of a hydrogen atom increases significantly, due to photo-excitation $1 \rightarrow 2$, by the scattering of the L$\alpha$-quanta, which are trapped for the case of the optically thick L$\alpha$-layer studied here. Hence, the increased importance of the ionization from the second level by both photospheric radiation (in rarified layers) and electron impact (in dense layers). In this case, the emission coefficient of hydrogen plasma in the Balmer lines increases significantly.

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It proves possible to include the field of the intrinsic Lα-emission since the solution for the equation of transfer in the case of a two-level atom is known in some specific cases (Ivanov, 1973). Despite the fact that the analytical methods are applied to idealized conditions (the physical parameters are constant with depth), while the numerical methods fail to deal with the diverse variety of the practical problems, one of the main results obtained by solving the equations of transfer for a two-level atom is that the second-to-first level population ratio in sufficiently deep layers is given by the Boltzmann formula with the excitation temperature equaling the kinetic temperature. This conclusion will be considered as satisfied at \( \tau_{Lα} > \tau_{cr} \). Then, after substituting the degree of hydrogen ionization \( x^* = n_e/n \) in the layers with \( \tau_{Lα} > \tau_{cr} \), we get in the case of sufficiently dense plasma:

\[
\frac{n_2}{n_1} = \frac{g_2}{g_1} \exp \left( -\frac{\chi_{12}}{kT} \right),
\]

\[n_1n_eq_1 + n_2n_eq_2 = n_en_p(\alpha - \alpha_1),\]

where \( n_2 \) is the number of hydrogen atoms at the second level; \( g_1, g_2 \) are the statistical weights; \( \chi_{12} \) is the excitation potential of the second level; \( q_2 \) is the rate of electron impact ionization from the second level. Recombination into the first level (whose rate is \( \alpha_1 \)) has been excluded here; i.e., for in the case of a noticeable optical depth in the Lyman continuum, a photon with wavelength smaller that 912 Å originated from the recombination will again ionize the neutral atom (similarly to the treatment of the Strömgren zones). The equations (A2) at known \( n \) and \( T \) determine \( x^* = n_e/n \) and \( n_2 \).

In the case complete escape of photons from the medium, the energy loss in the Lα-line is

\[\mathcal{L}_{Lα} = 4\pi\varepsilon_{Lα} = n_2A_{21}h\nu_{Lα} = n_1n_eC_{12}h\nu_{Lα},\]

where \( \varepsilon_{Lα} \) erg cm\(^{-3}\) s\(^{-1}\) sr\(^{-1}\) is the volume emission coefficient; \( A_{21} \) is the probability of spontaneous transition; \( \nu_{Lα} \) is the frequency of the transition in the line; \( C_{12} \) is the rate of electron impact excitation in cm\(^5\) s\(^{-1}\).

The Hα line loss in the lower layer was calculated as

\[\mathcal{L}_B = 4\pi\varepsilon_B = n_3A_{32}h\nu_{Hα} = n_2n_eC_{23}h\nu_{Hα},\]

where \( n_3 \) is the population of the third level; \( A_{32} \) is the probability of the transition in Hα; \( C_{23} \) is the rate of the electron impact excitation from the second level to the third one. The value of \( n_2 \) was determined from (A2).

It will be noted that the effect of the Sun’s photosphere radiation field is neglected in the Lα optically thick layers. The contribution of photoprocesses, as compared with electron impact, is given by the relations \( \varphi_2/n_eq_2 \approx (3 \times 10^{12})/n_e \) in (A2) and \( \varphi_{23}/(n_eC_{23}) \approx 10^{12}/n_e \) in (A4), where \( \varphi_2(\text{s}^{-1}) \) is the rate of photoionizations from the second level; \( \varphi_{23} \) is the rate of photoexcitations \( 2 \rightarrow 3 \). At the same time, the gasdynamic calculations show that the initial heating of rarified layers is very rapid.
and, after that $n_3 > 3 \times 10^{12} \text{ cm}^{-3}$ everywhere in the $\text{La}$ optically thick layers. In this case, therefore, the photoprocesses are neglected even for the case of Sun. This conclusion is more nearly valid for red dwarfs with $T_{\text{eff}} = 3250 \text{ K}$.

The probability of $\text{La}$-quantum escape from the medium was included when calculating the dynamic response by multiplying $L_\text{La}$ (see formula (A3)) by this probability. Besides that, we have explained the choice of $\tau_{\text{cr}} = 10^6$ for $\text{La}$ in the calculations of Kosovichev et al. (1980). The value $\tau_{\text{cr}} = 10^6$ occurs at $n = 10^{12} \text{ cm}^{-3}$ and $6 \times 10^{14} \text{ cm}^{-3}$ ($z = 2000 \text{ km}$ and $\approx 1000 \text{ km}$) in our quiet and flaring atmospheric models correspondingly.

The values of the rates of the elementary processes $q_1 = 6.0 \times 10^{-9} \exp (-\chi_\text{H}/kT)$ and $C_{12} = 2.0 \times 10^{-8} \exp (-\chi_{12}/kT)$ have been taken according to Vainshtein and Shevelko (1970); $\alpha$ and $\alpha_1$ have been taken from Burgess and Seaton (1960). The two-parameter approximations (Vainshtein et al., 1973) have been used for the rest coefficients.

The contribution made by ionization of metals has been added to the values $x = 2 \times 10^{-4}$ in the chromosphere and $x^* = 10^{-4}$ in the photosphere. The values of $x$, $x^*$ are presented in Figure 2. The total losses also include the $\text{H}^-$ ion emission (Henoux and Nakagawa, 1978):

$$L(\text{H}^-) = 1.5 \times 10^{-37} \frac{n_1 n_e}{T^{3/2}} (T^4 - T_{\text{min}}^4) \exp \left( \frac{8762}{T} \right),$$

where $T_{\text{min}} = 4200 \text{ K}$. In the regions where the metal-lines are formed the most significant $\text{Fe II}$ ion emission was taken into account according to Giovanelli (1978). The emission in the $\text{Ca II}$ and $\text{Mg II}$ resonance lines is neglected. This emission will effectively carry away the energy from the same layers in which the $\text{H}\alpha$ line quanta are emitted. However, the corresponding emission coefficient (per $1 \text{ cm}^{-3}$) in the above-mentioned lines, seems to be small as compared with the $\text{H}\alpha$ emission coefficient.

The free-bound, $L^p$, and free-free, $L^T$, emissions from the high-temperature layers have been included according to Sobelman (1977):

$$L^p + L^T = 5.5 \times 10^{-22} n_e^2 T^{-1/2} (1 + 0.25 \times 10^{-5} T) \text{ erg cm}^{-3} \text{ s}^{-1}.$$

The emission from the coronal ions have been calculated at the following abundances of the numbers of atoms:

<table>
<thead>
<tr>
<th>H</th>
<th>He</th>
<th>C</th>
<th>N</th>
<th>O</th>
<th>Ne</th>
<th>Mg</th>
<th>Si</th>
<th>S</th>
<th>Ca</th>
<th>Fe</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^4$</td>
<td>$10^3$</td>
<td>3</td>
<td>0.7</td>
<td>8</td>
<td>1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.03</td>
<td>0.5</td>
</tr>
</tbody>
</table>

(Figure 9, see also Raymond and Smith, 1977). The calculation techniques have been described by Vilkovisky and Obashev (1971). It should be noted that the choice of specific dependences of the energy loss at low ($T < 7000 \text{ K}$) and high ($T > 4 \times 10^4 \text{ K}$) temperatures is not essential to the given problem.
Fig. 9. The radiative loss by transparent plasma at $T > 3 \times 10^4$ K. The highly-ionized ions of the indicated elements make the major contribution to the loss at the corresponding temperatures.

Our calculations were carried out including the thermal conduction by neutral and ionized atoms. The specific expressions of the thermal conduction coefficient $x$ may be found in Kostyuk and Pikelner (1974).

The function of heating (according to Syrovatskii and Shmeleva, 1972) have been used in the form

$$P_1(\xi) = \frac{a}{2} F_0 E_{10} (2a \xi)^{-3/2} \varphi(\xi) \text{ erg s}^{-1},$$

where

$$\varphi(\xi) = \begin{cases} \frac{\pi}{2} \text{arctg} \sqrt{\frac{\xi - 1}{\xi_0 - 1}} - \frac{\xi}{\xi_0} \sqrt{\frac{\xi}{\xi_0} - 1} & \text{for } \xi < \xi_0, \\
\frac{\pi}{2} & \text{for } \xi \geq \xi_0. \end{cases}$$

Here, $\xi_0 = E_{10}^2/2a$; $a = 3.32 \times 10^{-37} (\ln (E_{10}/m_e c^2) - \frac{1}{2} \ln n + 38.7)$ is the ‘constant’ of ionization loss; $E_{10} = 10$ keV $= 1.6 \times 10^{-8}$ erg. In (1), the functions of heating and loss are substituted in units of erg g$^{-1}$ s$^{-1}$.
References