ALFVÉN WAVES IN THE SOLAR ATMOSPHERE

II: Open and Closed Magnetic Flux Tubes

JOSEPH V. HOLLWEG*

High Altitude Observatory, National Center for Atmospheric Research**, Boulder, Colo. 80307, U.S.A.

(Received 7 March, 1980)

Abstract. The linearized propagation of axisymmetric twists on axisymmetric vertical flux tubes is considered. Models corresponding to both open (coronal hole) and closed (active region loops) flux tubes are examined. Principal conclusions are: Open flux tubes: (1) With some reservations, the model can account for long-period ($T \approx 1$ hr) energy fluxes which are sufficient to drive solar wind streams. (2) The waves are predicted to exert ponderomotive forces on the chromosphere which are large enough to alter hydrostatic equilibrium or to drive upward flows. Spicules may be a consequence of these forces. (3) Higher frequency waves ($10 \leq T \leq \text{few min}$) are predicted to carry energy fluxes which are adequate to heat the chromosphere and corona. Nonlinear mechanisms may provide the damping. Closed flux tubes: (1) Long-period ($T \approx 1$ hr) twists do not appear to be energetically capable of providing the required heating of active regions. (2) 'Loop resonances' are found to occur as a result of waves being stored in the corona via reflections at the transition zones. The loop resonances act much in the manner of anti-reflectance coatings on camera lenses, and allow large energy fluxes to enter the coronal loops. The resonances may also be able to account for the observed fact that longer coronal loops require smaller energy flux densities entering them from below. (3) The waves exert large upward and downward forces on the chromosphere and corona.

1. Introduction

In a previous paper (Hollweg, 1978a; hereafter Paper I), the propagation of Alfvén waves in the solar atmosphere is discussed. Solution of the full wave equation in a full model of the solar atmosphere is given particular emphasis, because the small-wavelength WKB approximation is not appropriate for the wave periods of interest.

Paper I is motivated by in situ observations of Alfvén waves in the solar wind beyond 0.3 AU, by possible indirect evidence provided by Faraday rotation data for the presence of Alfvén waves in the corona between 2 and $10R_\odot$ ($R_\odot$ is the solar radius and $R$ is heliocentric distance), and by a body of theoretical work indicating that at least the high-speed solar wind flows may be driven by Alfvén waves (see Paper I; and Hollweg, 1978b, for references). If the waves in the solar wind are of solar origin, energy flux densities at the Sun of at least $4000 F_{\text{max}} \text{erg cm}^{-2} \text{s}^{-1}$ are implied by the in situ data in high-speed solar wind streams, while energy flux densities of about $5 \times 10^4 F_{\text{max}} \text{erg cm}^{-2} \text{s}^{-1}$ are suggested by the theoretical wave-driven models for the high-speed solar wind streams. ($F_{\text{max}}$ is the ratio of the solid angle subtended by a solar wind flow tube far from the Sun to that at $R = R_\odot$). The average value of $F_{\text{max}}$ for the hole studied by Munro and Jackson (1977) is 7.26, while $F_{\text{max}} = 4$ might be more appropriate near the center of the hole (G. Pneuman, private communication, 1976). In general, therefore, the data imply Alfvénic flux

* Present address: Department of Physics, University of New Hampshire, Durham, N.H., 03824, U.S.A.

** The National Center for Atmospheric Research is sponsored by the National Science Foundation.


Copyright © 1981 by D. Reidel Publishing Co., Dordrecht, Holland, and Boston, U.S.A.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System
densities of $10^4$–$10^5$ erg cm$^{-2}$ s$^{-1}$ at the Sun. Both the Faraday rotation and *in-situ* data show that most of the wave power is at long periods, the autocorrelation time being of the order of 1 hr.

Paper I therefore addresses the question of whether it is possible to construct a theory in which the convective motions at the sun generate Alfvén waves with flux densities of $10^4$–$10^5$ erg cm$^{-2}$ s$^{-1}$ at these long periods. Paper I demonstrates the existence of a series of peaks in the predicted energy flux at periods downwards from 1.6 hr, in which the flux densities could be in the range $10^4$–$10^5$ erg cm$^{-2}$ s$^{-1}$. However, there are three reasons why these peaks do not represent a satisfactory mechanism for providing the required energy flux densities:

(i) Even though the lowest frequency peaks are the broadest, they are nonetheless quite narrow. For example, Figure 3 of Paper I shows that the width of the broadest peak is only $\Delta f/f \approx 0.1$ at the level where $P_\odot = 10^5$ erg cm$^{-2}$ s$^{-1}$ ($P_\odot$ is time-averaged wave Poynting flux density at the Sun, and $f$ is frequency).

(ii) For $P_\odot = 10^4$–$10^5$ erg cm$^{-2}$ s$^{-1}$, the model of Paper I implies *horizontal* magnetic field fluctuation amplitudes of 350–1200 G at the long periods. Horizontal magnetic fields of this magnitude are not compatible with observations.

(iii) For long-period waves having $P_\odot = 10^4$–$10^5$ erg cm$^{-2}$ s$^{-1}$, the predicted variation with height of the horizontal wave velocity amplitude does not agree with observation.

Paper I assumes that the vertical background magnetic field strength in the lower solar atmosphere is constant with height. However, the magnetic field at the Sun is not uniform, but at photospheric height is concentrated into small regions (‘magnetic flux tubes’) having $|B_0| = 1500$–2000 G, which expand at greater heights to yield field strengths between several gauss (quiet regions) and a hundred gauss or so (active regions). The rapid decrease with height of $|B_0|$ in the magnetic flux tubes means that the Alfvén speed in the flux tubes has a very different variation with height than in Paper I. This, in turn, means that the propagation of Alfvén waves on flux tubes can differ greatly from the results found in Paper I. The purpose of the present paper is, therefore, to explore the properties of Alfvén waves in magnetic flux tubes in the solar atmosphere.

One of our principal emphases will still be to ask whether it is possible to construct a theory in which the solar convective motions generate long-period Alfvén waves with $\langle P_\odot \rangle = 10^4$–$10^5$ erg cm$^{-2}$ s$^{-1}$ (the angle brackets denote an average taking the ‘area filling factor’ of the flux tubes into account). It will be found that the magnetic flux tubes lead to a much more favorable picture in this regard than is the case in Paper I. With some reservations, the required energy flux densities at long periods can now be accounted for without the necessity of resorting to narrow resonance, the predicted horizontal magnetic field fluctuation amplitudes are now only a fraction of the background field in the flux tubes, and the predicted wave velocity amplitudes vary with height in a manner which is not inconsistent with observations. That wave propagation on magnetic flux tubes leads to an improved picture in these three respects is a consequence of the importance of finite-wavelength effects. Consider
the calculation of $\langle P_\odot \rangle$ as an illustration: In Paper I, $v_A$ is a very rapidly increasing function of height below and in the chromosphere-corona transition region. This leads to very strong reflections of the upward-propagating waves, with the result that the wave pattern is nearly standing and, except in the peaks, very little energy propagates upward to 'infinity'. (The peaks are themselves a consequence of the standing wave pattern. If the wave period is not too long, there are regions where the wave velocity amplitude is very small compared to the amplitudes elsewhere; these regions are in essence nodes. Now Paper I applies the boundary condition that $|\delta v| = 1$ km s$^{-1}$ at the base of the model. But if the base of the model corresponds to a node, then it is clear that $|\delta v|$ must be very large away from that node. The atmosphere becomes highly disturbed, and peaks with large wave energy flux densities result.) Now suppose that the magnetic field is concentrated into flux tubes. High in the solar atmosphere, where the flux tubes are essentially fully expanded to yield a vertical magnetic field whose strength is nearly constant with height, $v_A$ will have the same values as in Paper I if the atmospheric density and magnetic field strength are the same in both models. But at lower heights, $v_A$ will be larger in the flux tubes than in the model of Paper I, because the field is stronger there. Therefore, the gradient of $v_A$ with height will be considerably less in the flux tubes than in the model of Paper I. The most important consequence of this is that the wave reflection in the flux tubes is much less severe than in the model of Paper I, and more energy propagates upward to infinity, i.e. $\langle P_\odot \rangle$ is enhanced. Finite wave-length effects, as manifested by the reflections, are thus a crucial factor in determining $\langle P_\odot \rangle$.

This paper also discusses a number of features of Alfvén waves which are not considered in Paper I. The analysis is extended to include closed magnetic structures, in particular coronal active region loops. Some basic differences between Alfvén wave propagation on open and closed magnetic structures are emphasized; in particular, the possibility of 'loop resonances' is pointed out, and it is argued that they may represent a 'window' by which large fluxes of Alfvén waves can penetrate the photosphere and chromosphere and enter the corona. In addition, the forces exerted by the waves on the solar plasma are calculated taking into account the full finite-wavelength nature of the problem. It is found that the wave forces can be large, especially for wave periods less than a few minutes or so, with upward (and sometimes downward) forces comparable to or in excess of gravity, suggesting that hydrostatic equilibrium can be strongly violated and even that upward (and sometimes downward) motions can be driven by the waves, much in analogy with the way waves are thought to drive high-speed wind flows (see Hollweg, 1978b, for a review); spicules are one possible consequence of these forces. The analysis is extended to a discussion of waves with periods as short as about 10 s. It is found that if these short-period waves are driven by photospheric velocities with magnitudes comparable to those which Deubner (1976) has estimated for the 'short-period acoustic waves', then they may be able to supply the energy required by magnetic regions of the solar atmosphere and to drive atmospheric motions such as spicules. The present paper also makes a very preliminary analysis of the consequences of joule-frictional
damping of Alfvén waves in the photosphere and chromosphere; it is found that in active regions finite-wavelength effects enhance the heating due to these processes by two to three orders-of-magnitude over what other workers have previously estimated using WKB theory, and that joule-frictional damping of high-frequency Alfvén waves has the potential of satisfying the energy requirements of the magnetic regions of the photosphere and chromosphere. Finally, some speculations are offered concerning other possible damping mechanisms for the Alfvén waves. Although much work remains to be done, it appears at least possible that the energy requirements of the magnetic regions of the solar atmosphere, including spicules and coronal and chromospheric heating, may be supplied by Alfvén waves which originate in the convection zone.

The plan of this paper is as follows: The assumptions and basic equations are discussed in Section 2. Sections 3 and 4 present solutions to the wave equations on model representations of flux tubes which open into a coronal hole and which are closed into coronal active region loops, respectively; as was the case in Paper I, emphasis is given to solving the full wave equation in realistic models of the magnetic flux tubes with a proper boundary condition at ‘infinity’. Section 5 estimates the photospheric and chromospheric joule-frictional damping, and also speculates about other mechanisms by which the waves could be dissipated.

2. Basic Equations

Consider a potential magnetic field structure which exhibits axisymmetry. Take the axis of symmetry of the field to be vertical, so that if the usual cylindrical coordinates \((r, \theta, z)\) are employed, then \(z\) increases with height. The field is imagined to have an untwisted fleur-de-lis structure with \(B_{\theta\phi} = 0\). The field lines are concentrated in the photosphere, fan out in the chromosphere, and become essentially vertical in the upper chromosphere and corona, as in Gabriel (1976). Since we want to limit our discussion to pure linear Alfvén waves, which are noncompressive and do not couple to the radiation field or to gravity, we take \(\delta v_r = \delta v_z = 0\) and \(\delta v_\theta = \delta v_\phi(r, z, t)\); this corresponds simply to an axisymmetric twist of the flux tube. The corresponding magnetic field fluctuation obeys \(\delta B_r = \delta B_z = 0\) and \(\delta B_\theta = \delta B_\phi(r, z, t)\). Since the background motions of the solar atmosphere are probably only a small fraction of \(v_A\) everywhere in the magnetic flux tube (except perhaps on spicules), it is adequate to take \(v_0 = 0\); the effects of downflows on the wave propagation should be weak, since they are sub-Alfvénic. No allowance is made for dissipation at this stage. Finally, it is tacitly assumed that the flux tubes are stable over time scales comparable to or longer than the wave period.

The appropriate linearized equations for momentum conservation and the freezing-in of field lines are

\[
i\omega p_0 \delta v = (B_0 \cdot \nabla \delta B + \delta B \cdot \nabla B_0)/4\pi, \tag{1}
\]

\[
i\omega \delta B = \nabla \times (\delta v \times B_0), \tag{2}
\]
where the time-dependence is \( \exp (i \omega t) \). After a little manipulation, these become

\[
-\omega^2 x = \frac{B_0}{4 \pi \rho_0} \frac{\partial}{\partial s} \left[ r^2 B_0 \frac{\partial x}{\partial s} \right],
\]

(3)

\[
-\omega^2 y = r^2 B_0 \frac{\partial}{\partial s} \left[ \frac{B_0}{4 \pi \rho_0 r^2} \frac{\partial y}{\partial s} \right],
\]

(4)

\[
i \omega y = r^2 B_0 \frac{\partial x}{\partial s},
\]

(5)

where \( x = \delta v_\theta/r, \) \( y = r \delta B_\theta, \) and \( s \) is distance along a given field line when moving in the direction of \( +B_0 \). Equations (3) and (4) are wave equations for the velocity and magnetic field fluctuations. The only spatial variable is \( s \), because it is a property of Alfvén waves that information propagates only along \( B_0 \).

If the structure and geometry of the flux tube were known exactly, so that \( B_0, \rho_0, \) and \( r \) were known functions of \( s \), then it would be possible to solve Equations (3) and (4) exactly, subject to appropriate boundary conditions. We feel, however, that current knowledge of the structure and geometry of solar magnetic flux tubes is insufficient to warrant such a detailed approach. Instead we shall make the following approximation: If only field lines which are near the \( z \)-axis are considered, then it is easily shown that for them

\[
r(s)^2 B_0(s) \approx \text{constant}.
\]

(6)

In that case, (3) and (4) become

\[
-\omega^2 x = v_\lambda^2 \frac{\partial^2 x}{\partial s^2},
\]

(7)

\[
-\omega^2 y = \frac{\partial}{\partial s} \left( v_\lambda^2 \frac{\partial y}{\partial s} \right).
\]

(8)

The advantage of (7) and (8) is that \( r(s) \) is no longer a parameter. Similarly, \( r(s) \) is no longer a parameter in Equation (5). It is now only necessary to specify the variation of \( B_0 \) with height along the axis of the flux tube; \( r^2 \) is thereby implicitly specified via its proportionality to \( B_0^{-1} \).

The mathematical analysis of Paper I (see also Hollweg, 1972; Nye and Hollweg, 1980) can therefore be used without modification. As in those papers, a piecewise-exponential representation for \( v_\lambda(s) \) is made, and Equation (7) is solved analytically for \( x \) in each piece. Equation (5) determines \( y \) in each piece. The complete solution is then determined by requiring continuity of \( \delta v_\theta \) and \( \delta B_\theta \) at the boundaries between pieces, coupled with the specification of \( \delta v_\theta \) at the bottom of the flux tube and a boundary condition at 'infinity'. We arbitrarily take \( \delta v_\theta = (1 \text{ km s}^{-1}) \exp (i \omega t) \) at the bottom. On the open field lines it is required that there be only an outgoing wave at \( R = 6 R_\odot \), while on closed field lines it is required, somewhat arbitrarily, that the waves be absorbed at the far end of the flux tube; in each case the correct outer boundary condition is given by Equation (2) of Paper I. These boundary conditions
require some explanation. On the open field lines, the outer boundary condition is imposed high enough in the corona that the WKB approximation is well-satisfied. In that case the functional forms of outward- and inward-propagating waves can be precisely specified, and it is a simple matter to require that the inward-propagating component of the wave field vanish. The closed field line case is less straightforward. The WKB approximation is usually not valid at the far end of the flux tube, and there is no strong argument against the possibility of waves being reflected from the photosphere or convection zone at the far end of the flux tube. However, it seems at least possible that the waves could penetrate into the convection zone and be absorbed or scattered by the turbulent motions and tangled magnetic fields there. Moreover, we will be dealing with the flux tube models in which \( V_A \approx \text{constant near the top of the convection zone and in the low photosphere, and this near constancy of} V_A \text{ at the far end of the flux tube may at least minimize the wave reflections there. Accordingly, we feel there is some justification for imposing an absorbing boundary condition at the far end of a closed flux tube, but the reader should bear in mind that this is being done to some extent by default.}

The mathematical analysis appearing in our previous papers will not be repeated here. But the following additional features will be considered:

The perpendicular (to \( B_0 \)) component of the wave current density, denoted \( \delta j_\perp \), follows directly from the momentum equation

\[
i \omega \rho_0 \delta v_\theta \hat{\theta} = \delta j_\perp \times B_0 / c
\]  

(9)

which yields

\[
\delta j_\perp = i \omega \rho_0 c \delta v_\theta B_0 \times \hat{\theta}/B_0^2,
\]  

(10)

where \( c \) is the speed of light and \( \hat{\theta} \) is the unit vector in the \( \theta \)-direction. The amplitude of \( \delta j_\perp \) is then given by

\[
\|\delta j_\perp\| = \rho_0 \omega c |\delta v_\theta|/B_0,
\]  

(11)

where \( |\delta v_\theta| \) follows from Equations (21)–(22) of Paper I.

The time-averaged force per unit volume, \( F \), which the waves exert on the plasma consists of two terms. The first is simply the time-averaged second-order Lorentz force due to the waves, i.e. \( \langle \delta j \times \delta B \rangle / c \), where the angle brackets here refer to a time average. The component parallel to \( B_0 \) is

\[
\frac{\langle \delta j_\perp \times \delta B \rangle}{c} = -\frac{B_0}{B_0^2} \rho_0 \langle \delta B_\theta \partial \delta V_\theta / \partial t \rangle,
\]  

(12)

where only the real parts of \( \delta B_\theta \) and \( \partial \delta v_\theta / \partial t \) are to be taken. From Equations (21) and (24) of Paper I, there results

\[
\frac{\langle \delta j_\perp \times \delta B \rangle}{c} = \frac{B_0}{8 \pi} \omega B_\theta \varepsilon (I_1 I_2 + R_1 R_2)/v_\Lambda^3,
\]  

(13)

where \( I_1, I_2, R_1, \) and \( R_2 \) are defined by Equations (22) and (25) of Paper I, \( \varepsilon \) is
+1 (−1) when \( v_A \) increases (decreases) with \( s \), and \( B_b \) is the magnetic field strength at the base of the flux tube. The second contribution to \( \mathbf{F} \) is in essence the centrifugal force exerted on the plasma by the Alfvénic motions. Formally it is \(-\rho_0 (\delta \mathbf{v} \cdot \nabla \delta \mathbf{v})\). Utilizing orthogonal curvilinear coordinates with one of the base vectors parallel to \( \mathbf{B}_0 \) and one of the base vectors in the \( \theta \)-direction, it is easily shown that

\[
-\rho_0 \langle \delta \mathbf{v} \cdot \nabla \delta \mathbf{v} \rangle_s = \frac{1}{2} \rho_0 \langle \delta v_\theta^2 \rangle \frac{d \ln r^2}{ds} \tag{14}
\]

\[
= -\frac{1}{2} \rho_0 \langle \delta v_\theta^2 \rangle \frac{d \ln B_0}{ds}, \tag{15}
\]

where Equation (6) has been used to obtain (15). The component of \( \mathbf{F} \) parallel to \( \mathbf{B}_0 \) is given by the sum of Equations (13) and (15). We have only obtained the component of \( \mathbf{F} \) parallel to \( \mathbf{B}_0 \). However, \( \mathbf{F} \) will in general have a component transverse to \( \mathbf{B}_0 \). To calculate this component requires a more detailed theory than is being presented here. The reason is that the transverse component is determined by the transverse gradients of \( \langle \delta B_\theta^2 \rangle \) and \( \langle \delta v_\theta^2 \rangle \). Since an Alfvén wave propagates information only along \( \mathbf{B}_0 \), the transverse gradients of the wave field are not determined by any property of the waves themselves, but rather by the boundary conditions which determine how the wave is generated. The details of the mechanism for Alfvén wave generation in solar magnetic flux tubes are not known to this level of detail, however, and we shall henceforth consider only the parallel component of \( \mathbf{F} \).

Finally, it should be emphasized that the motion of the magnetic flux tube has here been taken to be an axisymmetric twist primarily out of a desire to present a specific and detailed set of equations describing pure noncompressive Alfvén waves. However, the mathematical description that results may apply to other situations as well. One example is where the flux tube contains motions whose horizontal scales are very small compared to some typical horizontal dimension of the flux tube; in that case the Alfvénic component of the motions will propagate according to the equations given above. A second example is the case where the periods and horizontal scales are long, so that the entire flux tube is moved as a unit; such a case might pertain if the flux tubes are moved back and forth in response to the supergranular motions. In that case the motions at the ‘edges’ of the flux tube would be primarily shears, and therefore primarily Alfvénic and subject to the same equations as given previously. At the ‘front’ and ‘back’ of the flux tube the motions would be compressive, and not Alfvénic. Figure 1 illustrates this point: ‘A’ and ‘C’ denote regions where the motions are Alfvénic and compressive, respectively.

It is also useful to note that in the case of solar magnetic flux tubes, approximation (6) may be only a weak constraint on the applicability of the numerical solutions which will be presented below. The reason is as follows: In the models of both open and closed magnetic flux tubes, the region between a height of about 500 km above \( \tau_{5000} = 1 \) and the base of the corona is the most important in determining the properties of the solutions of the Alfvénic wave equation. This is because \( v_A \) varies
most strongly with height in that region (see Figures 3 and 10). But from Figure 9 of Gabriel (1976), much of this region is occupied by field lines which are either approximately 'monopolar' or vertical and uniform. In both cases approximation (6) may be reasonably well satisfied even on magnetic field lines which are not close to the axis of the flux tube.

3. Open Flux Tubes

A. Flux Tube Model

This section presents numerical solutions for waves on a magnetic flux tube that opens into a coronal hole.

The density model for the lower solar atmosphere is based on the HSRA/VAL models (Gingerich et al., 1971; Vernazza et al., 1973). Above this is an (arbitrary) 200 km thick transition region, followed by a model for the upper transition region and coronal hole (R. Munro, private communication, 1977; Munro and Jackson, 1977). The density structure is specified at nineteen levels with values of neutral plus ionized hydrogen concentration, n_H, given in Table I. The atmosphere is assumed to be pure hydrogen. The height, h, is defined to be zero at \( \tau_{5000} = 1 \).

The variation of \( B_0 \) with height mimics the behavior suggested by Gabriel (1976) and Spruit (1976) for the magnetic flux tubes, and by Munro and Jackson (1977) for the coronal hole. Between \( h = 0 \) and \( h \approx 500 \) km, the flux tube is in pressure equilibrium with its surroundings, which we approximate by taking \( B_0(h)^2/p_0(h) = \) constant, where \( p_0(h) \) is the thermal pressure outside the tube. For simplicity we assume that \( p_0(h) \) falls off exponentially with height in this region with a scale height of 110 km. Between \( h \approx 500 \) km and \( h \approx 2000 \) km, the magnetic field behaves approximately like a monopolar field, and \( r \) increases linearly with height (Gabriel, 1976). Above \( h \approx 2000 \) km, the field becomes increasingly more vertical and uniform, except for the slow expansion in the coronal hole. In the coronal hole, the magnetic field is taken to have the same functional form found by Munro and Jackson (1977), except that \( F_{\text{max}} \) will here be taken to be 4.0 instead of their value of 7.26.
TABLE I
Hydrogen densities in the open flux tubes

\begin{tabular}{|c|c|}
\hline
\textbf{h} or \textbf{R} & \textbf{n}_H(\text{cm}^{-3}) \\
(km or solar radii) & \\
\hline
\text{h = 0} & 1.89 \times 10^{17} \\
260 & 3.83 \times 10^{16} \\
400 & 9.46 \times 10^{15} \\
600 & 1.26 \times 10^{15} \\
800 & 1.68 \times 10^{14} \\
1200 & 7.72 \times 10^{12} \\
1433 & 1.26 \times 10^{12} \\
2400 & 2.00 \times 10^{10} \\
2600 & 1.50 \times 10^{9} \\
3000 & 9.40 \times 10^{8} \\
4000 & 6.30 \times 10^{8} \\
7700 & 3.75 \times 10^{8} \\
\hline
\text{R = 1.04} & 1.60 \times 10^{8} \\
1.255 & 1.58 \times 10^{7} \\
1.6 & 1.58 \times 10^{6} \\
2.0 & 3.00 \times 10^{5} \\
2.5 & 8.50 \times 10^{4} \\
3.0 & 4.50 \times 10^{4} \\
6.0 & 2.50 \times 10^{3} \\
\hline
\end{tabular}

Figure 2 shows the behavior with height of $r(s)/r(0) = [B(0)/B(s)]^{1/2}$ between $h = 0$ and $h = 6000$ km (for field lines near the central axis of the flux tube, $h$ and $s$ can be regarded as identical). The exponential, monopolar, and uniform field regimes are evident in the figure. The parameters have been chosen to give a reasonable fit to the average field configuration presented by Gabriel (1976). If the expansion of the field lines in the coronal hole is ignored for the moment, the behavior shown in Figure 2 represents a flux tube whose area asymptotically increases by a factor of 270 between $h = 0$ and $h = \infty$. Since we are taking the magnetic field strength to be 1500 G at $h = 0$, this corresponds to a mean solar field of 5.56 G, which may not be unreasonable near the poles. At $h = 2500$ km, i.e. in the transition region, $B_0$ has fallen to 8.1 G. Combining the factor of 270 mentioned above with the value $F_{\text{max}} = 4$ in the corona and the factor $(R_{\text{1 AU}}/R_{\odot})^2 = (215)^2$ in the solar wind, the 1500 G field at $h = 0$ expands in this model to have a radial component of $3 \times 10^{-5}$ G at 1 AU, a value which agrees with in-situ measurements.

The nineteen values of $\rho_0$ represented by Table I can be combined with $B_0(s)$ to yield nineteen values of $v_A$. These values are then joined together to yield eighteen regions in which $v_A$ varies exponentially with height as shown in Figure 3. The rise of $v_A$ in the chromosphere and transition region is the principal factor determining the properties of the Alfvén waves. The waves are continuously reflected off of the atmospheric levels where $v_A$ changes rapidly. This affects the amount of energy which can be propagated upwards, and leads to a nearly standing wave pattern below
the corona. It is thus important to note that the increase in $v_A$ shown in Figure 3 is more than two orders of magnitude less than was found in the uniform field case of Paper I (compare the present Figure 3 with Figure 2 of Paper I).

B. ENERGY FLUX

Figure 4 illustrates several consequences of the reflections. The Poynting flux at $h = 0$ is displayed as function of wave period, $T$. $\langle P_\odot \rangle$ is obtained by dividing the values in Figure 4 by the area expansion factor, 270.) The horizontal line marked ‘WKB’ is the flux density which results in the high frequency limit, $P_\odot = \rho_0 |\delta v|^2 v_A/2$. For periods longer than about 500 s, $P_\odot$ is always considerably less than the WKB value. In the low-frequency limit $P_\odot$ is reduced by more than two orders-of-magnitude, and $\langle P_\odot \rangle = 1.9 \times 10^4$ erg cm$^{-2}$ s$^{-1}$. By comparison, Figure 3 of Paper I shows a reduction below the WKB value by about four orders-of-magnitude at $T = 3$ hr. The reflections manifest themselves also via the peaks appearing in Figure 4. As discussed above, the peaks result when a ‘node’ in the velocity pattern exists near $h = 0$. The peak-to-valley differences are much smaller in the present model than in Paper I, however. It is evident that the reflections are less severe in the present model.

C. LINEARITY

The values of $\langle \delta B^2 \rangle B_0^2$ and $\langle |\delta v|^2 \rangle v_A^2$ can be used to define the extent to which the waves become nonlinear. These quantities are equal to one another in the WKB
Fig. 3. The Alfvén speed vs height for the basic open flux tube model.

Fig. 4. Predicted Poynting flux at \( h = 0 \) vs wave period, for the basic open flux tube model. At each period \( |\delta\nu| = 1 \) km s\(^{-1}\), at \( h = 0 \).

approximation, but they differ from one another when the waves are of finite wavelength. They can be investigated most easily with the aid of Figure 5, where \( \langle \delta v^2 \rangle_{\text{WKB}} / \langle \delta v^2 \rangle_{\text{WKB}} \) and \( \langle \delta B^2 \rangle_{\text{WKB}} / \langle \delta B^2 \rangle_{\text{WKB}} \) are plotted as functions of \( T \) at eight representative heights. The quantities \( \langle \delta v^2 \rangle_{\text{WKB}} \) and \( \langle \delta B^2 \rangle_{\text{WKB}} \) are the velocity and magnetic field variances which would be required to carry the wave energy flux densities if the
Fig. 5. The normalized quantities $\langle \delta v^2 \rangle / \langle \delta v^2 \rangle_{WKB}$ (solid curve) and $\langle \delta B^2 \rangle / \langle \delta B^2 \rangle_{WKB}$ (dashed curve) vs wave period for the basic open flux tube mode, at eight heights. The light horizontal lines represent the values of $\langle \delta v^2 \rangle / \langle \delta v^2 \rangle_{WKB}$ and $\langle \delta B^2 \rangle / \langle \delta B^2 \rangle_{WKB}$ for which $\langle \delta v^2 \rangle / v_A^2 = 1$ or $\langle \delta B^2 \rangle / B_0^2 = 1$ if $P_\omega = 10^7$ erg cm$^{-2}$ s$^{-1}$. 
WKB approximation were valid; they are given by

$$\langle \delta v^2 \rangle_{\text{WKB}} = (4 \pi / \rho_0)^{1/2} P_\odot / B_\odot$$  \hfill (16a)$$

and

$$\langle \delta B^2 \rangle_{\text{WKB}} = 4 \pi (4 \pi \rho_0)^{1/2} P_\odot / B_\odot.$$  \hfill (16b)$$

First, arbitrarily take $\langle \delta B^2 \rangle / B_0^2 = 1$ as a rough guide as to the separation between the linear and nonlinear regimes, as defined by the magnetic field fluctuations. It is easily shown that when $\langle \delta B^2 \rangle / B_0^2 = 1$, then

$$\frac{\langle \delta B^2 \rangle}{\langle \delta B^2 \rangle_{\text{WKB}}} = \frac{B_0 v_A}{4 \pi P_\odot}.$$  \hfill (17)$$

Now take $\langle P_\odot \rangle = 10^7 \text{ erg cm}^{-2} \text{ s}^{-1}$, as a representative value; then $P_\odot = 2.7 \times 10^9 \text{ erg cm}^{-2} \text{ s}^{-1}$. Taking $B_\odot = 1500 \text{ G}$, we find from Equation (17) that the solutions will be in the linear (nonlinear) regime when $\langle \delta B^2 \rangle / \langle \delta B^2 \rangle_{\text{WKB}} < (>) 4.42 \times 10^{-8} B_0 v_A$. The quantity $B_0 v_A$ can be readily calculated at any height from the information in Figures 2 and 3. At $h = 0$, 800, 1433, and 2400 km, the divisions between the linear and non-linear regimes are given by $\langle \delta B^2 \rangle / \langle \delta B^2 \rangle_{\text{WKB}} = 49.7$, 4.46, 2.6, and 5.14, respectively; these values are shown by the light horizontal lines in Figure 5a. At $h = 0$, a flux density $\langle P_\odot \rangle = 10^7 \text{ erg cm}^{-2} \text{ s}^{-1}$ can be carried without strong nonlinearity if $T \gg 500 \text{ s}$ or $T \ll 500 \text{ s}$, but the vicinity of $T = 500 \text{ s}$ is marginal. At $h = 800 \text{ km}$, the flux density $\langle P_\odot \rangle = 10^7 \text{ erg cm}^{-2} \text{ s}^{-1}$ can only be carried by linear waves if $T \gg 600 \text{ s}$; shorter period waves must be nonlinear, although the shortest periods are only marginally so. The same general conclusion applies at $h = 1433 \text{ km}$, except that here the waves can be linear when $T \gg 180 \text{ s}$. At $h = 2400 \text{ km}$ (and above), no nonlinearity results for the specified energy flux density. From this discussion, the following conclusions can be drawn:

(i) The longest period waves are the most suitable for carrying large energy flux densities without introducing strong twists of the magnetic field. However, it is important to note from Figure 4 that the long-period waves are not likely to carry energy flux densities as large as $\langle P_\odot \rangle = 10^7 \text{ erg cm}^{-2} \text{ s}^{-1}$.

(ii) Waves of intermediate and high frequency can in principle carry $\langle P_\odot \rangle = 10^7 \text{ ergs cm}^{-2} \text{ s}^{-1}$, but they can only do so if moderate distortions of the magnetic field are allowed.

(iii) The wave-induced distortions of the magnetic field are most important in the middle chromosphere. They will probably be important in the photosphere and low chromosphere as well, and they will be unimportant in the upper chromosphere and corona.

If the division between the linear and nonlinear regimes is defined in terms of the velocity fluctuations as occurring when $\langle \delta v^2 \rangle / v_A^2 = 1$, then the division occurs when

$$\frac{\langle \delta v^2 \rangle}{\langle \delta v^2 \rangle_{\text{WKB}}} = \frac{B_0 v_A}{4 \pi P_\odot}.$$  \hfill (18)$$
which is also represented by the light horizontal lines in Figure 5a when $\langle P_\odot \rangle = 10^7$ erg cm$^{-2}$ s$^{-1}$. In this case the conclusions are quite different from when the nonlinearity is defined in terms of twist:

(i) The long-period waves are the least suitable for carrying large energy flux densities without allowing large relative velocity amplitudes. But the flux densities in the long-period waves will probably be low.

(ii) High-frequency waves can in principle carry $\langle P_\odot \rangle = 10^7$ erg cm$^{-2}$ s$^{-1}$, but only with moderate nonlinearity in the chromosphere. The relative velocity amplitudes will be small in the corona for the energy flux densities needed to supply the coronal energy losses, i.e. $\langle P_\odot \rangle \approx 10^5 - 10^6$ erg cm$^{-2}$ s$^{-1}$.

It is important to note, however, that large values of either $\langle \delta v^2 \rangle / v^2_\odot$ or $\langle \delta B^2 \rangle / B^2_0$ do not necessarily mean that the propagation characteristics of the wave will exhibit nonlinear effects. For example, in a uniform plasma with uniform $B_0$, it is possible to have Alfvén waves of arbitrarily large amplitude which are purely noncompressive, which do not steepen, and which do not undergo nonlinear damping (Barnes and Hollweg, 1974; Goldstein et al., 1974). This occurs when the polarization of the wave is such that $|B_0 + \delta B| = \text{constant}$ (e.g. circular polarization); in that case the magnetic pressure is constant and no nonlinear compressions are induced by the wave, regardless of the wave amplitude. On the other hand, if the polarization is such that the magnetic pressure is not constant (e.g. linear polarization), then the wave becomes compressive in second order, and steepening and damping can result (Barnes and Hollweg, 1974; Hollweg, 1971). Thus even though the wave may be nonlinear in the sense that $\langle \delta B^2 \rangle / B^2_0 \geq 1$ or $\langle \delta v^2 \rangle / v^2_\odot \geq 1$, this does not necessarily imply that the propagation characteristics of the wave will exhibit nonlinear effects.

To assess whether nonlinearity does in fact affect the wave propagation, it is necessary to evaluate the second-order force, $\delta j_1 \times \delta B; / c - \rho_0 \delta v_1 \cdot \nabla \delta v_1$, which the first-order waves exert on the plasma (the subscripts ‘1’ denote the first-order quantities). If this force is large then the second-order compressions and velocities can affect the wave propagation, but only in third order because it can be shown that no second-order terms appear in the $\theta$-components of either Equation (1) or (2). This aspect of the nonlinearity will be returned to later, when we discuss the second-order force.

It should also be noted that the ratio of $|\delta v_1|$ to the sound speed does not play any direct role in the nonlinearity of an Alfvén wave (Hollweg, 1971), i.e. $|\delta v_1|$ can be supersonic and still not have any nonlinear effects.

Figure 5 can be used to deduce the variation of $\langle \delta B^2 \rangle / B^2_0$ with height. For example, suppose $\langle P_\odot \rangle = 10^7$ erg cm$^{-2}$ s$^{-1}$ at $T = 0.15$ hr. Figure 5a shows that $\langle \delta B^2 \rangle / B^2_0 = 1$ at $h = 0$, rises to a value slightly in excess of 1 at $h = 800$ km, and then subsequently declines at greater heights. It is clear that $\langle \delta B^2 \rangle / B^2_0$ is not necessarily largest on the expanded parts of the flux tube, as one might have been tempted to deduce from Parker’s (1974) analysis. The reason is, of course, that Parker’s analysis is for force-free fields, which is not the case for the dynamic wave being studied here. At long enough wave periods a quasi-static forcefree picture will apply, but the
numerical results obtained here indicate that the quasistatic limit is valid only for \( T \geq 100 \) hr near \( h = 0 \). Since we feel that the supply of energy to the solar atmosphere is almost certainly determined by phenomena which evolve on time scales shorter than this value, we conclude that Parker's analysis may not apply to transport of energy by solar Alfvén waves, and that these waves can transport energy without becoming unduly 'wound up' in the expanded parts of the flux tube.

D. EQUIPARTITION

Since

\[
\frac{0.5 \rho_0 \langle \delta v^2 \rangle}{\langle \delta B^2 \rangle / 8 \pi} = \frac{\langle \delta v^2 \rangle / \langle \delta v^2 \rangle_{\text{WKB}}}{\langle \delta B^2 \rangle / \langle \delta B^2 \rangle_{\text{WKB}}} \tag{19}
\]

it is apparent from Figure 5 that at all periods above the transition region, and at long periods below the transition region, the wave kinetic energy density exceeds the wave magnetic energy density. The relatively low values of the magnetic energy density at long periods is a result of the field lines being open in the corona, where \( v_A \) is large. If the convection zone tries to impose a slow twist on the magnetic field, it rapidly unwinds, resulting in a low wave magnetic energy density. The rapid unwinding in turn yields a large wave kinetic energy density. On the other hand, at short periods below the transition region, either the kinetic or magnetic energy can predominate. This is a consequence of the standing wave pattern.

The failure of equipartition means that observations of \( \langle \delta v^2 \rangle \) or \( \langle \delta B^2 \rangle \) cannot be used to deduce the wave energy flux density via the WKB formulae \( P = \rho_0 \langle \delta v^2 \rangle v_A = \langle \delta B^2 \rangle v_A / 4 \pi \). For example, if \( \langle \delta v^2 \rangle / \langle \delta v^2 \rangle_{\text{WKB}} > 1 \), then observations of \( \langle \delta v^2 \rangle \) coupled with the WKB formula will lead to an overestimate of the energy flux density by the factor \( \langle \delta v^2 \rangle / \langle \delta v^2 \rangle_{\text{WKB}} \). Similarly, if \( \langle \delta B^2 \rangle / \langle \delta B^2 \rangle_{\text{WKB}} < 1 \), then observations of \( \langle \delta B^2 \rangle \) will underestimate the energy flux density by the factor \( \langle \delta B^2 \rangle / \langle \delta B^2 \rangle_{\text{WKB}} \). Figure 5 shows that these factors can differ substantially from unity below about 1.3\( R_\odot \). At greater heights the WKB approximation becomes increasingly better satisfied.

The departures from equipartition also have important consequences for the wave damping. The volumetric heating rates due to viscosity and ion-neutral friction are both proportional to \( \langle \delta v^2 \rangle \). If \( \langle \delta v^2 \rangle / \langle \delta v^2 \rangle_{\text{WKB}} > 1 \), then the e-folding distance for the energy flux is shorter than that which would be calculated using the WKB approximation, by the factor \( (\langle \delta v^2 / \langle \delta v^2 \rangle_{\text{WKB}})^{-1} \). A similar result follows for the joule heating, which is proportional to \( \langle \delta v^2 \rangle \), which is in turn proportional to \( \langle \delta B^2 \rangle \). Since \( \langle \delta B^2 \rangle / \langle \delta B^2 \rangle_{\text{WKB}} \neq 1 \), the joule heating can differ considerably from the WKB values, although the net effect in this case is not large.

E. WAVE-EXERTED FORCE

Figure 6 displays the magnitude of the parallel component of the time-averaged volumetric force which the waves exert in the plasma, calculated from Equations (13) and (15); the solid lines indicate upward force while the dashed lines indicate
Fig. 6. The time-averaged vertical volume force, normalized to local gravity, vs wave period for the basic open flux tube model, at nine heights. Each curve has been normalized to $P_\odot = 10^8 \, \text{erg cm}^{-2} \, \text{s}^{-1}$. The force scales as $P_\odot$. A solid (dashed) curve indicates upward (downward) force.

downward force. The force in each case has been calculated for $P_\odot = 10^8 \, \text{erg cm}^{-2} \, \text{s}^{-1}$, i.e. for $\langle P_\odot \rangle = 3.7 \times 10^5 \, \text{erg cm}^2 \, \text{s}^{-1}$, and normalized to the local gravitational volume force, $\rho_0 g$. (The calculation of the force requires that the scale height of the Alfvén speed be specified. As indicated in Figure 3, this quantity is discontinuous at the heights given in Figure 6. In each case, the scale height just above the indicated point is the one which was used in the calculation.) Consider first long periods, i.e. $T \geq 10 \, \text{min}$ or so. The force is then always upward. It is small
compared to gravity at $h = 0$, generally reaches a local maximum in the neighborhood of 1500 km, then declines through the transition region and low corona, and then starts rising again in the corona and solar wind, the rise aided in part by the decline of $g$ with increasing $r$. The magnitude of the relative force can be comparable to $g$ if $\langle P_\odot \rangle \approx 2 \times 10^5$ erg cm$^{-2}$ s$^{-1}$ at the long periods. This might be the case if long-period Alfvén waves of solar origin are responsible for the high-speed streams. In that case hydrostatic equilibrium can be significantly violated, and the atmosphere could even become gravitationally unbound. Consider next the high-frequency waves. Unlike the low-frequency waves, the high-frequency waves exhibit downward as well as upward forces. The upward forces always exceed the downward forces, and a broadband spectrum of waves should generally yield a net upward force. The fraction of the frequency domain occupied by downward force decreases with increasing height. This, coupled with the general increase of the magnitude of $F_\parallel / \rho_0 g$ with height below the transition region, means that the net upward force exerted by a spectrum of high-frequency waves should be a fairly strongly increasing function of height below the transition region. These forces may be quite large. Figure 4 predicts $P_\odot \approx 10^9$ erg cm$^{-2}$ s$^{-1}$ at high frequencies, if the power in the high-frequency motions which drive the waves corresponds to $|\delta v| = 1$ km s$^{-1}$. High-frequency motions of this amplitude could easily be accommodated by the microturbulence, for example. If $P_\odot = 10^9$ erg cm$^{-2}$ s$^{-1}$, i.e. if $\langle P_\odot \rangle = 3.7 \times 10^6$ erg cm$^{-2}$ s$^{-1}$, the values in Figure 6 must be multiplied by 10. At $h = 0$ the net upward force will then still be negligible compared to gravity. At $h = 800$ km, the net upward volume force could be as large as several tenths of $\rho_0 g$ (the exact value depends on the details of the frequency spectrum), and significant departures from hydrostatic equilibrium could result. At $h = 1433$ km, the net upward volume force could exceed $\rho_0 g$ by a substantial factor, and the atmosphere would then no longer be gravitationally bound. The high-frequency waves would in that case drive upward motions. Such motions could in fact manifest themselves as spicules, but a fuller theory is needed before such an identification can be made. Finally, a high-frequency wave energy flux density corresponding to $P_\odot = 10^9$ erg cm$^{-2}$ s$^{-1}$ would lead to vertical forces almost an order-of-magnitude greater than gravity in the vicinity of the transition region. But this value of $P_\odot$ would correspond to an average flux density $\langle P_\odot \rangle = 3.7 \times 10^6$ erg cm$^{-2}$ s$^{-1}$ entering the transition region and corona from below a coronal hole. This is perhaps an order-of-magnitude larger than the energy requirements of those regions, and an estimate of the vertical force based on $P_\odot = 10^8$ erg cm$^{-2}$ s$^{-1}$ would be more appropriate, assuming that high-frequency Alfvén waves are in fact the energy source for heating the transition region and corona. In that case the vertical force could be a substantial fraction of gravity, and hydrostatic equilibrium could be strongly violated.

In sum, long-period waves can significantly alter hydrostatic equilibrium if their energy flux density is sufficient to drive high-speed solar wind streams, while high-frequency waves will either alter hydrostatic equilibrium or even drive vertical flows if their energy flux density is comparable to what is needed to heat the
chromosphere and corona. However, it is important to note that the above estimates of $F_{||}/\rho_0 g$ depend sensitively on a number of poorly defined quantities. First, the geometry of $B_0$ will strongly affect the force. And, second, wave damping, which has not been included here, could increase the upward force exerted by the waves, by increasing the rate at which the wave energy density falls off with height. Figures 6a and 6b should therefore be regarded as estimates only.

Figure 6 can also be used to estimate the extent to which the waves become dynamically nonlinear. In addition to the time-averaged force exerted by the waves on the plasma, there will also be second-order forces which oscillate at twice the frequency and twice the wave-number of the first-order wave. These oscillating second-order forces drive second-order velocities and density fluctuations. For the non-WKB waves being considered here, the oscillating second-order forces will be of the same order as the time-averaged force discussed above. As a rough estimate, suppose that the amplitude of the oscillating acceleration is $eg = 0.27 \varepsilon$ km s$^{-2}$, where $\varepsilon$ is of order unity. If by high-frequency we mean $T < 100$ s, the second-order velocity amplitudes are less than 2 km s$^{-1}$. This result comes from taking $\varepsilon = 1$ and $|\delta v_2| = (T/2)ge/2\pi$, where $T/2$ is taken because the second-order force has twice the frequency of the first-order wave; this result ignores the effects of compressibility and the structure of the atmosphere and of $F_{||}$, and it should therefore be regarded only as an estimate. It will be found below that a second-order velocity amplitude of 2 km s$^{-1}$ is smaller than the first-order velocity amplitudes of the high-frequency waves at chromospheric and coronal heights if the first-order waves are carrying an energy flux density which is sufficient to heat the chromosphere and corona. Thus $|\delta v_2|/|\delta v_1| < 1$, and we conclude that even though they may not be totally negligible, the second-order velocities are not dominant for sizeable high-frequency wave flux densities.

The situation for the long-period waves is different. Figure 4 suggests that $\langle P_\odot \rangle = 2 \times 10^4$ erg cm$^{-2}$ s$^{-1}$ is a reasonable value for the long-period flux density in the model. Figure 6a then implies that the largest predicted value of $\varepsilon$ is about 0.1, at $h = 1433$ km. If $T = 1$ hr, then $|\delta v_2| = 8$ km s$^{-1}$, and $|\delta v_2|/|\delta v_1| \approx 1$. In addition, if $|\delta v_2| = 8$ km s$^{-1}$, the corresponding particle displacement amplitude, $|\delta \xi_2|$, is 2200 km if the period of the first-order wave is 1 hr; this result follows from $\delta \xi_2/\delta t = \delta v$, $|\delta \xi_2| = (T/2)|\delta v_2|/2\pi$. These values may indicate a difficulty with the theory. $|\delta v_2|$ is not negligible (but it is not dominant either). And rather large vertical excursions of matter, comparable to the thickness of the atmosphere, are implied. Of course, the estimate of $|\delta \xi_2|$ may be grossly in error because it now become necessary to take nonlinearity, compressibility, and the structures of the atmosphere and of $F_{||}$ into account. But there is some cause for concern. And the difficulties would be even worse if we required larger values of $\langle P_\odot \rangle$. For example, if $\langle P_\odot \rangle = 10^5$ erg cm$^{-2}$ s$^{-1}$, then $|\delta v_2| = 39$ km s$^{-1}$ and $|\delta \xi_2| = 11000$ km. Thus we feel that there may be a problem with the ability of our model to produce long-period fluxes with $\langle P_\odot \rangle = 2 \times 10^4$ erg cm$^{-2}$ s$^{-1}$, and even more so if larger flux densities are required.
On the other hand, the estimates of $|\delta v_2|$ and $|\delta \xi_3|$ may even be positive features of the theory. Even below a coronal hole, the solar atmosphere can hardly be regarded as plane-stratified. For example, Huber et al. (1974) show the height of formation of Ne vii $\lambda$ 465 varying by 5–10 arc sec in a polar coronal hole. It seems likely that this ‘corrugation’ of the transition region is time-dependent, and it therefore seems at least possible that transition region material may be moving up and down with displacements of the order of 5–10 arc sec. The forces associated with the long-period Alfvén waves being studied here have already been found to be compatible with such displacements, and it may in fact not be unreasonable to invoke the waves as a means of explaining the observed corrugation, but without temporal information nothing more can be said. The macroscopicules are fairly long-lived atmospheric motions which may also conceivably be related to the field-aligned forces associated with long-period waves, but a more likely interpretation would appear to be that the macroscopicule lifetimes are determined by the ballistic flight times rather than by some wave period (see Figures 2 and 3 of Withbroe et al., 1976).

The forces exerted by the long-period waves may help to resolve another problem. Figure 4 predicts $\langle P_\odot \rangle \approx 2 \times 10^4$ erg cm$^{-2}$ s$^{-1}$ at the long-periods. This value is still somewhat low to be consistent with wave-driven high-speed solar wind models, at least if $F_{\text{max}} > 1$. As noted earlier, $\langle P_\odot \rangle$ is low at long periods because of the reflections. Now suppose the atmosphere does in fact move up and down in response to the long-period waves. The reflecting boundary would then be flexible instead of rigid, and more wave energy should be able to penetrate to ‘infinity’. The correctness of this supposition can only be assessed via a fully nonlinear theory, however, which is beyond the scope of this analysis. (Other means of increasing $\langle P_\odot \rangle$ will be discussed below.)

Finally, it is interesting to note that $|\delta v_2|$ associated with the long-period waves can be comparable to or to exceed the sound speed below the transition region. In that case it is likely that shocks can form and the atmosphere can be thereby heated. We return to this question in Section 5.

F. HEIGHT DEPENDENCE OF $\delta v$

An important constraint on the possible Alfvénic energy flux is provided by measurements of $\langle \delta v^2 \rangle$. Figure 7a displays $\langle 2 \delta v^2 \rangle^{1/2}$ as a function of height for a long-period wave, $T = 1$ hr, with $P_\odot = 5 \times 10^6$ erg cm$^{-2}$ s$^{-2}$, i.e. $\langle P_\odot \rangle = 2 \times 10^4$ erg cm$^{-2}$ s$^{-1}$. Also shown, by the dark line, is the observed most probable value of the total horizontal velocity, given by Canfield and Beckers (1975). These authors point out that these data must be viewed with a good deal of skepticism, and that the chromospheric data may only represent an upper limit. In addition, it is not clear to what extent their data refer to the magnetic regions of the solar atmosphere. Finally, their data refer only to motions with $T \approx 10$ min (G. Athay, private communication, 1979). In any event, the horizontal velocity data do not contradict the theoretical prediction that significant fluxes of long-period waves can be generated by convectively-driven motions of magnetic flux tubes. This conclusion is
Fig. 7. The solid curves are the observed most probable value of the total horizontal velocity given by Canfield and Beckers (1975). The dashed curves are the predicted behavior for the basic open flux tube model: (a) if $\langle P_\theta \rangle = 2 \times 10^4$ erg cm$^{-2}$ s$^{-1}$ at $T = 1$ hr; (b) at $T = 60$ s, for the three indicated values of $\langle P_\theta \rangle$. 
in sharp contrast to Paper I, where the predicted height variation of \( \langle \delta v^2 \rangle^{1/2} \) is very different from the curve given by Canfield and Beckers.

Figure 7b again shows the data of Canfield and Beckers as the solid line, but they are now compared with the predicted height variation of \( \langle 2 \delta v^2 \rangle^{1/2} \) for waves with \( T = 60 \) s. (In plotting Figure 7b, only the envelope which encompasses the nodal pattern was considered.) The rate of increase with height of the high-frequency wave velocities is considerably greater than the observed rate. This may mean that the observed horizontal velocities can not be interpreted in terms of high-frequency Alfvén waves. An alternative explanation is that the data are indicating that the waves are damped. In that case it is possible to interpret Figure 7b as being roughly consistent with average flux densities of \( \langle P_\odot \rangle = 1.9 \times 10^6, 1.9 \times 10^5, 10^5, \) and \( 9.5 \times 10^4 \) erg cm\(^{-2}\) s\(^{-1}\) crossing the surfaces at \( h = 250, 1000, 1600, \) and \( 2500 \) km, respectively. Somewhat larger flux densities would be implied for waves with \( T < 60 \) s. And we expect that somewhat larger flux densities would be implied if the damping were self-consistently included in the model, since the (subtractive) effect of the reflections would be thereby reduced. Wave flux densities as large as those enumerated above could substantially contribute to the energy balance of the magnetic regions of the chromosphere and corona. But a theory which self-consistently includes damping, and reliable observations of the non-thermal velocities in the magnetic flux tubes, must be available before anything definitive can be said.

Thus, within the framework of the present model, the observed total horizontal velocities in the solar atmosphere are consistent with long-period Alfvénic flux densities which are sufficient to drive moderately active solar wind streams, or with damped high-frequency Alfvén waves which carry sufficient energy to contribute substantially to heating the magnetic regions of the chromosphere and corona, or with some intermediate combination. And if the reflections are weaker than our model implies (due to the 'flexibility' mentioned above, or to damping, or to the effects to be discussed below), then the observed \( \langle \delta v^2 \rangle \) could be consistent with larger values of \( \langle P_\odot \rangle \) than indicated in Figure 7.

G. OTHER PARAMETERS

How is this model affected by changes in the basic parameters? Consider first what happens when the flux tubes are packed twice as closely together, so that the expansion factor (excluding that in the coronal hole) is 135 instead of 270. The magnetic field at the base of the flux tube remains at 1500 G, but the coronal and solar wind fields are doubled. (Doubling \( F_{\max} \) to keep the solar wind field the same does not have any significant effect.) Consider the behavior with height of \( r(s)/r(0) \) to be similar to that shown in Figure 2, and the density structure of the atmosphere to be unchanged. For the long-period waves, \( T \geq 20 \) min, we have found that \( P_\odot = 2.8 \times 10^6 \) erg cm\(^{-2}\) s\(^{-1}\). This should be compared to the value \( 5.2 \times 10^6 \) erg cm\(^{-2}\) s\(^{-1}\) found in Figure 4. The reduction is a consequence of the reflections. The Alfvén speeds at the base of the flux tubes are the same in this model and in the first model,
but the Alfvén speed in the corona is about twice that in the first model. The gradient of the Alfvén speed is therefore greater in the present model, and more of the wave energy is reflected back down, resulting in a lower value of $P_\odot$. Remarkably, $\langle P_\odot \rangle$ is about the same in both models, $2.1 \times 10^4$ ergs cm$^{-2}$ s$^{-1}$ in this model, and $1.9 \times 10^4$ ergs cm$^{-2}$ s$^{-1}$ in Figure 4. Simply packing the flux tubes closer together does not substantially change the net long-period wave energy available to the solar atmosphere and solar wind. (The same result has also been obtained when the flux tubes are spaced twice as widely apart.) If the long-period wave flux is in fact being produced by the mechanism being studied here, we conclude that the net long-period wave flux is essentially independent of the average magnetic field strength at the Sun, if the only factor controlling the average field strength is the expansion factor of the flux tubes. If it is further assumed that the long-period waves are responsible for the high-speed solar wind streams, we would have to conclude that some factor other than the average solar field strength is responsible for varying the energetics of the streams. One possible factor is $F_{\text{max}}$ in the coronal hole; this follows because the Alfvén waves only deposit their energy in the solar wind beyond $2-3R_\odot$ (Hollweg, 1978b), where their energy flux density, which is fixed at the Sun, has already been diluted by the divergence of the coronal hole. Levine (1978) has provided evidence that $F_{\text{max}}$ is an important factor controlling the formation of high-speed streams. This is not inconsistent with the scenario just presented, but a definitive verification would require showing that Levine’s data is in fact consistent with a fixed value of $\langle P_\odot \rangle$. And it would also require showing that other factors, such as the coronal density or temperature, are themselves not correlated with $F_{\text{max}}$ in such a way as to produce the effect observed by Levine.

The effect of varying the local magnetic field strength is different from varying only the average field via the filling factor. Consider first the effect of changing nothing but the magnetic field strength everywhere. In that case the curve for $P_\odot$ vs $T$ is the same as that given in Figure 4, except that it is displaced up and to the left in proportion to $\langle B_0 \rangle$. Changing the magnetic field strength everywhere can increase $P_\odot$ and $\langle P_\odot \rangle$ in the same proportion, because there are no changes in the reflections.

Consider next the effect of changing the magnetic field strength at the base of the flux tube, but at the same time changing the spacing between the flux tubes in such a way as to keep the average field strength at the Sun constant. We have investigated the behavior of $P_\odot$ vs $T$ which results when the magnetic field at the base is doubled to 3000 G, and the expansion factor below the coronal hole is doubled from 270 to 540. The interesting point is that at long periods $\langle P_\odot \rangle = 3.7 \times 10^4$ erg cm$^{-2}$ s$^{-1}$, compared to $1.9 \times 10^4$ erg cm$^{-2}$ s$^{-1}$ in Figure 4. The increase of $\langle P_\odot \rangle$ results because $v_A$ at the base of the flux tube is increased, while $v_A$ in the corona is unchanged, and the reflections are reduced as a consequence.

The conclusion of the above exercise is that at long-periods, $\langle P_\odot \rangle$ is only weakly affected by the flux tube filling factor, but $\langle P_\odot \rangle$ does vary in response to global changes in $\langle B_0 \rangle$, and also in response to changes in the value of $\langle B_0 \rangle$ at the base even when the average solar field strength is unchanged.
One other possibility for changing \( \langle P_\odot \rangle \) suggests itself. At the long periods, \( \langle P_\odot \rangle \) is heavily influenced by the reflections off of the gradient in \( v_A \). If the gradient of \( v_A \) can be reduced, so too will the reflections, and \( \langle P_\odot \rangle \) can be enhanced. Now, as shown in Section 2, the gradient is along a given field line, i.e. \( dv_A/ds \). We have been considering only the central (vertical) field lines in the flux tube, but it seems likely that the central field lines are those on which \( dv_A/ds \) is largest, for two reasons:

(i) Since \( \rho_0 \) varies only in the vertical direction, \( |d\rho_0/ds| \) is largest on the central field lines;

(ii) in the potential field model of Gabriel (1976) the ratio of the magnetic field strength high in the atmosphere to that at \( h = 0 \) is largest on the central field lines, and smallest on the outer field lines.

Thus by considering only the central field lines, we may have been overestimating the reflections and underestimating \( \langle P_\odot \rangle \) at the long-periods, a possibility which seems particularly likely when it is recalled that the ‘outer’ field lines will in general represent a larger fraction of the flux tube area than the ‘central’ field lines. On the other hand, this effect may be counteracted by lower values of \( |B_0| \) near the edge of the flux tube than near the center. Thus only a detailed model of the structure of the flux tube, which is not currently available, can allow these effects to be assessed. In view of the sensitivity of all the above solutions to the reflections, our prejudice is that the reduced reflections will overwhelm the lower values of \( |B_0| \) near the edge, and allow larger values of \( \langle P_\odot \rangle \) to be generated at long periods than the above calculations indicate. In that case it seems possible that long-period wave flux densities sufficient to drive high-speed solar wind flows can be explained by this theory.

In this sub-section we have considered only the response of the wave energy flux to changes in the basic parameters. The responses of other quantities, such as \( \langle \delta B^2 \rangle/\langle \delta B^2 \rangle_{\text{WKB}} \) etc., are not dramatic, and will not be considered here.

4. Closed Flux Tubes

A. FLUX TUBE MODEL

The previous section considered flux tubes which are open to ‘infinity’. In this section flux tubes of finite length, which are rooted into the top of the convection zone at both ends, are considered. But only a very restricted class of such tubes will be explored. First, only tubes which are symmetric about their midpoints will be examined. Second, only tubes which extend a moderate distance into the corona will be studied, such as, for example, the coronal active region loops. Two loops, 33 400 and 66 800 km in length, will be studied. (Truncation errors in the numerical code precluded study of longer loops.) Third, only Alfvénic motions which are noncompressive in first order will be considered. This means that any coupling with gravity is excluded. Formally, this is accomplished by deforming the loops into a vertical configuration, as sketched in Figure 8, and then applying the analysis of Section 2.

Consider first a loop which is 33 400 km long. Table II displays its density structure (the structure is symmetric about \( h = 16 700 \) km). The transition region in this case
begins at $h = 1500$ km. The transition region is lower in this model than below a coronal hole, since the coronal pressure in the short loop is large. The pressure in the corona in this case is 6 dyne cm$^{-2}$ if $T = 2.5 \times 10^6$ K and if the molecular weight is 0.5.

The behavior of $r(s)/r(0)$ vs $h$ is shown in Figure 9. The expansion factor in this case is 15, so that a field of 1500 G at the base of the flux tube becomes 100 G in the coronal loop.

Figure 9 and Table II combine to yield the piecewise-exponential behavior of $v_A$ vs $h$ shown in Figure 10.

B. ENERGY FLUX AND LOOP RESONANCES

Figure 11 displays $P_\odot$ vs $T$. Several differences from Figure 4 should be noted. First, the peaks (near the middle of figure) are now much narrower and higher than in Figure 4, and the valleys are much broader and deeper. One reason for these differences can be seen by comparing Figures 3 and 10. The Alfvén speed increases more rapidly with height in the present model, and as a consequence the reflections
Fig. 9. Same as Figure 2, but for the basic closed flux tube model.

Fig. 10. Same as Figure 3, but for the basic closed flux tube model.
are stronger. In addition, the waves now encounter another region of rapidly changing Alfvén speed at the far end of the loop (not shown in Figure 10), and this region also contributes strongly to the reflections. The stronger reflections in the present case are responsible for the behavior of Figure 11. Second, \( P_\odot \) increases linearly with \( T \) at the long periods in Figure 11, whereas \( P_\odot \) is independent of \( T \) at the long periods in Figure 4. The reason is as follows: Even though an absorbing boundary condition has been imposed at the far end of the flux tube, it turns out that, because of the large density there, the magnetic field lines are almost immobile at the top of the convection zone at the far end. Even an absorbing boundary condition acts in some ways like field-line tying. (This can be understood by imagining the waves at the far end of the flux tube to be generated by the waves near the middle of the tube. The latter waves exist in a region of very low mass density, and their energy density is therefore low. Consequently they are unable to drive large motions at the far end of the flux tube, where the mass density is high. But the principal reason is the loss of wave energy due to reflections as the waves propagate from their source.) On the other hand, at the end of the flux tubes where the waves are generated, the field lines move back and forth with an excursion amplitude \( |\delta \mathbf{E}| = |\delta \mathbf{v}|/T = (1 \text{ km s}^{-1}) \times (T/2\pi) \). Now with one end of a field line almost fixed, and the other end undergoing excursions which increase as \( T \), it is clear that \( |\delta \mathbf{B}| \) must increase as \( T \). But since the time-averaged Poynting flux varies as \( \langle \delta v_\theta \delta B_\theta \rangle \), it is clear that \( P_\odot \sim T \) if both \( |\delta v_\theta| \) and the phase between \( \delta v_\theta \) and \( \delta B_\theta \) are independent of \( T \). This in fact turns out to be the case in the present model. The constancy of the phase with \( T \) is not easy to understand physically, but the constancy of \( |\delta v_\theta| \) with \( T \) is simply a consequence of our having taken \( |\delta \mathbf{v}| = \text{constant} = 1 \text{ km s}^{-1} \) at \( h = 0 \), at all \( T \).

It thus appears that whereas the open flux tubes seem to favor high-frequency waves as a means of generating large values of \( P_\odot \), the closed configuration tends to
favor the long-period waves. However, even at \( T = 3 \) hr, Figure 11 only gives \( P_\odot = 5.5 \times 10^6 \text{ erg cm}^{-2} \text{s}^{-1} \), or \( \langle P_\odot \rangle = 3.7 \times 10^5 \text{ erg cm}^{-2} \text{s}^{-1} \). This value of \( \langle P_\odot \rangle \) is more than an order-of-magnitude too small to provide the energy requirements of the short high-pressure coronal active region loops discussed by Rosner et al. (1978a). Long-period twists do not appear to be a suitable means for heating coronal active region loops.

The same conclusion can be reached also with regard to the intermediate periods in Figure 11. The average level of \( P_\odot \) at periods between 0.01 and 0.3 hr is less than \( 10^7 \text{ erg cm}^{-2} \text{s}^{-1} \), implying that \( \langle P_\odot \rangle \) is less than \( 7 \times 10^5 \text{ erg cm}^{-2} \text{s}^{-1} \), which is again too small to heat coronal active region loops.

Figure 11 contains a new feature, which does not appear on open flux tubes. At high frequencies the lower envelope of the \( P_\odot \) vs \( T \) curve rises, reaching the WKB value near certain frequencies. These frequencies are resonant frequencies of the coronal part of the loop. The resonant periods are given roughly by \( T_{\text{res}} = 2L_{\text{cor}}/nv_A \), where \( L_{\text{cor}} \) is the length of the coronal part of the loop, and \( n = 1, 2, 3, \) etc. For example, a coronal active region loop with \( B_0 = 100 \text{ G} \), \( L_{\text{cor}} = 30 \text{ 000 km} \), \( p = 6 \text{ dyne cm}^{-2} \), and temperature \( = 2.5 \times 10^6 \text{ K} \), will have \( T_{\text{res}} = (26/n)s \), which may be in the range of periods characterizing the microturbulence (Deubner, 1976). In contrast to the peaks which have been discussed previously, the new resonances found here are a result of there being a region of the atmosphere where wave energy can be stored. The waves that are stored in the corona reflect off the transition region from above, while the waves that are entering the corona from below normally reflect off the chromosphere and transition region from below. What happens at the loop resonances is that the reflections from above and below cancel each other, and a large wave energy flux can enter the corona from below. The effect is analogous to the behavior of anti-reflectance coatings on camera lenses. These resonances may be the key to understanding how coronal active region loops are heated, since they may provide particularly easy access of energy from the convection zone to higher levels.

The WKB value of \( P_\odot \) in this model is \( 1.2 \times 10^9 \text{ erg cm}^{-2} \text{s}^{-1} \), corresponding to \( \langle P_\odot \rangle = 8 \times 10^7 \text{ erg cm}^{-2} \text{s}^{-1} \). The width of the resonances in the frequency domain is between \( \Delta f/f = 0.05-0.1 \) and the Poynting flux carried by the resonances is therefore about \( (4-8) \times 10^6 \text{ erg cm}^{-2} \text{s}^{-1} \). Such an energy flux could be enough to contribute substantially to the loop heating.

C. EQUIPARTITION AND LINEARITY

Figure 12 displays \( \langle \delta v^2 \rangle/\langle \delta v^2 \rangle_{\text{WKB}} \) and \( \langle \delta B^2 \rangle/\langle \delta B^2 \rangle_{\text{WKB}} \) as functions of \( T \) at four heights. The most noteworthy feature of the curves is the violation of equipartition in the sense of \( \langle \delta B^2 \rangle/(4\pi \langle \delta v^2 \rangle \rho_0) > 1 \) at long periods in the chromosphere, and at most periods in the corona. This is a direct consequence of the magnetic field lines being almost tied at the far end of the flux tube, as discussed above. The field lines have no way of unwinding, the field can become highly twisted, and the magnetic energy becomes large.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System
The light horizontal lines in Figure 12 are obtained from Equations (17) and (18) and, as in Section 3, can be used to assess when $\langle \delta B^2 \rangle^{1/2}/B_0 = 1$ or $\langle \delta v^2 \rangle^{1/2}/v_A = 1$. The horizontal lines have been defined by taking $B_0 = 1500 \text{ G}$ and $P_0 = 1.5 \times 10^8 \text{ erg cm}^{-2} \text{ s}^{-1}$, i.e., $\langle P_0 \rangle = 10^7 \text{ erg cm}^{-2} \text{ s}^{-1}$, and the values of $B_0$ and $v_A$ follow from Figures 9 and 10. It can be concluded from Figure 12 that a flux density of $\langle P_0 \rangle = 10^7 \text{ erg cm}^{-2} \text{ s}^{-1}$ can be carried by the waves if some weak nonlinearity vis-a-vis both $\langle \delta B^2 \rangle^{1/2}/B_0$ and $\langle \delta v^2 \rangle^{1/2}/v_A$ in the photosphere and low chromosphere is allowed, but no nonlinearity is required at greater heights.

Figure 12 shows extremely large values of $\langle \delta B^2 \rangle/\langle \delta B^2 \rangle_{\text{WKB}}$; compare with the values appearing in Figure 5a for the open flux tube. The largeness of $\langle \delta B^2 \rangle/\langle \delta B^2 \rangle_{\text{WKB}}$ suggests that the joule heating for waves on closed field lines may be...
orders-of-magnitude larger than for waves on open field lines, and orders-of-magnitude larger than would be calculated on the basis of the WKB approximation. The joule heating exceeds the WKB value roughly by the factor $\langle \delta B^2 \rangle_{\text{WKB}} / \langle \delta B^2 \rangle_{\text{WKB}}$, which can be $10^2$ to $10^3$ in the photosphere and chromosphere at high frequencies (the loop resonances are an exception, however). Figure 12 also indicates that $\langle \delta v^2 \rangle_{\text{WKB}} / \langle \delta v^2 \rangle_{\text{WKB}}$ can be in the range $10^2$ to $10^3$ below the transition region at high frequencies (the loop resonances are again an exception). The viscous and frictional heating will be correspondingly larger than would be calculated according to the WKB approximation. These points will be examined again in Section 5.

Figure 12 shows that $\langle \delta v^2 \rangle_{\text{WKB}} / \langle \delta v^2 \rangle_{\text{WKB}}$ and $\langle \delta B^2 \rangle_{\text{WKB}} / \langle \delta B^2 \rangle_{\text{WKB}}$ approach 1 in the loop resonances below the transition region. This means that the above-mentioned enhancements of the joule, viscous and frictional damping rates do not occur in the loop resonances. We will in fact find in Section 5 that the chromospheric and photospheric damping of the loop resonances is weak. In this sense the loop resonances are particularly suited for transporting energy from the convection zone into the corona.

D. WAVE-EXERTED FORCE

Figure 13 displays $F_\parallel / \rho_0 g$ as a function of $T$ at five heights. Solid (dashed) lines indicate forces in the direction of increasing (decreasing) $h$. Thus for $h < 16700$ km, a solid line indicates a force which acts against gravity, and conversely for the dashed lines; see Figure 8. Figure 13 is normalized to $P_\odot = 10^8$ erg cm$^{-2}$ s$^{-1}$, i.e., $\langle P_\odot \rangle = 6.7 \times 10^6$ erg cm$^{-2}$ s$^{-1}$. Such a value of $\langle P_\odot \rangle$ would be roughly appropriate if the waves were significantly contributing to the chromospheric or coronal energy balance, but Figure 11 indicates that except in the loop resonances this model may fall short of supplying this energy flux density by one to two orders-of-magnitude; $F_\parallel / \rho_0 g$ would have to be scaled down accordingly. Figure 13 indicates that the net upward force can be a substantial fraction of gravity at $h = 0$ and that it can substantially exceed gravity in the chromosphere, if the required energy flux density is carried by a spectrum of high or intermediate frequency waves outside of the loop resonances. Such forces could cause strong departures from hydrostatic equilibrium and even drive strong motions along the magnetic field. The upward force decreases in magnitude in the upper chromosphere, and changes sign to become a downward force in the near side of the coronal loop. The downward force can in fact be comparable to gravity if $\langle P_\odot \rangle = 6.7 \times 10^6$ erg cm$^{-2}$ s$^{-1}$. On the farside of the loop, the force remains in the direction of decreasing $h$ but is upward against gravity. The relative force becomes weaker as the far end of the loop is approached.

The appearance of both upward and downward net forces on closed flux tubes makes it difficult to reach any general conclusions about their possible physical manifestations. We merely suggest that the appearance of this dichotomy on closed field lines, and its absence on open ones, may at least in part account for the richness of motions which appear on the closed field configurations of solar active regions.
Fig. 13. Same as Figure 6, but for the basic closed flux tube model. Solid (dashed) curves are for forces in the direction of increasing (decreasing) s.

The $F_\parallel/\rho g$ vs $T$ curve (at constant $P_\odot$) exhibits sharp dips below the transition region, at the frequencies of the loop resonances. This means that the loop resonances can be particularly effective in transporting energy from the top of the convection zone into the corona, without developing dynamical nonlinearities and the consequent damping (see below and Section 5) in the chromosphere. In this sense
too the loop resonances may serve as a window whereby energy can reach coronal loops from the convection zone.

In contrast to the open field lines (Figure 6), the forces exerted by long-period waves on closed field lines decrease with increasing periods, and are probably unimportant.

The forces on the chromosphere for the same values of \( \langle P_\odot \rangle \) are roughly the same in the open and closed field cases \( T \leq 100 \) s, and we can assert that the chromospheric values of \( |\delta v_2| \) will also be roughly the same in both cases. The values of \( |\delta v_2| \) on closed flux tubes can therefore approach the sound speed if \( \langle P_\odot \rangle \) is large enough to contribute significant energy flux to the atmosphere, and some dissipation and heating via shock formation could in that case result. The loop resonances are an exception, however, for the reason given above.

E. OTHER PARAMETERS AND SCALING LAWS

Thus it appears so far that Alfvén waves can supply significant amounts of energy to closed flux tubes only via the loop resonances. Is it possible to alter the model in such a way as to alter this conclusion?

One possibility is to pack the flux tubes more closely together. We have examined a model which is the same as that just discussed, except that the area expansion factor is reduced from 15 to 6, i.e., the coronal field strength is increased from 100 G to 250 G. At periods greater than 200 s or so, the \( P_\odot \) vs \( T \) curve is essentially identical to that in Figure 11. In contrast to what was found in Section 3, on a closed flux tube the long-period part of \( \langle P_\odot \rangle \) can be increased by packing the flux tubes more closely together, essentially in proportion to the packing. The reason is that the long-period flux is determined by the approximate line tying at the far end of the flux tube, which is unaffected by the packing. The situation is different for shorter period waves, however. They are affected by the reflections much in the same way as was found in Section 3. For \( T \) between 10 and 100 s, the \( P_\odot \) vs \( T \) curve for an expansion factor of 6 lies below the curves for an expansion factor of 15, the difference becoming more pronounced at higher frequencies. \( \langle P_\odot \rangle \) can be increased slightly by increasing the packing, but the dependence is weak. Finally, increasing the packing moves the loop resonances to higher frequencies.

The value of \( \langle P_\odot \rangle \) can be increased also by increasing the magnetic field strength at the base of the flux tube, while simultaneously increasing the area expansion factor so as to keep the coronal field strength constant. Except for the loop resonances, the curve in Figure 11 moves to the left in proportion to the magnetic field strength at the base. At the long periods, \( T \geq 200 \) s or so, the curve also moves up by the same factor. At higher frequencies, \( 10 \leq T \leq 100 \) s or so, the curve moves up by a larger factor than direct proportionality to the magnetic field strength at the base, because the reflections are reduced, much in the same fashion as was found in Section 3. In both cases \( \langle P_\odot \rangle \) is increased, but the effect is pronounced only at the long periods. The loop resonances themselves stay essentially at the same frequency, because the coronal magnetic field strength is unchanged. But the energy flux density at
resonance, which is essentially the WKB value, is increased in direct proportion to the magnetic field strength at the base. Thus the value of $\langle P_\odot \rangle$ in the loop resonances should remain unchanged in this case.

We have also investigated the effect of the length of the loop on $\langle P_\odot \rangle$. This has been done by considering a loop which is in every respect identical to the one which we have been discussing, except that the coronal part of the loop has been extended so as to double the total length of the loop, from 33 400 km to 66 800 km. The most interesting effect of doubling the length of the loop is a reduction in the predicted value of $\langle P_\odot \rangle$ at long periods ($T \approx 5$ min) by a factor of 4. That is, at long periods $\langle P_\odot \rangle$ varies in inverse proportion to the square of the loop length. There is a simple reason for this. Imagine a uniform magnetic field which is tied to $z = L$ and slowly oscillated back and forth in the transverse direction with a fixed amplitude at $z = 0$. For small amplitudes, it is clear that $|\delta B|/B_0 \sim L^{-1}$. Thus lengthening the loop means that less energy appears in the magnetic field fluctuations, in inverse proportion to the square of the loop length.

It is interesting to note that longer coronal active region loops are observed to require less energy input than shorter loops. Suppose that the coronal loops exhibit a correlation between temperature, $\tau$, and pressure, $p$, of the form $(\tau/\tau_0) = (p/p_0)^\alpha$ where $\alpha$, $\tau_0$ and $p_0$ are constants. Then Equations (4.3) and (4.4) of Rosner et al. (1978a) imply that the energy flux density entering the coronal and transition region part of the loop from below scales as $L_{\text{cor}}^{(1+\alpha/2)/(3\alpha-1)}$. Setting $(1+\alpha/2)/(3\alpha-1) = -2$ gives $\alpha = 2/13$, which is consistent with the temperature and pressure data for coronal active region loops reported by Rosner et al. Thus the predicted dependence of the long-period Alfvénic Poynting flux on $L_{\text{cor}}$ is consistent with the observed $\tau - p$ relation, at least as far as the slope of the curve goes. But the difficulty remains, that the predicted flux density in Figure 11 is too small to provide the required energy.

Moreover, there is a fundamental inconsistency. The analysis of the preceding paragraph makes use of the fact that coronal active region loops undergo quasi-continuous heating. But our calculations for long-period waves critically depend on the approximate line-tying at the far end of the flux tube. Clearly, these calculations cannot be appropriate for waves which are dissipated before they reach the far end of the tube. In particular, it is the line-tying which make $\langle P_\odot \rangle$ proportional to $T$ at long-periods. And it is the line-tying which is responsible for the $L$-dependence of $\langle P_\odot \rangle$. If the line-tying is removed by dissipation in the coronal part of the loop, then the model can no longer produce large values of $\langle P_\odot \rangle$ at long periods. This point will be documented in Figure 14 below. And it can also be shown that dissipation in the coronal part of the loop destroys the dependence of $\langle P_\odot \rangle$ on $L$ at long periods. The conclusion is that the long-period waves are not a likely candidate for heating coronal active region loops.

This conclusion bears also on the possibility of coronal active region loops being heated by parallel currents which are presumed to be generated by convective motions at one end of the flux tube (e.g., Tucker, 1973; Rosner et al., 1978b). The twisting motions which are being invoked here are one means of generating parallel
currents. In particular, the long-period Alfvénic motions, $T \geq 5$ min or so, will produce what may be regarded as quasi-static currents flowing from one end of the loop to the other. As stated above, the parallel currents cannot be calculated without specifying in detail the transverse structure of the flux tube and the transverse behavior of the twisting motions themselves, which is beyond the scope of this study. But the actual parallel currents may be largely irrelevant. To the extent that the parallel currents are generated by twists, which obey the Alfvénic wave equation, the important quantity for assessing the energy supply is the Poynting flux into a loop, and not the parallel current itself. If we conclude that long-period Alfvén waves which dissipate in a coronal active region loop are unable to supply the required Poynting flux, then it must also be concluded that the associated parallel currents cannot be responsible for heating the loop.

Two further points should be emphasized here. First, although it is fashionable to invoke parallel currents as a means of heating coronal loops, we feel that it is meaningless to do so without specifying the detailed mechanism by which the currents are generated, so that the associated energy flux can be computed. Second, we feel that it may also be meaningless to invoke steady motions (i.e., $T = \infty$) to generate parallel currents. Both Figures 11 and 14 show that the long-period Poynting flux depends on $T$. Thus a proper estimate of the available energy supply requires specification of the time-scales of the motions which drive the currents. To the extent that the solar wind data can be used to infer the time-scales in active regions, most of the long-period power should appear at time scales of hours, at which periods the model seems incapable of providing the required Poynting fluxes, regardless of whether the heating occurs via parallel currents or not.

A second important effect of increasing $L_{\text{cor}}$ is the appearance of the loop resonances at lower frequencies. But it is not clear whether the loop resonances can explain the dependence of $\langle P_\odot \rangle$ on $L_{\text{cor}}$ implied by the $\tau$-$p$ relation. Superficially, if
the convective power increases with increasing period, then one might expect that increasing \( L_{\text{cor}} \) would increase \( \langle P_{\odot} \rangle \) in the loop resonances, contrary to the \( \tau-p \) relation. On the other hand, we have found that the loop resonances are substantially narrower on the longer loop. By itself, this would lead to a negative correlation between \( \langle P_{\odot} \rangle \) and \( L_{\text{cor}} \), as the data seem to imply. Another possibility is that the wave damping associated with the loop resonances depends on \( L_{\text{cor}} \), so that the energy actually deposited in the loop depends on \( L_{\text{cor}} \). One possibility notes that the fundamental period of the loop resonance varies as \( L_{\text{cor}}^{-2} \). Now joule, viscous, heat conduction, and frictional damping rates vary as \( T^{-2} \), implying heating proportional to \( L_{\text{cor}}^{-2} \), if all other factors are unchanged. A heating rate proportional to \( L_{\text{cor}}^{-2} \) is consistent with the observed \( \tau-p \) relation. Finally, \( B_0 \) itself is probably smaller in the longer loops. This could affect the coronal heating as follows: A smaller value of \( B_0 \) in the corona implies that the area expansion factor is larger on the longer loops, so that even for a fixed value of \( P_{\odot} \), the value of \( \langle P_{\odot} \rangle \) will be smaller on the longer loops. Whether the loop resonances can account in detail for the \( \langle P_{\odot} \rangle - L_{\text{cor}} \) relation can only be assessed concomitantly with knowledge of the frequency spectrum and damping mechanism of the waves, which are not known. All that can be said is that the loop resonances may very well be able to supply the energy requirements of the coronal active region loops, and the possibility at least exists that they can also account for the \( \langle P_{\odot} \rangle - L_{\text{cor}} \) correlation of the coronal loops.

The point has already been made that the solutions represented by Figures 11–13 depend strongly on the reflections and approximate line-tying at the far end of the flux tube. But it was also mentioned that the waves can have no contact with the far end of the flux tube if they are dissipated before propagating the length of the tube. The consequences of such dissipation have been crudely modelled by assuming that the dissipation occurs totally at the midpoint of the flux tube. That is, the flux tube structure represented by Table II and Figures 9 and 10 has been used, except that the absorbing boundary condition has been moved to \( h = 16,700 \) km and the part of the flux tube at \( h \geq 16,700 \) km has been removed from consideration. The resulting behavior of \( P_{\odot} \) is shown in Figure 14 which should be compared with Figure 11. One consequence of moving the absorption to the middle of the loop is a reduction of the slope of the \( P_{\odot} \) vs \( T \) curve at \( T \geq 0.3 \) hr. It will be recalled that the increase of \( P_{\odot} \) with \( T \) at long periods in Figure 11 was a consequence of the line-tying at the far end of the flux tube, which does not occur in Figure 14.

On the other hand, Figure 14 shows that moving the absorption to the middle of the loop substantially raises the \( P_{\odot} \) vs \( T \) curve at all periods less than 3 hr. This behavior results because the reflections at the far end of the flux tube are eliminated, and more energy is able to propagate upwards. The long-period waves in Figure 14 are still insufficient to supply the energy requirements of the solar atmosphere, however. But moving the absorption to the middle of the flux tube increases the permitted wave energy flux density for \( 10 \leq T \leq 100 \) s to such an extent that these waves could supply a significant energy flux to the atmosphere. Moreover, it will be seen in Section 5 that waves with \( T \leq 60 \) s can undergo appreciable joule-frictional
damping in the photosphere and chromosphere of closed flux tubes. Damping low in the atmosphere should reduce the effect of reflection to a greater degree than does absorption at the middle of the coronal loop, and it seems possible that the energy supply to the atmosphere can be even larger than implied by Figure 14. In that case waves with $10 \, \text{s} \leq T \leq 100 \, \text{s}$ could conceivably supply the energy requirements of the atmosphere below the transition region. A model which self-consistently includes damping is needed to assess this possibility, however.

F. CONCLUSION

In sum, the loop resonances offer an attractive possibility for heating coronal active region loops, and they offer the possibility of explaining the $(P_{\odot})^{-1} - L_{\text{cor}}$ relation. Long-period waves, $T >$ a few hundred seconds or so, do not appear to be a feasible means of heating the corona or lower solar atmosphere. Excluding the loop resonances, higher frequency waves may be able to supply significant energy to the lower solar atmosphere, and to produce heating via joule-frictional damping, but the damping in this case plays a crucial role in determining the energy supply, and a model which self-consistently includes damping is required.

5. Damping

The results of the previous section indicate that damping can strongly influence the nature of the wave solutions. But it was also argued that the nature of the wave solutions can strongly affect the damping. Clearly, if damping is present at all, which must of course be the case if the waves are responsible for the heating, a fully self-consistent solution for the wave including their damping must be constructed. Such a solution is not our intent here. Instead, we will merely try to use the undamped solutions to assess the conditions under which damping might be important.

At present, models for solar magnetic flux tubes are too uncertain to allow detailed calculation of the electrical conductivity, viscosity, and ion-neutral collision frequencies as functions of position in a flux tube. Instead, the values obtained by Osterbrock (1961) will be used to provide crude estimates of the damping. Osterbrock's values are based on the old solar atmosphere model of van de Hulst (1953), however, in which the height scale differs from current solar models. To partially correct for this different behavior, we have written Osterbrock's values for the conductivity, etc. as function of of $n_{\text{H}}$ rather than of $h$, and then used the height behavior of $n_{\text{H}}$ in the models of this paper to estimate the height variation of the damping rates. The results thereby obtained are given in Table III for the basic open flux tube model (Figure 3) and the basic closed flux tube model (Figure 10) of this paper. The quantities $\xi_1$ and $\xi_2$ are the damping lengths (the distance over which energy falls by a factor $e^{-1}$) due to joule and frictional damping, respectively, for $T = 10 \, \text{s}$; the viscous damping is negligible compared to the joule damping for the entries in Table III. The values of $\xi$ scale with frequency as $T^2$. The values of $\xi$ given in the table are based on the assumption that the waves propagate according to the
TABLE III
The WKB damping lengths due to joule ($\xi_1$) and frictional ($\xi_2$) damping, on the basic open and closed flux tube models. All values are for $T = 10$ s, and scale as $T^2$. 

<table>
<thead>
<tr>
<th>$h$ (km)</th>
<th>$\xi_{1,\text{WKB}}$ (km)</th>
<th>$\xi_{2,\text{WKB}}$ (km)</th>
<th>$\xi_{1,\text{WKB}}$ (km)</th>
<th>$\xi_{2,\text{WKB}}$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$4.7 \times 10^5$</td>
<td>$3.0 \times 10^7$</td>
<td>$4.7 \times 10^5$</td>
<td>$3.0 \times 10^7$</td>
</tr>
<tr>
<td>175</td>
<td>$3.4 \times 10^6$</td>
<td>$2.5 \times 10^5$</td>
<td>$2.2 \times 10^6$</td>
<td>$2.2 \times 10^5$</td>
</tr>
<tr>
<td>440</td>
<td>$6.8 \times 10^4$</td>
<td>$2.0 \times 10^4$</td>
<td>$6.0 \times 10^4$</td>
<td>$1.9 \times 10^4$</td>
</tr>
<tr>
<td>710</td>
<td>$3.3 \times 10^5$</td>
<td>$5.5 \times 10^3$</td>
<td>$4.0 \times 10^6$</td>
<td>$1.2 \times 10^4$</td>
</tr>
<tr>
<td>900</td>
<td></td>
<td></td>
<td>$4.5 \times 10^7$</td>
<td>$2.0 \times 10^4$</td>
</tr>
<tr>
<td>1035</td>
<td>$2.1 \times 10^6$</td>
<td>$6.4 \times 10^3$</td>
<td></td>
<td>$3.1 \times 10^4$</td>
</tr>
<tr>
<td>1150</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1750</td>
<td>$1.1 \times 10^4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2340</td>
<td>$1.2 \times 10^4$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

WKB approximation. As already emphasized, this approximation is not appropriate in the solar atmosphere and corrections must be applied to the values in the table in order to obtain the correct damping lengths. The correct value of $\xi_1$ is obtained by dividing the value in the table by $\langle \delta j_{1,\text{WKB}} \rangle^2 / \langle \delta j_{1,\text{WKB}} \rangle$, and the correct value of $\xi_2$ is obtained by dividing the value in the table by $\langle \delta v^2 \rangle / \langle \delta v^2 \rangle_{\text{WKB}}$. In addition, $\xi_1$ is based on the assumption that the waves propagate parallel to $B_0$. If transverse gradients were important, $\xi_1$ would be reduced roughly by the factor $[1 + (\nabla_v / \nabla_B)^2]$. Finally, the net damping length, $\xi$, resulting from simultaneous action of joule and frictional damping, is given by $\xi^{-1} = \xi_1^{-1} + \xi_2^{-1}$.

Using Figure 5 to obtain the corrected values of $\xi$ on the open flux tubes shows that waves with $T = 10$ s should undergo moderate joule-frictional damping only near $h = 1000$ km. Only waves with $T \approx 10$ s can contribute to chromospheric heating via this damping mechanism. Longer period waves, even with large values of $\langle P_\odot \rangle$, should pass through the atmosphere without undergoing joule-frictional damping and without the consequent heating.

In Section 3 it was found that high-frequency waves on open flux tubes could carry energy flux densities sufficient to supply the energy requirements of the solar atmosphere, but it now appears that the energy can be made available to the atmosphere via joule-frictional damping only if most of the flux is carried by waves with $T \approx 10$ s. Not enough is known about the frequency spectrum of the solar atmospheric motions to rule out this possibility, but it seems unlikely. If most of the energy flux is at $T \approx 10$ s, there are three possibilities:

(i) Alfvén waves are not responsible for chromospheric heating;
(ii) $\xi_1$ and $\xi_2$ in Table III are significantly overestimated (an overestimate by a factor of 36 would mean that waves with $T \approx 2$ min would be damped);
(iii) some other damping mechanisms are operating. Possibility (iii) will be
discussed below.

The larger Alfvén speeds on the closed flux tubes result in larger WKB values of $\xi_1$
and $\xi_2$ than on open tubes. Except for the non-WKB corrections, one would be
tempted to conclude that joule-frictional damping of Alfvén waves is weaker on the
closed flux tubes. However, the non-WKB corrections are much larger on the closed
flux tubes than on the open ones, as can be seen by comparing Figure 12 with Figure
5, and it turns out that the damping is actually stronger on the closed flux tubes. (The
loop resonances are an exception.) For example, near $T = 10$ s, Figure 12 indicates
that the non-WKB correction factor can reduce $\xi_1$ and $\xi_2$ by a factor of roughly 200 in
the photosphere and chromosphere. Table III then shows that $\xi$ can be as small as a
few hundred kilometers at $T = 10$ s, and the damping will be severe between
$h \approx 100$ km and $h \approx 1500$ km. In addition, below $h \approx 1000$ km Figure 12 shows
that the non-WKB correction factor increases with $T$ for $T \leq 2$ min. It can be concluded
that, except for the loop resonances, waves with $T \leq 1$ min will suffer appreciable
damping in the photosphere and chromosphere. If the energy flux were large for
waves with $T \approx 1$ min, appreciable photospheric and chromospheric heating could
result.

It is important to note that the non-WKB correction to the damping is small in the
loop resonances. The resonant waves below the transition region propagate almost
according to the WKB approximation, and the damping lengths in Table III are then
appropriate. The damping lengths for $T \approx 4$ s are then always in excess of the
thickness of the atmosphere below the transition region, and the waves can propa-
gate from the convection zone into the corona without suffering appreciable
joule-frictional damping.

Even though joule-frictional damping may be strong in the photosphere and
chromosphere of a closed flux tube such as might exist in an active region, the
joule-frictional damping is weak in the photosphere and chromosphere of open flux
tubes, and weak in both open and closed coronae. Are there other damping
mechanisms whereby the large Alfvén wave fluxes predicted by this paper can be
damped and the atmosphere heated? In this regard we will merely present a few
speculations, to be the subjects of future studies.

Figure 5a shows that moderate nonlinearities vis-à-vis $\langle \delta v^2 \rangle / v_A^2$ can result on open
flux tubes below the corona if $\langle P_\odot \rangle \geq 10^7$ erg cm$^{-2}$ s$^{-1}$. Similarly, Figure 12 shows
nonlinearity on closed flux tubes below the middle chromosphere if $\langle P_\odot \rangle \geq
\approx 10^7$ erg cm$^{-2}$ s$^{-1}$. This means that if the waves on ‘adjacent’ field lines are not
related with one another, then large relative shearing velocities (of the order of
2$v_A$) can occur from field line to field line. And above $h = a$ few hundred kilometers
or so, where the Alfvén speed exceeds the sound speed, the relative shearing
velocities can substantially exceed the sound speed. It is tempting to speculate that
the large shears could be Kelvin-Helmholz unstable. If a Kelvin-Hemholz instability
were to occur, it could result in energy being taken out of the Alfvén wave and put
into smaller scale motions, which could subsequently damp via joule, viscous, or
frictional processes. And if the driving velocity shears are supersonic, it is conceivable that the small-scale unstable motions could rapidly steepen into shocks and subsequently dissipate.

It is also tempting to speculate that the relative shearing of 'adjacent' field lines could be unstable to tearing instabilities, especially when \( \langle \delta B^2 \rangle / B_0^2 \approx 1 \), which can be the case in the photosphere and chromosphere of open and closed flux tubes when \( \langle P_o \rangle = 10^7 \text{ erg cm}^{-2} \text{ s}^{-1} \) (except in the loop resonances). However, a preliminary estimate for the efficacy of the collisional tearing mode (using Table I of Drake and Lee, 1977) indicates that, below the middle chromosphere, tearing instabilities will grow on a time scale shorter than the time scale over which the driving magnetic field pattern is altered only if the transverse (to \( B_0 \)) spatial scale of the Alfvén waves is less than about 15 km. (In obtaining this estimate a wave period 300 s has been assumed, and we have taken \( (c^2/4 \pi \Sigma) = 10^8 \text{ cm}^{-2} \text{ s}^{-1} \) (\( \Sigma \) is the electrical conductivity), \( n_e = 4 \times 10^{11} \text{ cm}^{-3} \), \( n_H/n_e = 10^4 \), \( B_0 = 800 \text{ G} \), and \( \langle \delta B^2 \rangle / B_0^2 \approx 1 \). It turns out that the tearing instability growth rate does not vary strongly with height between the photosphere and middle chromosphere. It seems remotely possible that the transverse spatial scale of the Alfvén waves could be as small as 15 km in the lower solar atmosphere, but some dissipation of Alfvén waves via tearing instabilities could occur. For reasonable transverse scales, tearing will be unimportant in the corona, however, because the electrical conductivity is too large and \( \langle \delta B^2 \rangle / B_0^2 \) is small in our models.

Another possibility for damping Alfvén waves involves their non-linear conversion into compressive modes. This conversion is effected by the \( \delta j \times \delta B - \rho_0 \delta v \cdot \nabla \delta v \) pondermotive force which the waves exert on the plasma (e.g., Barnes and Hollweg, 1974; Chin and Wentzel, 1972; Kaplan and Tsytovich, 1973; Wentzel, 1977). Even though damping of the first-order Alfvén wave by itself may be weak, the damping of the compressive wave which is nonlinearly driven by the Alfvén wave may be strong, and nonlinear damping of the Alfvén wave in effect occurs. The degree to which the Alfvén waves is damped depends in part on the damping rate of the driven compressive mode, and in part on the extent to which the Alfvén wave can drive the compressive wave. Both of these factors may be significant below the transition region in the models of this paper. A particularly interesting possibility is that the nonlinearities could result in the Alfvén waves steepening into fast shocks below the transition region. The shocks could then heat the chromosphere. Moreover, shocks formed below the transition region would maintain themselves as shocks as they penetrate into the corona, and coronal heating would occur. Finally, the interaction of a fast shock with the transition region (a contact surface) will in general set the transition region into motion, which may manifest itself as a spicule. A nonlinear model for the wave propagation is currently being developed, and these possibilities can be tested numerically.

Finally, mode conversion of the Alfvén waves into compressional waves as the former propagate on the curved field lines of the coronal loops can also occur (Hollweg and Liliequist, 1978). Subsequent damping of the compressional waves

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System
can result in an effective damping of the Alfvén waves. Mode conversion of the Alfvén mode into the fact mode may be particularly efficient, and the subsequent viscous and heat conduction damping of the fast mode can be strong if the fast wave has a sufficiently small wavelength transverse to $B_0$, $\lambda_\perp$. For example, if the driving Alfvén waves are confined to coronal active region loops, $\lambda_\perp$ might be the loop diameter, i.e., $\approx 2000$ km. Unfortunately, a small $\lambda_\perp$ implies propagation nearly perpendicular to $B_0$, and that is when the conversion of Alfvén waves to fast waves is weakest.

6. Summary

The foregoing attempts to illuminate the most important properties of linearized Alfvén waves on solar magnetic flux tubes. The study is motivated primarily by a belief, based on several lines of evidence gleaned from theoretical and observational studies of the solar wind, that the Sun is a radiator of an energetically significant flux of long-period Alfvén waves into the solar wind. Our principal goal is to see to what extent the generation of those waves can be understood theoretically. Since we believe that the Sun’s Alfvénic emission can strongly affect the dynamics and thermodynamics of the solar wind, we are led naturally to ask also whether the Sun can generate Alfvén waves at other frequencies and on other magnetic field configurations, and whether those waves can influence the dynamics and thermodynamics of magnetic regions of the lower solar atmosphere and corona. This study is motivated also by the recognition that even though Alfvén waves have for more than three decades been invoked as playing a variety of roles in solar atmosphere, a proper study of the non-WKB nature of the waves has not yet been carried out; without such a study, it is impossible to assess the effects of Alfvén waves in the solar atmosphere. Finally, even though it is currently fashionable to invoke a variety of anomalous processes for heating the corona, especially coronal active region loops (e.g., Ipson, 1978; Rosner et al., 1978b; Wentzel, 1978), it is almost never asked how the energy can reach the corona from the solar convection zone, except perhaps to invoke magnetic field ‘twists’. We believe that a broader outlook is necessary, and that it may be pointless to invoke a variety of dissipation processes in the corona without first asking how the postulated energy can be made available to the corona in the first place. This paper therefore examines in some detail the extent to which propagating time-dependent twists, i.e. Alfvén waves, can supply energy to the solar atmosphere.

On flux tubes which open into the solar wind, it is found that the long-period waves ($T \approx 1$ hr or so) propagating near the axis of the tube can carry an energy flux density of a few times $10^4$ erg cm$^{-2}$ s$^{-1}$ at the Sun. This is at the low end of what we consider to be the ‘observed’ range of long-period Alfvén wave flux densities at the base of the solar wind. There are reasons to believe that even larger long-period flux densities can be generated on field lines not near the central axis, or carried via nonlinear effects.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System
If the average magnetic field strength of the Sun below open regions of the corona, such as coronal holes, is controlled only by the filling factor of the magnetic flux tubes, then it is found that \( \langle P_{\odot} \rangle \) at the long periods is essentially independent of the filling factor. The energy flux which Alfvén waves supply to the solar wind may therefore be essentially independent of \( \langle B_{\odot} \rangle \), rather dependent primarily on the fraction of the solar surface which opens into the solar wind. This conclusion is qualitatively consistent with observational results which indicate that the energy in high speed streams is positively correlated with the area of open fields at the Sun.

It is also found that high-frequency waves \( (T \approx 500 \text{ s or so}) \) on open flux tubes can in principle carry an energy flux density which is sufficient to account for the non-radiative energy supply of the solar atmosphere, i.e., \( \langle P_{\odot} \rangle = 10^6 - 10^7 \text{ erg cm}^{-2} \text{ s}^{-1} \); this is the predicted energy flux if \( |\delta v| = 1 \text{ km s}^{-1} \) at high-frequencies at \( h = 0 \), which does not seem unreasonable in view of the large microturbulent velocities at \( h = 0 \). Unfortunately, joule-viscous dissipation will only be effective for waves with \( T \approx 10 \text{ s} \), unless our crude estimates of the damping coefficients are in error. It is unlikely that most of the wave power exists at such short periods, and some other damping mechanisms must be found if the waves are to heat the atmosphere. We were able to show that the nonlinearity of the waves in the photosphere and chromosphere offers several possibilities for damping the waves there. Further work is needed in this regard, but we at least have some grounds to be optimistic that Alfvénic heating of the magnetic regions of the solar atmosphere can occur.

The model for wave propagation on open flux tubes predicts that the waves should exert large net upward forces on the chromosphere and lower corona. Hydrostatic equilibrium can be significantly violated, and in some cases net upward forces in excess of \( \rho_0 g \) are predicted. Alfvén waves therefore offer the possibility of an explanation for observed solar motions such as spicules and macrospicules. A fully nonlinear calculation for the effect of the Alfvén waves on the flow and the effect of the flow on the Alfvén wave propagation is needed to test this possibility. Such a calculation is currently in progress.

The solutions on closed flux tubes are found to exhibit strong differences from the open solutions. Long-period waves \( (T \geq 500 \text{ s or so}) \) do not seem capable of supplying enough energy to account for the heating of the lower solar atmosphere or corona. Even anomalous dissipation of the currents associated with long-period twists does not represent an attractive possibility for coronal heating, because the energy carried from the convection zone into the corona by the long-period twists is too small. An attractive possibility for getting energy from the convection zone into the corona is via the loop resonances. Even though the loop resonances exist via energy storage on the coronal part of the flux tube, the resonances affect the propagation in the photosphere and chromosphere as well in such a way that large energy flux densities into the corona can be obtained without being impeded by appreciable damping between the convection zone and the corona. The resonances serve therefore as ‘windows’ which allow energy to reach the coronal parts of closed flux tubes. It is not clear how the waves damp in the corona, however.
Except for the loop resonances, high-frequency waves ($T \leq 500$ s or so) on closed flux tubes pose a problem. If damping is not taken into account, it is found that these waves cannot supply a significant energy flux to the solar atmosphere. However, it is also found that the joule-frictional damping of these waves in the photosphere and chromosphere cannot be neglected. We were able to argue that the damping should be able to increase the energy flux, by reducing the reflections which are responsible for the low predicted flux of the undamped waves. On the other hand, reducing the reflections reduces the damping. The possibility exists that a fully self-consistent model for the wave propagation, with damping included, could lead to a situation in which non-loop-resonant high-frequency waves can simultaneously carry a significant energy flux and heat the magnetic regions of the photosphere and chromosphere. Such a self-consistent model still needs to be carried out.

The forces exerted by long-period waves on closed flux tubes are probably unimportant. But the loop resonances and the high-frequency waves can exert forces which are strong enough either to alter hydrostatic equilibrium or to drive flows. Remarkably, whereas the time-averaged forces on open flux tubes are upward, the forces on closed flux tubes can be either upward or downward. It is difficult to draw any unique conclusions about the possible wave-driven flows on closed flux tubes. Indeed, the complex structure of the wave forces may in part be responsible for the complexity and variability of the observed active region mass flows.

Our study particularly emphasizes the non-WKB nature of the waves. Observations of $\langle \delta v^2 \rangle$ or $\langle \delta B^2 \rangle$ cannot be used to determine the wave energy flux without additional knowledge of the nature of the wave propagation; errors of several orders-of-magnitude can result if the usual WKB formulas are used to deduce the energy flux density. But even more importantly, the ordinary linear joule-frictional damping rates, the time-averaged wave forces, and the nonlinear damping rates due to coupling into compressional modes cannot be properly assessed without treating the full wave solutions. For example, the joule-frictional damping length in the photosphere and chromosphere of closed flux tubes is smaller than the WKB value by two to three orders-of-magnitude. We conclude that the entire question of wave energy transport and its linear and nonlinear dissipation must be re-examined.

The necessity for full wave solutions means that little can be learned without having full models for the magnetic regions of the solar atmosphere, including $B_0(s)$, $r(s)$, $\rho_0(s)$, and the joule, viscous and frictional damping coefficients in the flux tubes. Similarly, if theory and observation are to be compared in any meaningful way, it will be necessary to have data at least on the spatial distribution of $|\delta v|$ and the volumetric heating rates inside flux tubes; data on $|\delta B|$ would be equally useful. Equally important will be the distribution of wave variables in the frequency domain, because frequency is an important variable governing the behavior of non-WKB phenomena.

The non-WKB nature of the wave propagation means that the heating of the solar atmosphere may be globally controlled. The fact that longer coronal active region loops have a smaller energy input than shorter ones may be one observational indication that the heating is in fact globally, rather than locally, controlled. If the
heating is indeed globally controlled, then full wave solutions in full models of the solar atmosphere will be necessary if the heating is to be understood. And local analyses, such as local stability analyses of coronal active region loops, may be meaningless.

Acknowledgements

This work has benefitted immeasurably from the Skylab Active Region Workshop, in which the author was a participant. The author is grateful also to R. G. Athay for a critical reading of an earlier version of this manuscript.

References