Density sensitive C\textsc{II} lines in cool stars of low gravity

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Summary. It is shown that the relative intensities of emission lines within the multiplet uv 0.01 of C\textsc{II}, around 2325 Å, are sensitive to electron density in the range $10^9 > N_e > 10^7$ cm$^{-3}$.

The lines therefore offer a valuable method for measuring $N_e$ in the chromospheres of late-type giants and supergiants. Calculated line ratios are compared with those observed in a range of objects.

1 Introduction

Late-type giant and supergiant stars are currently being observed using the International Ultraviolet Explorer (IUE) satellite, for the purpose of modelling the structure and energy requirements of their chromospheres.

Early spectra obtained (e.g. Brown, Jordan & Wilson 1979; Carpenter & Wing 1979; Dupree et al. 1979; Linsky & Haisch 1979) have shown that in contrast to near main sequence objects the late-type low gravity stars show no evidence of chromospheric material at $T_e > 2 \times 10^4$ K. Models of the chromospheres of stars such as α Boo (Ayres & Linsky 1975) and α Tau (Kelch et al. 1978), made from the lines of Ca\textsc{II} and Mg\textsc{II}, suggest that the electron density should be $\sim 10^8$ cm$^{-3}$, at $T_e \sim 10^4$ K. Also there is evidence that photo-excitation (Haisch et al. 1977) and radiation trapping (Brown & Jordan 1980; Brown, Ferraz & Jordan 1980) play an important role in line formation in these cool extended envelopes. Given the complexity of the spectra and their interpretation it is particularly important to establish spectroscopic methods of directly evaluating atmospheric parameters such as the electron density.

A preliminary estimate of $N_e$ in the M giant β Gru (M3 II) has already been made using the ratio of lines within the C\textsc{II} multiplet $2s^2 2p^2 ^4P - 2s 2p^2 ^4P$ at $\sim 2325$ Å (Brown et al. 1980). In the present Communication the calculations are extended over a wider range of...
$N_e$ and $T_e$ and comparisons are made with previously unpublished stellar spectra and with objects with $N_e$ at the high and low density limits, the latter serving to cross-check the calculations.

The method is also valuable since the multiplet lies within the range of the long-wavelength camera of IUE and thus the necessary high resolution observations can be obtained with moderate exposures (less than ~ 4 hr).

At present the accuracy of the density measurements is limited by a lack of knowledge concerning the collision rates between the $^4P$ levels. The exposures available to date are in general not optimum, usually being shorter than necessary to obtain sufficiently reliable flux ratios. In addition to presenting new data and calculations, the purpose of the present Communication is to point out the value of the C II 2325 Å multiplet and to encourage further high resolution observations and calculations of atomic data for the C II transition.

2 The C II $^2P-^4P$ multiplet as a density diagnostic

The sensitivity of lines within the $2s^2 2p^2 2P - 2s 2p^2 4P$ multiplet of the Boron i-like ions to the electron density has been realized for some years. Previous work has considered N III, O IV and higher ions in the context of the solar atmosphere (e.g. Flower & Nussbaumer 1975; Feldman & Doschek 1979; Nussbaumer & Storey 1979).

The transition probabilities for the five members of the multiplet have been calculated for ions between C II and Fe XXII by Dankwort & Trefftz (1978) and recently by Nussbaumer & Storey (1981). The total collision strength for the $2P-4P$ multiplet in C II has been calculated by Jackson (1972). The individual collision strengths for this multiplet and for transitions between the $^4P$ J-levels have been calculated for N III and O IV by Flower & Nussbaumer (1975) and by Nussbaumer & Storey (1979). No $^4P-^4P$ collision strengths are available for C II.

In order to investigate the likely range of sensitivity to $N_e$ the individual $^2P-^4P$ collision strengths in C II were taken to be in the same ratios as in N III and O IV. The $^4P-^4P$ collision strengths were extrapolated from N III and O IV, maintaining the same relative values.

Table 1. Atomic parameters adopted.

| Transition       | $\Delta J$ | $\lambda$ (Å) (air) | $\Omega \eta$ | $A^\dagger$ | $A^\ddagger$ | $A^\ddagger$ | $A^\ddagger$ |
|------------------|------------|----------------------|---------------|--------------|--------------|--------------|
|                  |            |                      |               | $s^{-1}$     | $s^{-1}$     | $s^{-1}$     | $s^{-1}$     |
| $2s^2 2p^2 2P-2s 2p^2 4P$ | 3/2–5/2    | 2325.40              | 1.33          | 34.4         | 34.4         | 43.2         | 37.6         |
|                  | 3/2–3/2    | 2326.93              | 0.60          | 8.11         | 9.4          | 5.24         | 8.5          |
|                  | 3/2–1/2    | 2328.12              | 0.21          | 40.2         | 59.0         | 65.5         | 65.7         |
|                  | 1/2–5/2    |                      | 0.27          | –            | –            | –            | –            |
|                  | 1/2–3/2    | 2323.50              | 0.47          | 1.01         | 1.2          | 1.71         | 2.8          |
|                  | 1/2–1/2    | 2324.69              | 0.32          | 42.5         | 63.0         | 55.3         | 56.4         |
|                  |            |                      |               | Total: 3.2*  |              |              |              |
| $2s 2p^2 4P-2s 2p^2 4P$ | 1/2–5/2    | 1.4                  |               |              |              |              |              |
|                  | 1/2–3/2    | 2.4                  |               |              |              |              |              |
|                  | 3/2–5/2    | 4.3                  |               |              |              |              |              |

* From Jackson (1972).
† From Dankwort & Trefftz (1978).
‡ Empirical – see text.
§ From Nussbaumer & Storey (1981).
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Figure 1. Calculated line ratios for \( R_1 = E(3/2-5/2)/E(3/2-1/2) \), \( R_2 = E(3/2-5/2)/E(3/2-3/2) \), and \( R_3 = E(1/2-1/2)/E(3/2-3/2) \) as a function of \( N_e \) and \( T_e \). The \( T_e \) values used were 7000 K (dotted line), \( 10^4 \text{K} \) (full line) and \( 2 \times 10^4 \text{K} \) (dashed line). Ratios observed in the low density limit (NGC 6572) and high density limit (the Sun) are also shown.

Because of the uncertainty of this procedure and some systematic differences between the observed and calculated ratios the sensitivity of the calculated ratios to the collision strengths and \( A \)-values, as well as to \( N_e \) and \( T_e \) has been investigated.

At the temperatures considered loss of population through collisions to higher levels is estimated to be unimportant compared with collisional de-excitation to the ground state. Recombination could in principle be significant, but the stars concerned do not show evidence of \( \text{C} \text{III} \) emission (at 1909 Å) and recombination is estimated to be unimportant.

The atomic parameters adopted are given in Table 1. The wavelengths are from Moore (1970).

The results of the calculations using the transition probabilities of Dankwort & Trefftz are shown in Fig. 1. Three ratios

\[
R_1 = E(3/2-5/2)/E(3/2-1/2), \\
R_2 = E(3/2-5/2)/E(3/2-3/2), \\
R_3 = E(1/2-1/2)/E(3/2-3/2),
\]

are shown as a function of \( N_e \) and \( T_e \). The temperatures used were 7000 K (dotted line), \( 10^4 \text{K} \) (full line) and \( 2 \times 10^4 \text{K} \) (dashed line).

It can be seen that the ratios are not unduly sensitive to \( T_e \). Uncertainties in observed line ratios and atomic data produce more significant uncertainties in \( N_e \).

3 Comparisons with observed line ratios

Fig. 1 shows line ratios observed in a planetary nebula, NGC 6572, and in the Sun (Doschek, Feldman & Cohen 1972). The planetary nebula is used as an example of an object where ratios should be as in the low density limit, when they depend only on relative excitation rates within \( ^2P-^4P \) and branching ratios from individual \( ^4P \) levels. No density is attributed to NGC 6572, it is simply assumed to be \( \leq 10^5 \text{cm}^{-3} \). Flower & Penn (1981) refer to this nebula as a low density object. It can be seen that with error bars of at least ±10 per cent in the intensity ratios there is no significant difference between the observed and calculated ratios.
The high density limits are given by relative Boltzmann populations and the relative $A$-values of the transitions concerned. The solar density at around $10^4$K is usually taken as $\sim 10^{11}$ cm$^{-3}$. The solar ratios are less satisfactory. Error bars of $\pm 10$ per cent are shown, but the actual uncertainties may be larger since it is difficult to obtain accurate fluxes of the C II emission lines above the solar continuum around 2325 Å. In particular, accepting the branching ratios and the relative $A$-values from the $^4P$ levels as being correct then, at least one of the solar measured ratios must be incorrect.

The observed ratios in a range of objects are illustrated in Fig. 2 (wavelength scales are not absolute), and intensities are listed in Table 2.

When the observed ratios are compared with the calculated values shown in Fig. 1 it is apparent that even allowing for uncertainties of $\pm 15$ per cent in the measured line ratios, the three different ratios do not lead to the same value of $N_e$. Instead, densities measured from $R_1$ are systematically higher than those from $R_2$ or $R_3$. Apart from a few exceptions, which lie within experimental uncertainties, there is a systematic decrease in $N_e$ with later spectral type and lower gravity. The observed ratios are clearly different from either the low or high density limit values. Thus there exists the potential for $N_e$ measurements in stars where $N_e \sim 10^8$ cm$^{-3}$. 

Figure 2. Line of C II uv multiplet 0.01 as observed in several late-type stars and a planetary nebula.
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Table 2. Observed line intensities in selected cool stars and density extremes.

<table>
<thead>
<tr>
<th>Star</th>
<th>Spectral type</th>
<th>LWR time (min)*</th>
<th>Relative intensities</th>
<th>Total flux$^8$ (erg cm$^{-2}$ s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>High $N_e$</td>
<td>4 stable</td>
<td>$\lambda 2323.5$ $\lambda 2324.7$ $\lambda 2325.4$ $\lambda 2326.9$ $\lambda 2328.1$</td>
<td></td>
</tr>
<tr>
<td>(1) $\alpha$ Boo</td>
<td>K2 III</td>
<td>3741 40</td>
<td>4 50 100 15 58</td>
<td>1.6 ($-1$)</td>
</tr>
<tr>
<td>(2) $\alpha$ Tau</td>
<td>K5 III</td>
<td>9644 210</td>
<td>17 $^\dagger$ 34 100 25 $^\dagger$ 40</td>
<td>9.2 ($-12$)</td>
</tr>
<tr>
<td>(3) $\gamma$ Cru</td>
<td>M3 III</td>
<td>1356 120</td>
<td>9 $^\dagger$ 35 100 28 $^\dagger$ &lt;38</td>
<td>7.0 ($-12$)</td>
</tr>
<tr>
<td>(4) $\beta$ Gru</td>
<td>M3 II</td>
<td>9677 85</td>
<td>11 $^\dagger$ 37 100 27 28</td>
<td>8.2 ($-12$)</td>
</tr>
<tr>
<td>(5) $\alpha$ Ori</td>
<td>M2 I ab</td>
<td>1371 120</td>
<td>11 $^\dagger$ 31 100 51 27</td>
<td>7.7 ($-12$)</td>
</tr>
<tr>
<td>NGC 6572  Low $N_e$</td>
<td>7422 120</td>
<td>34 18 100 60 16</td>
<td>1.64 ($-11$)</td>
<td>9.5 ($-12$)</td>
</tr>
</tbody>
</table>

* All exposures through the 10 X 20 arcsec aperture.
† Blended with Fe II (UV3) emission at 2327.2 Â.
‡ Weakly exposed, very low signal to noise data.
$^8$ Observed from Earth orbit, calibration factor $1.67 \times 10^{-14}$ erg cm$^{-2}$ s$^{-1}$ Â$^{-1}$ FN$^{-1}$ min. Uncertainty in the relative intensities is estimated to be at least $\pm$ 15 per cent.

To investigate the sensitivity of the ratios to the collision strengths for the $^4P - ^4P$ transitions several sets of calculations were made. It is, however, difficult to change the ratios $R_1$ and $R_2$ in this way (e.g. changes of 25 per cent in the collision strengths lead to changes of less than 5 per cent in $R_1$ and $R_2$).

The observed solar and stellar ratios suggest that $R_1$ should be reduced at $N_e \leq 10^8$ cm$^{-3}$ in order to lead to the same value of $N_e$ for the different ratios. Simply as an illustration the high density limits for $R_3$ and $R_1$ were forced to fit the solar ratios — this implying the first set of ‘empirical’ $A$-values given in Table 1. Then as discussed above, the third solar ratio, $R_2$, would not fit the calculated value. The empirical dependence of the ratios on $N_e$ are shown in Fig. 3, and the observed ratios are also marked on this figure. Although still not perfect, there is, for the stellar observations, much closer agreement between the values of $N_e$ from the three ratios.

If the more recent transition probabilities calculated by Nussbaumer & Storey (1981) are adopted then in the low density limit the ratios shown in Fig. 4 result. These are still in

![Figure 3](https://example.com/figure3.png)

Figure 3. Line ratios as a function of $N_e$, with $T_e = 10^4$K, forcing agreement between calculated and observed solar values for $R_1$ and $R_3$ (see text). The stellar ratios shown are for (1) $\alpha$ Boo, (2) $\alpha$ Tau, (3) $\gamma$ Cru, (4) $\beta$ Gru and (5) $\alpha$ Ori. Typical error bars of $\pm$ 15 per cent are shown.

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moderately good agreement with those observed in NGC 6572. The most significant improvement is that the theoretical branching ratio of 3:1 for decays from the $^4P_{3/2}$ level, agrees more closely with the average observed value than does the ratio of 8:1 calculated by Dankwort & Trefftz. The branching ratio from $^4P_{1/2}$ is not substantially different between the two sets of calculations.

In the high density limit the ratios $R_2$ and $R_3$ are 12.4 and 5.3, respectively, significantly larger than the observed solar values. However, as suggested by Storey (private communication) an alternative set of 'empirical' $A$-values may be found by forcing a fit to the solar ratios whilst retaining the calculated branching ratios and normalizing to the total $A$-value. This procedure results in the values given in the last column of Table 1.

The line ratios as a function of $N_e$ are then as shown in Fig. 4. It can be seen that whilst the low and high density limits are satisfactory the discrepancy between the value of $N_e$ derived from the three ratios in a given star still remains.

Further progress on the accurate determination of $N_e$ must await more accurate collision strength calculations for C II. Improved observed ratios in the low and high density limits would also further constrain the $A$-values.

4 Discussion

The electron densities implied for $\alpha$ Boo (point 1) and $\alpha$ Tau (point 2) are $\sim 3 \times 10^8$ cm$^{-3}$ and $2 \times 10^8$ cm$^{-3}$, respectively, for $T_e \sim 10^4$ K. The chromospheric models by Ayres & Linsky (1975) and Kelch et al. (1978) give $N_e \sim 5.4 \times 10^8$ cm$^{-3}$ and $8.4 \times 10^7$ cm$^{-3}$, respectively, in reasonable agreement considering the large inherent uncertainties.

Using the $N_e$ values of $3 \times 10^8$ cm$^{-3}$ and $2 \times 10^8$ cm$^{-3}$ and the observed total flux in the $^2P - ^4P$ multiplet, the corresponding values of $\int N_{\text{HI}} \, dh$ can be obtained, and are $\sim 6 \times 10^{20}$ cm$^{-2}$ and $3 \times 10^{20}$ cm$^{-2}$. (We adopt $N($C II)/$N($C) = 0.28 at $10^4$ K (Jordan 1969), $N($C)/$N($H) = 2.5 $\times$ $10^{-4}$ and stellar parameters from the above papers.) With $N_{\text{HI}} \sim N_e$ at $10^4$ K, the resulting thickness of the emitting regions is $\sim 2 \times 10^{12}$ cm, i.e. of the order of the stellar radius in contrast to the much narrower regions found for the Sun and near main sequence stars.

In principle, the ratio of the C II lines at 1335 and 2325 Å may also be used to give an independent measurement of $T_e$, and the 1335 Å flux a value of $\int N_e \, N_{\text{HI}} \, dh$. As pointed out in an earlier report (Brown et al. 1980), in $\alpha$ Tau the emission measure from the 2325 Å multiplet and $T_e = 10^4$ K predicts an order of magnitude more flux in the 1335 Å multiplet.
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than is observed. A discussion of the absolute fluxes and the ratio of the lines at 1335 and 2325 Å, including possible effects of line opacities, is postponed to a later paper.

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References