AXISYMMETRIC AND MIXED POLOIDAL-TOROIDAL
MAGNETIC FIELDS FOR HD 215441 AND 53 CAM

M. GOOSENS and W. VAN ASSCHE
Astronomisch Instituut
Katholieke Universiteit Leuven
Naamsestraat 61
B-3000 LEUVEN (Belgium)

SUMMARY. - The oblique rotator model is adopted, and axisymmetric,
mixed poloidal-toroidal magnetic fields are considered for the des-
cription of the periodic magnetic variations observed in many Ap-
stars. A schema is presented for the determination of the surface structure
of the magnetic field from $H_e$ and $H_o$ observations. This schema has
been successfully applied to the magnetic observations of HD 215441
and 53 Cam. It was found that several magnetic surface structures
fit the observational data.

1. INTRODUCTION

The oblique rotator model has been widely accepted as an adequate
description of the periodic magnetic variations observed in many Ap-
stars. Considerable effort has been invested in determining surface
magnetic field geometries from observations of $H_e$ and $H_o$ time vari-
ations. These magnetic field geometries are irrotational in almost
all cases. However, the assumption of a virtual electrodynamic
vacuum for the outer layers of a star may not be a good approximation
of reality (Mestel, 1974), and the distribution of the magnetic field
in the outer layers of the star is probably not irrotational, but
almost force-free (Milsom and Wright, 1976). This has led us to con-
sider general axisymmetric magnetic field models for the observed
magnetic variations. Such magnetic fields have both poloidal and to-
oroidal components and are probably dynamically stable. Moreover, we
assume the toroidal component to be a non-linear expression of the
magnetic stream function as in force-free magnetospheres (Raadu, 1971;
Milsom and Wright, 1976). In Section 2, a schema is set up to deter-
mine the surface structure of the magnetic field from $H_e$ and $H_o$ ob-
servations. In Section 3, several surface magnetic field geometries
are derived for HD 215441 and 53 Cam.

2. OUTLINE OF THE METHOD

The oblique rotator model is applied to the observed periodic
magnetic variations. Consider a spherical star that rotates rigidly
about an axis of rotation $\hat{z}$ and that possesses a frozen-in magnetic
field. The magnetic field is axisymmetric about a magnetic axis $\hat{m}$
through the stellar center, and inclined by an angle $\beta$ to the axis
of rotation $\hat{z}$. The notation is as in Goossens (1979) : $i$ denotes
the angle included by the rotation axis and the line of sight $\hat{B}$.

A natural representation of a general axisymmetric magnetic field
is

$$\hat{H} = \nabla \wedge \left( \frac{S(r, \phi)}{r \sin \theta} \hat{\phi} \right) + \hat{H}_T \hat{\phi},$$

$$i$$

Upper Main Sequence CP Stars
Université de Liège, June 1981 277

© IAGL • Provided by the NASA Astrophysics Data System
where $S(r, \theta')$ is the magnetic stream function of the poloidal component $H_p$. $H_T$ is the toroidal component, which is chosen to be related to the magnetic stream function as in force-free magnetospheres

$$H_T = aS^k.$$  

(2)

$a$ is the parameter that describes the twist of the field, and $k$ is a constant that has to be chosen so that the poloidal component behaves like a dipole at large distances from the star (Radu, 1971). This implies $k > 2$.

We expand the magnetic stream function in terms of associated Legendre functions $P_k^l(\cos \theta')$ to arrive at the following expressions for the distribution of the magnetic field components over the stellar surface

$$H_r = \sum_{n=1}^{\infty} S_n P_n^1(\cos \theta'),$$
$$H_{\theta'} = -\sum_{n=1}^{\infty} T_n P_n^1(\cos \theta'),$$
$$H_{\phi'} = \sum_{n=1}^{\infty} F_n P_n^1(\cos \theta'),$$

(3)

where $S_n$, $T_n$, and $F_n$ are constants. The constant $F_n$ can be expressed in terms of $\alpha$ and of $S_{\ell}$ ($\ell = 1, \infty$) for each value of $k$:

$$F_n = a\alpha_n(S_{\ell}) \text{ with } \ell = 1, \infty.$$  

(4)

The constants $S_n$ and $T_n$ determine the distribution of the poloidal components, and the distribution of $H_T$ is fixed by $\alpha$ and $S_n$.

The parameters $S_n$, $T_n$, and $\alpha$, which describe the surface magnetic field geometry, and the angles $i$ and $\beta$ must be chosen such that the theoretical $H_e$ and $H_S$ curves are good approximations of the observed curves. First consider the $H_e$ observations. Goossens et al. (1981) derived an analytical expression for the theoretical variation of $H_e$ in a star with an axisymmetric magnetic field surface structure. They found that

$$H_e = \sum_{j=0}^{\infty} D_j \cos 2\pi(f - f_0)$$

(5)

where $D_j$ are constants and $f$ is the phase. The constants $D_j$ are expressed in terms of the constants $S_n$ and $T_n$, which describe the surface structure of the poloidal magnetic field component, and the angles $i$ and $\beta$. The condition that the theoretical $H_e$ curves coincide with the least-squares fit of the form of equation (5) to the $H_e$ observations yields equations for the quantities $S_n$, $T_n$, $i$, and $\beta$. The adequacy of the approximation is given by the quantity

$$\sigma_{H_e} = \left[ \frac{1}{N_e-1} \sum_{i=1}^{N_e} (H_{e,i} - (H_{e,i})c)^2 \right]^{1/2},$$

(6)

where $N_e$ is the number of $H_e$ observations, and $H_{e,i}$ and $(H_{e,i})c$ represent the observed and computed values of $H_e$ at phase $f_i$ respectively. A numerical method of trial and error to minimize
\[ \sigma_{H_S} = \frac{N_S}{N_S - 1} \left( \sum_{i=1}^{N_S} (H_{S,i} - (H_{S,i})_c)^2 \right)^{1/2} \]

is used to obtain estimates for \( i, \beta, S_n, T_R \) and \( \alpha \) in order to ensure a good reproduction of the \( N_S \) \( (f_j, H_{S,j}) \) observations of \( H_S \). \( (H_{S,i})_c \) denotes the computed value.

For each solution, we computed the polar strength, \( H_P \), the difference in magnetic field strength at the two poles, \( \Delta H_P \), and the quantities \( \tilde{\alpha} \) and \( M \), which are two measures of the relative strengths of the toroidal and poloidal magnetic field components. They are defined as the ratios

\[ \tilde{\alpha} = \frac{H_{T,eq}}{B}, \quad M = \frac{H_{S,T}}{H_{S,P}} \]

where \( H_{T,eq} \) is the strength of the toroidal component at the equator, \( B \) is the total flux through a hemisphere divided by \( 2 \pi R^2 \) (\( R \) = radius), and \( H_{S,T} \) and \( H_{S,P} \) are the means over one cycle of the hypothetical surface magnetic field that is produced by \( H_T \) and \( H_P \) alone.

3. APPLICATION TO HD 215441 AND 53 CAM

The schema presented in the previous Section has been succesfully applied to the magnetic observations of HD 215441 (\( H_e \) observations by Borra and Landstreet, 1978, \( H_S \) observations by Preston, 1969), and of 53 Cam (\( H_e \) observations by Borra and Landstreet, 1977, \( H_S \) observations by Huchra, 1972). The existence of different axisymmetric surface structures of magnetic fields that reproduce the observations equally well has been established for both stars. Two values of \( k \) have been considered: \( k = 3, k = 4 \). For both values of \( k \) for HD 215441, we obtained solutions for two slightly different sets of \( i \) and \( \beta \), namely \( i = 11^\circ, \beta = 62^\circ \) and \( i = 14.5^\circ \) and \( \beta = 52.5^\circ \). By way of illustration, we give in Table I some characteristic quantities of our solutions for \( i = 14.5^\circ, \beta = 52.5^\circ \) and \( k = 4 \). The solutions obtained for \( i = 14.5^\circ, \beta = 52.5 \) with \( k = 3 \), and those for \( i = 11^\circ \) and \( \beta = 62^\circ \) with \( k = 3, 4 \) are qualitatively the same.

Table I. Characteristic quantities of solutions for HD 215441 with \( k = 4, i = 14.5^\circ \) and \( \beta = 52.5^\circ \)

<table>
<thead>
<tr>
<th>( H_P )</th>
<th>( \Delta H_P )</th>
<th>( M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>-40</td>
<td>0.12</td>
</tr>
<tr>
<td>44</td>
<td>-30</td>
<td>0.31</td>
</tr>
<tr>
<td>45</td>
<td>-20</td>
<td>0.50</td>
</tr>
<tr>
<td>45</td>
<td>-10</td>
<td>0.70</td>
</tr>
<tr>
<td>45.5</td>
<td>-5</td>
<td>0.76</td>
</tr>
</tbody>
</table>

The solutions for HD 215441 are characterized by a predominant dipole-like surface structure of the poloidal component with values of \( H_1 \) in the range 40 kGs to 50 kGs and values of \( \Delta H_P \) in the range -40 kGs to -5 kGs. There is also a clear relationship between \( |\Delta H_P| \) and the relative importance of the toroidal component. Solutions with large values of \( |\Delta H_P| \), in which there is a relatively important
quadrupole-like contribution to the poloidal component, have small toroidal components, while solutions with small values of $\Delta H_p$, in which the poloidal component is almost purely dipole-like, have strong toroidal components. The values of $\sigma_{He}$ and $\sigma_{HS}$ are for HD 215441

$$\sigma_{He} = 1.43 \text{ kGS}, \quad \sigma_{HS} = 0.6 \text{ kGS}. \quad (9)$$

The theoretical curves are in good approximation of the observations (see Fig. 1).

![Graph showing $H_e$ and $H_S$ variations for HD 215441](image)

_Fig. 1. $H_e$ and $H_S$ variations for HD 215441_

We have obtained solutions for 53 Cam for three sets of the angles $i$ and $\beta$: namely $i = 65^\circ$, $\beta = 100^\circ$; $i = 75^\circ6$, $\beta = 139^\circ$; $i = 75^\circ6$, $\beta = 102^\circ$. Unlike HD 215441, the solutions for 53 Cam are all characterized by poloidal components that in addition to a dipole-like contribution, have a strong quadrupole-like contribution, and by relatively strong toroidal components. The range in $|\Delta H_p|$ values seems to be much smaller than for HD 215441. The values of $\sigma_{He}$ and $\sigma_{HS}$ for 53 Cam are:

$$\sigma_{He} = 0.36 \text{ kGS}, \quad \sigma_{HS} = 0.66 \text{ kGS}.$$ 

The theoretical curves are good approximations of the observations (see Fig. 2).
Fig. 2. $H_e$ and $H_s$ variations for 53 Cam

A more detailed description of this investigation is being prepared for publication.

References