EFFECTS OF PARTICLE DRIFT ON THE TRANSPORT OF COSMIC RAYS. IV. MORE REALISTIC DIFFUSION COEFFICIENTS

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ABSTRACT

New results from numerical simulations of cosmic-ray modulation by the solar wind are presented. It is argued that the scattering mean free path should be larger than the particle gyroradius in the average magnetic field. Since this constraint was violated in our previous paper, we discuss here simulations which incorporate the larger diffusion coefficients. We find that these simulations exhibit a greater influence of diffusion, as anticipated, and the difference between drift and no-drift solutions is not so great as before. Nonetheless, we still find profound effects of the drifts. The drifts still determine the origin of the bulk of the cosmic rays seen at any given time in the inner solar system. Thus, during the 1975 solar minimum, positively charged cosmic rays seen in the inner solar system came primarily from the outer boundary near the heliospheric poles, and negative particles came from the equatorial regions of the boundary. The situation reverses during alternate solar cycles and with the sign of the particle charge. The calculated energy spectra agree reasonably well with observations and are insensitive to the magnitude of the diffusion coefficient.

Subject headings: cosmic rays: general — particle acceleration — Sun: solar wind

I. INTRODUCTION

Previous work in this series of papers has demonstrated the importance of including drift effects in models of the transport of cosmic rays in the heliosphere. The third paper in the series (Jokipii and Kopriva 1979, hereafter Paper III) presented the first detailed numerical simulations of solar modulation of galactic cosmic rays, based on a realistic heliospheric model. These calculations verified the importance of drifts and revealed the existence of a new type of modulation based on the balance between drift and energy loss, in which diffusion plays a negligible role. It was regarded as encouraging that the computed energy spectra and other features of the model were close to those observed, although no claim was made that our model parameters were realistic.

It is the purpose of the present paper to present the results of further simulations for more realistic diffusion coefficients. The coefficients $k_\parallel$ and $k_\perp$ used in Paper III were assumed to be independent of heliospheric radius and latitude. Jones (1980) pointed out that this has the undesirable feature that the cyclotron radius becomes larger than the scattering mean free path in the outer solar system and near the heliographic poles. One expects that in actuality, for reasonably fluctuating magnetic fields, the mean free path would be somewhat larger than the cyclotron radius in the average magnetic field. The opposite would require, at a minimum, magnetic field intensity fluctuations larger than the average field on the scale of a cyclotron radius, which are not observed. Note that calculations based on quasi-linear theory, of necessity, satisfy the above constraint (e.g., Morfill and Völk 1979).

Unfortunately, there is no consensus as to what a "reasonable" diffusion coefficient in the heliosphere would be. Therefore, we have computed the modulation for a variety of diffusion coefficients, for which the scattering mean free path is greater than the cyclotron radius. We conclude that many of our earlier qualitative conclusions also hold for the models considered here, although the quantitative results differ somewhat. In particular, we find that the "small diffusion coefficient" approximation is not as accurate an approximation as in some previously computed models.

II. PARAMETERS

The model, computational procedure, and program used in the present calculations are exactly the same as those used in Paper III, except for the parameters used. The interested reader is referred to Paper III for a summary. We have used a spherical heliosphere boundary at a heliocentric radius of 25 or 50 AU instead of the 10 AU used in Paper III to represent more accurately the actual situation. Of course, the actual radius of the boundary is known only to be greater than...
about 20 AU, and the boundary may well be nonspherical. In view of our present lack of evidence concerning the shape and distance of the boundary, the choice of a simple spherical boundary at 25 to 50 AU seems a plausible choice. We also describe briefly results for a 10 AU boundary, in order to provide an easier comparison with Paper III. The energy spectrum at the outer boundary, as in Paper III, was chosen to correspond to a density which is an inverse power law in total energy with exponent $-2.6$, as in Paper III. Also as in Paper III, we chose an absorbing inner boundary condition at 0.01 AU.

With the inner and outer boundary conditions fixed, it remains to choose the transport parameters inside the heliosphere. Again, to keep the parametrization at a manageable level, the present results are for a constant solar wind velocity $V_w$ equal to 400 km s$^{-1}$. There is some evidence that the velocity is larger at high heliographic latitudes, but the precise functional dependence is not known. The main conclusions of the present paper have been determined to be insensitive to moderate variations in the wind velocity with latitude (factor of $\sim 2$ from equator to pole). The interplanetary magnetic field is assumed to be an Archimedean spiral with a magnitude of 5 $\gamma$ at 1 AU in the solar equatorial plane.

Finally we specify the form of the diffusion coefficients used in the simulations. The calculations discussed here were carried out for two general types of diffusion coefficients. We emphasize again the absence of a generally accepted spatial or energy dependence of the diffusion coefficient. Hence, the first form we use corresponds to the simple assumption that the parallel mean free path was somewhat larger than the cyclotron radius at the orbit of Earth, and that its spatial dependence at any given kinetic energy was proportional to the gyroradius elsewhere. This form clearly avoids the mean free path problem discussed in the Introduction. The value and energy dependence at the orbit of Earth was taken to be essentially the same as that used in Paper III. Hence the functional form of the parallel diffusion coefficient is given by

$$\kappa_p = \kappa_0 P^{1/2} \beta \left[ B(E)/B(r) \right] \text{cm}^2 \text{s}^{-1},$$

$$\kappa_0 = 1.5 \times 10^{22},$$

(1)

for $r > 1$ AU and independent of $r$ (but otherwise varying with theta as $1/B$) for $r < 1$ AU. This latter condition is imposed to avoid a numerical instability which occurs for small $\kappa_p$. In the above $B(r)$ is the magnetic field intensity as a function of position in the heliosphere, and $B(E)$ is the intensity at the orbit of Earth. Also, $P$ is the rigidity in units of GV, and $\beta$ is the ratio of the particle speed to the speed of light. The value of the perpendicular diffusion coefficient is given by

$$\kappa_\perp = 0.05 \kappa_p,$$

with some results presented for $\kappa_\perp = 0.1 \kappa_p$. These values are similar to those used in Paper III at the orbit of Earth, but they are some two to three orders of magnitude larger at the outer boundary.

A second form of $\kappa_j$ was used in some of the simulations. Morfill and Volk (1979) carried out detailed calculations of the spatial dependence of using results from a WKB analysis of fluctuations and quasi-linear theory. We have evaluated polynomial approximations to their curves and used them in some of our simulations. It is of interest that their calculated values of the parallel mean free path are of the same general order as those obtained from the simple $1/B(r)$ assumption discussed above. In fact, the simulations carried out using their coefficients are remarkably similar to those obtained using the simpler approximation.

We do not regard the above choices for kappa as definitive in any way but as probably somewhat better approximations than those used previously in Paper III. The purpose of the present paper is to explore briefly the consequences of these large coefficients in the outer solar system for the modulation of galactic cosmic rays.

### III. Computational Results

Figures 1-5 present the results of a typical set of simulations for which $\kappa_j$ is given by equation (1), $\kappa_0 = 1.5 \times 10^{22}$ and $\kappa_\perp / \kappa_p = 0.05$. The outer boundary was set at 50 AU. In all cases care was taken to assure convergence and to avoid instabilities. Varying the mesh size and momentum step produced no sensible changes in the solutions presented here. The figure legends give a description of the parameters corresponding to each figure.

We note the following generalizations, based on these and other simulations:

1. We find, as in Paper III, that the energy spectrum at the orbit of Earth is quite insensitive to the diffusion coefficient, wind velocity, and boundary radius. Varying $\kappa_0$ in equation (1) over the range $0.5 \times 10^{22} - 1.5 \times 10^{22}$ produced a change in the spectrum which is less than the spread in the observed spectra.

2. For the case $qA > 0$, we still see evidence for the flat "plateau" of small radial gradient, although it is not nearly as pronounced as for lower values of $\kappa$.

3. The radial gradients for $qA < 0$ are substantially greater than for $qA > 0$, as before.

4. The greatest difference from our previous results are for the variation of the intensity with heliographic latitude. The present simulations have a much smaller latitude gradient for $qA < 0$ than was found in Paper III. Although the intensity at the poles is still less than at the equator at 1 AU, the difference is quite small and may
Fig. 1.—The solid line is the energy spectrum calculated in the solar equatorial plane at a heliocentric radius of 1 AU for $q_A < 0$. The parameters used are $\kappa_1 = 1.5 \times 10^{-2} \mu_0/\beta$ at the orbit of Earth, $\kappa_1 = 0.05 \mu_1, V_e = 4 \times 10^7 \text{cm} \text{s}^{-1}$ and a boundary distance of 50 AU. The dashed line is the assumed galactic spectrum at the outer boundary. The data points are for the period around the 1965 solar minimum as compiled by Meyer (1969).

Fig. 2.—As in Fig. 1 for $q_A > 0$. The data points are 1975 observations of Garcia-Munoz, Mason, and Simpson (1977).

Fig. 3.—The calculated intensity of 1.7 GeV protons as a function of heliocentric radius for polar angles of 0° to 90° at 10° intervals normalized to 1 at the outer boundary. Parameters as in Fig. 1; $q_A < 0$.

Fig. 4.—As in Fig. 3 for $q_A > 0$. 

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even change sign as parameters are changed. However, there is still a large difference in latitude gradient between the $qA<0$ and $qA>0$ simulations for all the curves we have run.

5. We have also carried out simulations using the diffusion coefficients calculated by Morfill and Volk (1979). They differ little from those presented here.

6. To provide a basis for comparison with Paper III, we discuss briefly simulations carried out with a boundary at 10 AU and $K_r = 0.5 \times 10^{22}$. These correspond to Figure 2 of Paper III, except for the spatial variation of $\kappa_{ij}$. We find that the energy spectra for the drift cases are surprisingly close (within a factor of 2) to those in Paper III. The case $V_c = 0$, however, now is only a factor of 5–10 below the drift cases at an energy of 0.1 GeV. The radial intensity plots do not show so pronounced a plateau, although it is still quite evident for $qA > 0$.

Finally, we wish to emphasize the close similarity between our calculated energy spectra and those observed near the last solar minima for a broad range of parameters. The best fit we have for protons is shown in Figures 6 and 7 and corresponds to changing $\kappa_0$ to $0.5 \times 10^{22}$. These results on the energy spectrum are expected to be relatively insensitive to the assumed energy spectra at the boundary for reasons discussed below.

IV. LATITUDE ORIGIN OF MODULATED PARTICLES IN THE INNER HELIOSPHERE

In Paper III, we concluded that drift was a dominant process bringing galactic cosmic rays into the inner heliosphere. This would argue that for $qA > 0$, most particles seen at Earth come in from the boundary near the heliographic poles and for $qA < 0$ they come in from the equatorial regions. It is of interest to determine whether this is the case even for the large diffusion coefficients used here, where diffusion is more competitive with drift.

To check this, we carried out simulations in which the boundary condition was nonzero only in a region in latitude centered at a chosen latitude $\theta_0$. In the simulations reported here, the distribution is a Gaussian with a...
half-width of 10°. Figures 8 and 9 display some typical results for the same sets of parameters as used in Figures 1–5, and $\theta_0 = 0$ (polar injection). In each case the intensity is normalized to the value at the boundary, at the maximum of the Gaussian.

It is quite clear from the figures that even in these cases where $k_1$ is quite large in the outer heliosphere, the drifts are very effective in transporting the particles. For $qA > 0$ the particles are effectively drawn into the inner heliosphere by the drifts, whereas for $qA < 0$ the particles are quite effectively excluded. Note that the gradient in the equatorial plane is positive in the inner solar system and becomes negative at large radii. The position of the maximum can be varied in or out by changing the drifts or the radius of the outer boundary. The situation for equatorial injection is similar to polar injection with the equator and pole interchanged.

We may conclude that, in this model, over a very broad range of parameters, the drifts determine the origin of the bulk of the particles seen in the inner heliosphere. This conclusion is important in interpreting various phenomena observed in the heliosphere and has been used recently in a new theory of the anomalous component (Pesses, Jokipii, and Eichler 1981).

V. ENERGY CHANGES

As a final modulation parameter to be examined, we consider the energy change experienced by the particles as they propagate into the inner heliosphere. In Paper III we were able to calculate the energy loss directly in the small diffusion coefficient approximation and obtain 230 MeV for protons. However, it is clear that this approximation is not valid for the more realistic parameters discussed here.

In order to study the energy loss in these simulations, we introduced an energy dependence at the outer boundary which is the superposition of a series of Gaussians in momentum. The upper envelope of the maxima is the same boundary spectrum as was used in the previous simulations. This procedure gives identifiable features in the energy spectrum which may then be followed into the modulation region. In particular, the new energy loss may be estimated by determining the shift of a feature in energy. Sample spectra are given in Figures 10 and 11 for $qA > 0$, at 1 AU for an observer at the pole and at the equator. The energy loss is substantially less at the pole than at the equator. For the case $qA < 0$, we find a similar value for the energy loss at the equator, but the polar energy loss is approximately the same as the equatorial value in this case. These numbers are again not very sensitive to the parameters.

We conclude that in these cases, where diffusion is much more important than in Paper III, the mean energy loss of particles observed at the orbit of Earth is still 200–250 MeV. Clearly, the spectrum at Earth will be little influenced by particles having energies less than 200–250 MeV in the Galaxy.
VI. SUMMARY AND CONCLUSIONS

We have presented the results of numerical simulations of the modulation of galactic cosmic rays, including drift. The simulations presented here are for more realistic values of the diffusion coefficient and for more varied boundary conditions than those discussed in Paper III.

The general conclusions of Paper III still apply, although the effects of diffusion are clearly more important.

These conclusions should apply over a broad range of parameters if the effective drift velocity is approximately that given by a straightforward application of the standard expression (Jokipii, Levy, and Hubbard 1977). Variation of the drift velocity by a factor of ~2 should not significantly change our conclusions. However, we have no direct knowledge of the magnetic field structure over most of the heliosphere, so the present conclusions must be regarded as illustrative of one quite plausible field configuration.

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