THERMAL BIFURCATION IN THE SOLAR OUTER ATMOSPHERE

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ABSTRACT

Two distinct plasma thermal states are possible in the solar outer atmosphere, owing to the bifurcated character of the low-temperature cooling function at small optical depths. In radiative equilibrium, the plasma is strongly cooled to temperatures well below 4000 K by surface emission in the $v_f = 1$ fundamental vibration-rotation bands of carbon monoxide. However, when significant mechanical energy deposition is present in addition to the radiative heating component, the only effective cooling channel available to stabilize the plasma is optically thin emission in the recombination continuum of $H^-$. As a result, thermal equilibrium in a mechanically heated atmospheric zone can be attained only for temperatures above $T_{\text{crit}} \approx 4900$ K because $H^-$ is itself a net radiative heating agent for temperatures cooler than $T_{\text{crit}}$. Thermal bifurcation effects in the solar outer atmosphere are encouraged by the likelihood that mechanical energy deposition is significantly enhanced in small-scale, discrete structures, namely, magnetic flux tubes.

Subject headings: Sun: atmosphere

I. INTRODUCTION

A longstanding problem posed by empirical studies of the solar photosphere and chromosphere is the troubling disagreement among thermal structure models derived using different spectral diagnostics that are thought to be formed at similar levels of the atmosphere. For example, the inner damping wings of the Ca II λ3934 (K) and Mg II λ2796 (k) resonance lines imply that the temperature minimum region at the photosphere-chromosphere interface is comparatively hot ($T \approx 4500$ K) (Ayres and Linsky 1976), while the infrared overtone and fundamental vibration-rotation band systems of carbon monoxide suggest the presence of very cool material ($T < 4000$ K) at comparable or somewhat higher levels of the photosphere (Ayres and Testerman 1980).

A straightforward empirical resolution of the diagnostic dilemma is to suppose that the solar outer atmosphere is composed of two (or more) thermally distinct zones. On the one hand, the Ca II and Mg II emission cores and damping wings would be formed preferentially in the hot zones because the ultraviolet thermal emissivities weight exponentially toward hotter temperatures. On the other hand, the carbon monoxide absorption spectrum would be formed preferentially in the cool structures because the molecular dissociative equilibrium favors cooler temperatures.

In this paper, I propose a natural explanation why the solar outer atmosphere might exhibit two distinct thermal states, one hot, the other cool. The scenario is based on the bifurcated character of the low-temperature plasma cooling function at small optical depths, and the notion that the mechanical heating responsible for the chromospheric temperature inversion is significantly enhanced in small-scale structures—magnetic flux tubes—while the energy balance of the surrounding atmosphere is governed largely by radiative equilibrium.

II. OUTER ATMOSPHERE ENERGY BALANCE

The plasma energy balance in the outer layers of the solar photosphere is controlled by a variety of competing heating and cooling processes. The most important sources of heating are (1) absorption of background photospheric radiation fields by local opacity sources, i.e., radiative heating; and (2) deposition of energy by the dissipation of waves or other forms of material disturbances, i.e., mechanical heating. The single most important cooling process is radiative emission by lines and continua: conduction and convention are ineffective at the temperatures and densities of the outer photosphere and low chromosphere. At high temperatures in the middle chromosphere ($T > 6000$ K), the radiative cooling channels are few in number and easily identified. The most important coolants are the Mg II and Ca II resonance lines, the subordinate infrared triplet of Ca II, and the hydrogen Balmer lines and continua (e.g., Athay 1976; Linsky and Ayres 1978; Vernazza, Avrett, and Loeser 1980). At cooler temperatures ($T < 5000$ K), it has usually been presumed that the dominant radiative cooling process is optically thin $H^-$ recombination with a small contribution from effectively thin cores of strong neutral and ionic lines (e.g., Athay 1976). However, it is straightforward to...
show that at temperatures below $T_{\text{crit}} \approx 4900$ K in the outer photosphere, H$^-$ is in fact a rather strong net radiative heating agent (Ayres 1980, and references therein to previous work). The H$^-$ heating results from an imbalance between the energy absorbed from the background photospheric radiation field, which when expressed per H$^-$ ion is independent of the local temperature, and the energy returned to the radiation field by photorecombinations, which decreases as the fourth power of the local temperature.

The fact that H$^-$ is a strong heating agent for temperatures below $T_{\text{crit}} \approx 4900$ K already poses a dilemma because conventional semiempirical models of the outer photosphere (Avrett 1977) require the presence of material much cooler than $T_{\text{crit}}$, and chromospheric formation theories require substantial mechanical heating of those layers as well (Ulmschneider 1974). An important question, then, is: if H$^-$ does not cool the solar outer photosphere, what does?

Elaborate numerical simulations of the radiative energy balance of the outer photosphere (Kurucz 1974) have implicated atomic line blanketing as the major source of plasma cooling in the higher layers. However, the conventional atomic line-blanketed radiative equilibrium (RE) simulations in local thermodynamic equilibrium (LTE) are misleading in the sense that the "LTE" assumption tends to promote maximum cooling by each of the lines considered. In the actual solar situation, many of the important atomic line cores are driven far from LTE by the low collision rates in the tenuous layers of the outer atmosphere, and the net radiative cooling rate may be orders of magnitude reduced from the LTE case. That behavior is illustrated straightforwardly as follows.

The energy balance of a strong resonance line (two-level atom approximation) is (e.g., Athay 1976)

$$ e = 4\pi \kappa (S_{\lambda_0} - \bar{J}) = \frac{4\pi^2 e^2 \lambda^2}{mc^2} f_{n_l} e(B_{\lambda_0} - \bar{J}) \text{ergs cm}^{-3} \text{s}^{-1}. \quad (1) $$

Here $S_{\lambda_0}$ is the line source function (which is assumed to be constant over the line profile and equal to the value at line center $\lambda_0$), $B_{\lambda_0}$ is the Planck function, $\bar{J}$ is the mean intensity $J_\lambda$ averaged over the absorption profile, $f$ is the oscillator strength, $n_l$ is the lower level population, and $e = C_{ul}/(A_{ul} + C_{ul})$ is the collisional destruction probability. When $e$ is positive, the line is a net coolant. When $e$ is negative, the line is a net heating source.

The "LTE" assumption in the conventional line-blanketed radiative equilibrium simulations fixes the collisional epsilon at unity. This is equivalent to forcing the atomic levels to be populated according to a Boltzmann distribution at the local temperature. (Note, incidentally, that the "LTE" assumption used in the LTE-RE models is not strict LTE, otherwise the mean intensity $\bar{J}$ would identically equal the local Planck function $B$, and the net radiative cooling by the line would be zero. This, of course, is the essence of LTE which requires detailed balance among plasma thermodynamic processes, in a local sense.) In practice, however, the density dependent $e$ is generally very small ($\approx 10^{-2}$) for strong, permitted transitions in the low-pressure layers of the outer photosphere. Consequently, the "LTE"-RE assumption can potentially overestimate the actual atomic line core cooling by as much as several orders of magnitude. Therefore, one expects that the LTE-RE simulations overestimate the temperature drop from the gray H$^-$ case that would be produced by a correct treatment of the atomic line blanketing in statistical equilibrium. Nevertheless, the superficial similarity between the temperature-pressure stratifications predicted by the LTE-RE calculations and those inferred by empirical modelers strongly suggests the presence of an additional cooling agent that is not incorporated in the conventional treatments but which strongly mimics the surface cooling produced by LTE atomic line blanketing. I propose that the "missing" cooling agent is molecular line blanketing, specifically the 5 $\mu$m fundamental vibration-rotation bands of carbon monoxide. In particular, CO surface cooling is well known to compete strongly with LTE atomic line blanketing in stars much cooler than the Sun (Johnson 1973).

a) CO Cooling

Carbon monoxide is an obvious choice for a potent plasma coolant in the solar outer atmosphere. First, CO is extremely abundant where conditions favor its formation: the CO concentration near the $T_{\text{min}}$ of the VAL model (Vernazza, Avrett and Loeser 1973, 1976) is 10 times that of Fe$^+$ (the dominant ionization stage of iron in the photosphere), 100 times that of Ca$^+$, and 10$^5$ times that of neutral calcium. Secondly, the solar CO lines are formed very close to LTE (Tsuji 1964; Thompson 1973; Hinkle and Lambert 1975). By their nature, the vibration-rotation transitions have exceedingly small radiative transition probabilities, while the V-R collisional cross sections are appreciable, and the transitions can be induced by collisions with neutral hydrogen and helium atoms. (Atomic transitions typically can be excited only by electron collisions, and electrons are outnumbered by neutral hydrogen atoms near $T_{\text{min}}$ by a factor of 10$^4$.) As a result of the small Einstein $A$ values, but large collision rates, the effective epsilons for the CO transitions are very nearly unity. Finally, the CO vibration-rotation bands contain an enormous number of strong, largely nonoverlapping, lines.
The major atomic contribution to the plasma cooling therefore must be the resonance lines of minority abundance ions of the abundant ionization stages, namely, Mg i \lambda 2852 and Ca i \lambda 4227 for example. (Note that X = 0.01 for typical plasma parameters, T=4000 K, n_e = 10^14 cm^{-3}, and \epsilon = 10^3. If one assumes LTE ionization equilibrium, one finds that the ratio is \sim 0.02 for Mg i \lambda 2852 and 3 \times 10^{-4} for Ca i \lambda 4227. Since these particular features are among the strongest in the spectrum, it is clear that CO must be the dominant cooling agent in the high photosphere, providing that the equilibrium temperature is low enough that the CO is not appreciably dissociated. Note that in LTE the atomic/CO cooling ratios would be a factor of \epsilon^{-1} = 10^3 larger. It is not surprising, then, that CO would not be recognized as an important plasma coolant compared with LTE atomic line blanketing, at least in solar-temperature RE simulations.

b) The Low Temperature Cooling Function

One can estimate straightforwardly the relative importance of CO and atomic line cooling in the outer layers of the photosphere where the CO bands are optically thin. Near T=4000 K, the LTE CO cooling is proportional to (Appendix A)

\[ e_{CO} \sim \lambda_0^2 f_{CO} A C B_\lambda(4000 \text{ K}), \]  

where \( f_{CO} \approx 2 \times 10^{-5} \) is the mean \( \Delta V = 1 \) vibration-rotation oscillator strength, \( A_C \approx 5 \times 10^{-4} \) is the carbon abundance (relative to hydrogen), and \( \lambda_0 \approx 4.7 \mu \text{m} \) is the mean wavelength of the CO fundamental bands.

The corresponding atomic line cooling is

\[ e_{i} \sim \lambda_0^2 f_{e,i} \left[ \frac{n_i}{n_{el}} \right] e B_\lambda(4000 \text{ K}), \]  

where the quantity in brackets is the fractional population of the lower level of the line relative to the total concentration of that element in all ionization stages. (Note that \( \lambda_0 \) is typically \leq 0.5 \mu m here.

In the high photosphere, near the location of \( T_{\text{min}} \) in conventional single-component models, the strong resonance lines of the abundant ionization stages, namely, Mg ii H and k and Ca ii H and K, are effectively thick (i.e., \( J \approx B \)) and, consequently, are inefficient coolants. The major atomic contribution to the plasma cooling therefore must be the resonance lines of minority species, Mg i \lambda 2852 and Ca i \lambda 4227 for example.

The ratio of the atomic line cooling (eq. [3]) to the CO cooling (eq. [2]) can be evaluated straightforwardly for typical plasma parameters, \( T = 4000 \text{ K}, n_e = 10^{14} \text{ cm}^{-3}, n_H = 10^{15} \text{ cm}^{-3}, \) and \( \epsilon = 10^3. \) If one assumes LTE ionization equilibrium, one finds that the ratio is \sim 0.02 for Mg i \lambda 2852 and 3 \times 10^{-4} for Ca i \lambda 4227. Since these particular features are among the strongest in the spectrum, it is clear that CO must be the dominant cooling agent in the high photosphere, providing that the equilibrium temperature is low enough that the CO is not appreciably dissociated. Note that in LTE the atomic/CO cooling ratios would be a factor of \epsilon^{-1} = 10^3 larger. It is not surprising, then, that CO would not be recognized as an important plasma coolant compared with LTE atomic line blanketing, at least in solar-temperature RE simulations.

c) Implications for the Plasma Thermal State

In short, the thermal structure of the outer photosphere likely will be dictated by a balance between the strong surface cooling by optically thin CO and the radiative heating by \( H^- \).
The bifurcated character of the low temperature cooling function leads me to conclude that the solar outer atmosphere is itself thermally bifurcated: it is "cold" \((T \lesssim 4000 \text{ K})\) where mechanical energy deposition is weak and radiation transport dominates the local energy balance, and it is "hot" \((T \gtrsim 5000 \text{ K})\) where the mechanical energy deposition is substantial.

\textit{d) The Importance of Small-Scale Structures}

The possibility of a thermally bifurcated atmosphere is encouraged by the likelihood that mechanical energy transport, and deposition, in the solar outer atmosphere is significantly enhanced in small-scale structures: the so-called magnetic flux tubes (see Chapman 1980; Spruit 1980). The tubes are few hundred kilometer diameter concentrations of magnetic flux that bloom outward with increasing height above the optical continuum forming layers. Chromospheric brightness in conventional diagnostics, such as the Ca \(\Pi\) \(K\) and Mg \(\Pi\) \(k\) emission cores, is well correlated with the mean surface magnetic field strength (Skumanich, Smythe, and Frazier 1975), which in turn is related to the fractional area covered by the flux tubes (Chapman 1980). Empirically, then, it appears that the presence of chromospheric material is intimately related to the presence of magnetic field concentrations. Possible reasons why the flux tubes might be sites of amplified chromospheric heating are summarized by Stein and Leibacher (1980).

1 The \(H^-\) recombination cooling above \(T_{\text{crit}}\) is reinforced by the enhancement of the electron density owing to the partial ionization of hydrogen.

If the mechanical energy deposition is significantly enhanced within flux tube interiors, the entrained plasma will be considerably hotter—essentially chromospheric—than the surrounding atmosphere, owing to the cooling instability described above. Consequently, the internal gas pressure scale height will be larger than that of the external photosphere, which is comparatively cold owing to the \(CO\) cooling mechanism that operates effectively when the mechanical heating is weak (and when the \(CO\) is optically thin). Because the pressure scale height is larger inside the hot flux tubes, horizontal pressure equilibrium will force the tube envelopes to expand rapidly with increasing height above \(\tau_{5000} \approx 10^{-4}\), where the \(CO\) surface cooling begins to play an important role. The tube fields likely fill much of the available space at the base of the corona, although the brightness enhancements associated with mechanical energy deposition in the tube outer atmospheres may be spatially concentrated toward the individual tube centers and consequently have a smaller effective filling factor than the magnetic field itself (e.g., Chapman 1980). The atmospheric zones controlled by the \(CO\) cooling mechanism would appear as cold fingers of plasma protruding into the hotter but more tenuous chromospheric envelopes of the merged flux tubes.

\textbf{III. DISCUSSION}

The implications of the thermal bifurcation scenario, if it is correct, are several.

First, mean atmospheric models, such as the VAL, that are constructed to match spatially-averaged diagnostics are likely to be quite unreliable in the outer photospheric layers, where the potential bifurcation effects are large. For example, the structure of a mean model that incorporates a chromospheric temperature inversion near \(\tau_{5000} = 10^{-4}\) does not account for the potentially large volumes of cool plasma that may be present at those levels of the atmosphere. The cool plasma can affect the optical depth scales of medium-strength Fraunhofer lines, particularly those of minority species such as Fe \(\Pi\), Ti \(\Pi\), Ca \(\Pi\), etc., and thereby adversely influence estimates of chemical abundances and formation heights (the latter are used in velocity and magnetic field studies, for example) based on conventional mean models.

Second, simulations of center-to-limb behavior in strong and medium-strength lines using single-component mean models have questionable validity. For example, the cool fingers of plasma protruding into the chromospheric layers will have an amplified visibility in "cool" spectral features near the limb, since the line of sight can pass through several of these relatively opaque structures, whereas the optical or infrared opacity of the hot, low-density chromospheric material enveloping the fingers is very small. Consequently, the
limb behavior of molecular and low-excitation atomic lines will be controlled largely by the cool atmospheric zones, while the disk center behavior will be a more nearly genuine spatial average of the hot and cold components.

Third, the interpretation of chromospheric line profiles, such as the doubly reversed emission cores of the Ca II and Mg II resonance lines will be significantly modified since the shapes of the spatially averaged emission cores would be determined by a rather large dilution effect. Namely, only a fraction of the solar surface is covered by bright chromospheric emission (from the flux tubes), while the majority of the surface emits what is very likely a pure absorption profile of the K (or k) line core. Owing to the dilution effect, a conventional single-component interpretation of the Ca II or Mg II core emission strength and shape (e.g., Ayres and Linsky 1976) will likely produce a model that is not representative of either of the distinct atmospheric components.

A similar caveat applies to interpretations of continuum intensities formed near $T_{\text{min}}$. For example, the microwave free-free continuum of H$^+$ and neutral hydrogen invariably has been treated as a benchmark diagnostic for the "mean" structure of the $T_{\text{min}}$ region because the emissivity is purely thermal and the far-infrared Planck function averages linearly over temperature fluctuations (e.g., Avrett 1977). Unfortunately, the linearity of the temperature-intensity mapping of the microwave continuum is essentially irrelevant to the situation at hand, in which the two distinct atmospheric zones have very different temperature-pressure stratifications. In particular, at a fixed wavelength in the f-f continuum, one samples the atmospheric temperatures at comparable gas pressures in the hot and cold components. At the pressures corresponding to the location of $T_{\text{min}}$ in a homogeneous model such as the VAL, one is essentially averaging photospheric temperatures in the cool component ($T \lesssim 4000$ K) with chromospheric temperatures ($T \gtrsim 6000$ K) in the hot flux-tube envelopes. Consequently, even though the fractional area covered by chromospheric material may be small, the chromospheric emission will still contribute heavily to the spatially averaged spectrum. Again, a one-dimensional interpretation will produce a mean thermal model that is not characteristic of either of the major atmospheric constituents.

The largely illustrative arguments I have presented here must be tested by observations (with the Solar Maximum Mission, for example) and by detailed numerical simulations. Nevertheless, the notion of a thermally bistable and spatially bifurcated outer photosphere provides a ready way to reconcile the otherwise conflicting pictures of the solar atmosphere found in spectral thermometers such as the infrared bands of carbon monoxide and the ultraviolet resonance lines of Ca II and Mg II.

APPENDIX A

ANALYTICAL REPRESENTATION OF THE CO COOLING CURVE

The plasma cooling by a single CO fundamental ($\Delta V=1$) feature in LTE has the following form:

$$
\phi_{\nu J} = \frac{\pi e^2 \lambda^2}{mc} f_{\nu J} \frac{n_{\nu J}}{n_{e}n_{H}} 4\pi (B_{\lambda}(T) - \bar{J}).
$$

(A1)

Here $n_{\nu J}$ is the population of the lower vibration-rotation (V, J) level of the transition, and $f_{\nu J}$ is the corresponding oscillator strength.

Consider the layers of the atmosphere where the CO $\Delta V=1$ lines are optically thin, $\tau_{\text{9000}} \lesssim 10^{-4}$, and the temperature regime where the CO concentration is at maximum, namely, $T \lesssim 4000$ K. At these temperatures, the CO density is saturated in the sense that all of the available carbon atoms have associated into CO molecules (note that $A_{\text{C}} < A_{\text{O}}$; Ross and Aller 1976); hence,

$$
n_{\text{CO}} \approx A_{\text{C}} n_{\text{H}} \approx 5 \times 10^{-4} n_{\text{H}}.
$$

(A2)

Furthermore, the disk brightness temperatures of the strong CO features typically are less than or of order 4000 K (Ayres and Testerman 1980); hence, the mean intensity in the optically thin layers can be approximated as

$$
\bar{J} \approx \frac{1}{2} B_{\lambda}(4000 \text{ K}).
$$

(A3)

In addition, the electron density is determined by metal ionization and can be approximated as

$$
n_{e} \approx 10^{-4} n_{\text{H}}.
$$

(A4)
where the proportionality constant is the combined abundance of the easily ionized metals, Fe, Si, and Mg (Vernazza, Avrett, and Loeser 1976).

If one sums equation (A1) over all of the possible vibration-rotation levels in the ground electronic state, one obtains

\[ \phi_{\text{CO}} = \frac{\pi e^2}{mc} \frac{\lambda^2}{c} \langle f \rangle n_{\text{CO}} A \Delta \lambda(T) \times \left[ 1 - \frac{1}{2} \frac{B_s(4000)}{B_s(T)} \right], \tag{A5} \]

where the population-mean \( \Delta V = 1 \) oscillator strength is

\[ \langle f \rangle = \frac{\sum f_{VJ} n_{VJ}}{\sum n_{VJ}} \approx 2.2 \times 10^{-5} \hat{\tau}^{0.6}, \tag{A6} \]

based on the oscillator strength distribution given by Kirby-Docken and Liu (1978). Note that two radiative transitions—the \( P \) and \( R \) branches—are possible for each \((V, J \geq 1)\) level and that \( \sum_{VJ} n_{VJ} \equiv n_{\text{CO}} \) since very few of the molecules are in excited electronic states.

Taking \( \lambda \approx 4.7 \mu \text{m} \), approximating the infrared Planck function as a power law in \( \hat{T} = T/5000 \), and combining the several relations yields an analytical representation for the collective CO cooling curve,

\[ \phi_{\text{CO}} \approx 4.8 \times 10^{-27} \hat{T}^{3.2} (1 - 0.36 \hat{T}^{-1.5}) m^{-1} \text{ ergs cm}^{-3} \text{ s}^{-1}. \tag{A7} \]

Here the column mass density \( m \) (g cm\(^{-2}\)) was substituted for the hydrogen density \( n_{\text{H}} \) (cm\(^{-3}\)), assuming hydrostatic equilibrium and the perfect gas law,

\[ P = 1.1 n_{\text{H}} kT = gm, \]

or

\[ n_{\text{H}} = 3.6 \times 10^{16} m \hat{T}^{-1} \text{ cm}^{-3}, \tag{A8} \]

where \( P \) is the gas pressure, \( g \) is the solar surface gravity, and a 10% helium abundance (by number) has been assumed.

The analytical expression for the CO cooling should be accurate above \( \tau_{5000} = 10^{-4} \), where the majority of the \( \Delta V = 1 \) vibration-rotation lines are optically thin. However, the CO cooling will be reduced in the deeper layers by a factor related to the fraction of CO lines that are optically thick at those levels. Furthermore, the CO cooling at temperatures above 4000 K will be reduced by dissociation since the molecular concentration will be smaller than the “saturated” limit, \( n_{\text{CO}}^{\text{max}} = A_C n_{\text{H}} \).

### APPENDIX B

**ANALYTICAL REPRESENTATION OF H\(^-\) ENERGY BALANCE**

Like LTE carbon monoxide, H\(^-\) is a comparatively simple system that is amenable to analytical treatment, including departures from LTE.

The energy balance relation for H\(^-\) has the following form (e.g., Ayres 1980 and references to previous work therein): \( \phi \)

\[ \phi_{\text{H}} = \frac{n_{\text{H}}}{n_e n_{\text{H}}} (\delta T - b_{\text{H}} \delta T^0). \tag{B1} \]

Here \( b_{\text{H}} \) is the departure coefficient,

\[ b_{\text{H}} = \frac{R T + \bar{C}}{R T^0 + \bar{C}}, \tag{B2} \]
where \( R^i \) and \( R^{(0)} \) are radiative recombination and photodetachment rates, respectively, and

\[
C = C_r + \frac{C_H}{1 + C_H n_H / C_{3H} n_H}
\] (B3)

is a term that describes the collisional side of the joint \( H^{-} \)-\( H_2 \) statistical equilibrium (e.g., Vernazza, Avrett, and Loeser 1973). Using the VAL rate coefficients and \( n_e \approx 10^{-4} n_H \), equation (B3) reduces to

\[
C \approx 2 \times 10^{-9} n_H = 7.2 \times 10^7 T^{-1} \text{ s}^{-1}
\] (B4)

The radiative rates can be approximated as

\[
R^i \approx 4 \pi \bar{a} N(\hat{T}) \text{ s}^{-1},
\] (B5)

where \( \bar{a} \approx 2.4 \times 10^{-17} \text{ cm}^2 \) is the Planck mean photodetachment cross section, and

\[
\mathcal{P}(\hat{T}) \approx 6 \times 10^{21} \hat{T}^3 \text{ photons cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1},
\] (B6)

is the frequency-integrated photon distribution function (e.g., Allen 1973).

The remaining terms in equation (B1) are the energy emission and absorption rates, respectively, associated with recombinations and photodetachments. These can be expressed as

\[
E^i = 4 \pi \bar{a} B(\hat{T}),
\] (B7)

where

\[
\mathcal{P}(\hat{T}) \approx 1.1 \times 10^{10} \hat{T}^4 \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}
\] (B8)

is the frequency-integrated Planck function (Allen 1973).

Both \( R^{(0)} \) and \( E^{(0)} \) are fixed rates that depend on the background photospheric radiation field. To a good approximation,

\[
R^{(0)} \approx \frac{1}{2} R^i(T_{\text{eff}}),
\] (B9)

and

\[
E^{(0)} \approx \frac{1}{2} E^i(T_{\text{eff}}),
\]

where \( T_{\text{eff}} = 5770 \text{ K} \) is the solar effective temperature. (The \( \frac{1}{2} \) accounts for the fact that in the optically thin layers of the outer photosphere, only one hemisphere is illuminated.)

Combining these several relations yields

\[
\phi_{H} = 5.6 \times 10^{-27} \hat{T}^{0.2} \left[ 1 - 0.88 \hat{T}^{-1} \times \left( \frac{1.3 + 51 m \hat{T}^{-4}}{1 + 51 m \hat{T}^{-1}} \right) \right] \text{ ergs cm}^2 \text{ s}^{-1}.
\] (B10)

Note that owing to the \( n_e n_H \) normalization of the energy balance relation (cf. Cox and Tucker 1969), the density enters only in the term associated with departures from LTE. In this regard, the large associative detachment rate (e.g., VAL),

\[
H^{-} + H \leftrightarrow H_2 + e^{-},
\] (B11)

assures that NLTE effects will be small, except in the highest layers of the photosphere (i.e., \( m \leq 0.01 \text{ g cm}^{-2} \)).

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