CONVECTION AND MAGNETIC FIELDS IN STARS

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ABSTRACT

Recent observations have demonstrated the unity of the study of stellar and solar magnetic fields. Results from numerical experiments on magnetoconvection are presented and used to discuss the concentration of magnetic flux into isolated ropes in the turbulent convective zones of the Sun or other late-type stars. Arguments are given for siting the solar dynamo at the base of the convective zone. Magnetic buoyancy leads to the emergence of magnetic flux in active regions, but weaker flux ropes are shredded and dispersed throughout the convective zone. The observed maximum field strengths in late-type stars should be comparable with the field $\left(8\pi p\right)^{1/2}$ that balances the photospheric pressure.

Subject headings: convection — hydrodynamics — stars: magnetic — Sun: magnetic fields

I. INTRODUCTION

The Sun is unique in having a magnetic field that can be mapped in detail, but magnetic activity seems to be a standard feature of stars with deep convective envelopes. Fields of 2000 gauss or more have been found in two late-type main sequence stars (Robinson, Worden, and Harvey 1980). Wilson (1978) has detected chromospheric Ca ii emissions from many G, K, and M stars, some of which show cyclic variations of activity. Some, perhaps all, dMe stars flare, and the unexpectedly high X-ray luminosities of K and M stars (e.g., Vaiana 1979) suggest that they have strong magnetic fields. Although the Sun's field is comparatively feeble, all attempts to explain magnetic activity in stars are based on solar observations. Over the last decade the structure of large-scale solar magnetic fields has been clarified by measurements from space, while high resolution ground-based observations have revealed the interaction between small-scale magnetic features and cellular convection in the photosphere.

Theory also has made considerable progress (Parker 1979a). Since magnetic activity is associated with turbulent convection, it is important to study the interaction between magnetic fields and convection (for a review, see Weiss 1977). Nonlinear magnetoconvection has recently been investigated in several series of numerical experiments (Peckover and Weiss 1978; Galloway and Moore 1979; Weiss 1981a, b). The aim of this paper is to relate these idealized model calculations to the structure of magnetic fields at the surface and in the interior of the Sun and other late-type stars.

II. CONCENTRATION OF MAGNETIC FLUX

In § II we summarize the results obtained for simplified models of hydromagnetic convection. The structure of turbulent magnetic fields is then described in § III. Theory and observation are brought together in § IV, where we discuss the formation of isolated flux tubes in the interior of the Sun. Photospheric fields are covered in § V. Finally, we consider the origin of magnetic fields in the Sun and other stars with vigorous convective zones.

Our conclusions emphasize the need for observational programs, covering stars of differing mass, that can establish the correlation between magnetic activity and rotation or convective velocity; in addition, we need better estimates of the field strengths in starspots and the areas they occupy.
Fig. 1.—Expulsion of magnetic flux: the evolution of the magnetic field in a kinematic calculation with $R_m = 250$. Lines of force are shown at times $T/t_i = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,$ and $20$, where $t_i = T/8$. 
when $R_m = 250$, with boundary conditions such that $B_x = 0$ on $x, z = 0, L$, so that lines of force can move freely at the upper and lower boundaries. (For a similar calculation with different choices of velocity and boundary conditions, see Weiss 1966). As the eddy turns over, magnetic flux is quickly concentrated into sheets at the lateral boundaries, while the central field grows progressively more distorted until reconnection occurs and flux is expelled from the convective eddy. In the final state the field at the edges of the cell is amplified by a factor of order $R_m^{1/2}$ (e.g., Proctor and Weiss 1978). This amplification proceeds rapidly: After one turnover time $t_0 = L/U$, the field can be increased by a factor of up to 200 (Weiss 1966). On the other hand, flux expulsion takes much longer: The process is completed by $t \approx 5.5t_0$ when $R_m = 250$, but this time increases as $R_m^{-1/3}$ (Weiss 1966).

In a three-dimensional flow, the flux is concentrated into an isolated tube and the field is amplified by a factor of order $R_m$ (Galloway, Proctor, and Weiss 1978; Galloway and Moore 1979).

These strong fields exert forces which modify the pattern of convection, so it is necessary to consider the dynamical problem and solve the equations of motion. Linear theory has been extensively discussed (Chandrasekhar 1961; Danielson 1961). In the astrophysically interesting case, when the thermal diffusivity $\kappa \gg \eta$ and $B_0$ is sufficiently large, convection sets in as overstable oscillations. Heat transport by oscillatory convection is relatively inefficient so it is important to ascertain when steady convection can occur. The configuration is described by dimensionless parameters

$$R = \frac{g\beta d^4}{\kappa v} \quad \text{and} \quad Q = \frac{B_0^2 d^2}{4\pi \mu \nu} \quad (3)$$

(the Rayleigh and Chandrasekhar numbers), and by the ratios

$$\sigma = v/\kappa \quad \text{and} \quad \zeta = \eta/\kappa \quad (4)$$

of the diffusivities. Here $v$ is the viscous diffusivity, $d$ the layer depth, $\beta$ the superadiabatic temperature gradient, and $\sigma$ the coefficient of expansion. For $\zeta < 1$ but $Q \gg 1$, overstability sets in when $R = R^{(e)} \sim \pi^2 \sigma(\eta/Q)(\sigma + 1)$ and steady solutions bifurcate from the static conducting solution at $R = R^{(e)} \sim \pi^2 Q$.

Nonlinear calculations confirm the existence of subcritical steady convection in the range $R_{\text{min}} \leq R < R^{(e)}$. As $R$ is increased, steady convection first appears with finite amplitude at $R = R_{\text{min}}$. Figure 2 shows streamlines and lines of force for steady two-dimensional convection with periodic boundary conditions (cf. Weiss 1981a). Once again, the flux is confined to narrow sheets, allowing convection to take place in the field-free central region; within these sheets, the field is strong enough to exclude the motion. Similar results have been obtained for an axisymmetric configuration, where the flux is confined to a stagnant axial tube within which the field is nearly uniform (Galloway and Moore 1979).

Although linear theory predicts that convection should first appear in narrow cells, elongated in the direction of the field $B_0$, the nonlinear results indicate that steady convection occurs more readily in wider cells, which allow more space for convective motion between the concentrated fields. The numerical experiments suggest that $R_{\text{min}} \sim \pi^2 Q \ll R^{(e)}$, as conjectured by Danielson (1961). For a given Rayleigh number, steady convection occurs for $Q \leq Q_{\text{max}} \sim R/\zeta$; hence only oscillatory motion is possible when the field exceeds a critical value

$$P_{\text{max}} \sim (g\sigma \beta^2 d^2)^{1/2} \quad (5)$$

The solutions displayed in Figure 2 are symmetrical about the center of the cell. As $Q$ approaches $Q_{\text{max}}$ asymmetrical solutions are preferred. Figure 3 shows the final steady state for convection in a box whose width is 4 times its depth. The run was started from noisy initial conditions but settled down to give two cells (with the same sense of rotation) on either side of a stagnant central slab. In this solution the central region contains over 60% of the total flux, but with more extreme parameters 90% of the flux can be concentrated on one side of a convection cell (Weiss 1981b). Heat transport is apparently made more efficient by assembling as much magnetic flux as possible into a single isolated sheet or tube, so liberating the rest of the region for convection to take place. In a wider layer this provides a mechanism for amalgamating small flux tubes into a single concentration when the imposed field is relatively strong.
III. TURBULENT MAGNETIC FIELDS

If the average field \( B_0 \) is sufficiently weak, flux concentration is limited by diffusion. The transition from this kinematic regime to a dynamic regime occurs when the field is strong enough to exclude motion from the flux tubes. For two-dimensional convection this happens for a field \( B_m \) in the flux sheet such that

\[
B_m^2 \sim 4\pi \rho U^2 R_m^{-1/2} (v/\eta) \tag{6}
\]

(Peckover and Weiss 1978). This peak field therefore depends on the ratio of the viscous to the magnetic diffusivity (Busse 1975). In three dimensions, the transition occurs when

\[
B_m^2 \sim 4\pi \rho U^2 (v/\eta)/\ln R_m \tag{7}
\]

(Galloway, Proctor, and Weiss 1978), so that the field in a flux tube is stronger than that from equation (6). This field is reached when the flux through a cell reaches the critical value

\[
F_c = L^2 B_0 \sim (4\pi \rho \eta v/\ln R_m)^{1/2} L. \tag{8}
\]

It is difficult to analyze the stability of these nonlinear equilibria, but the following crude argument suggests that two-dimensional flux sheets should be unstable in the dynamic regime. Consider a sheet, of half-width \( \epsilon \) and with a vertical field \( B^* \). Suppose that, in the absence of this field, the existing density gradients would generate a flow with a velocity \( U_0 \) at the flux sheet. The magnetic field generates a counterflow with velocity \( U_1 \sim (B^* /4\pi \eta v) \) (Galloway, Proctor, and Weiss 1978), which is proportional to \( \epsilon \). In equilibrium, \( U_1 \sim U_0 \) and the flow is excluded from the flux sheet. Now suppose that the sheet suffers a slight varicose distortion with a wavelength (in the \( y \) direction) much greater than \( \epsilon \), while the field \( B^* \) remains unchanged. Where the sheet contracts, \( \epsilon \) decreases, and \( U_1 \) is reduced; hence the external velocity presses the flux inward. Where the sheet swells, \( U_1 \) increases and the external velocity is reduced. Thus the perturbation should grow until the sheet splits into isolated tubes. This simple argument suggests that, in a three-dimensional configuration where flux is concentrated into sheets at the boundaries of convection cells, these sheets will fragment into discrete ropes which will congregate preferentially at the corners.

So far, we have considered only steady, persistent patterns of motion. In a turbulent regime, individual convection cells only survive for a period comparable with the turnover time \( t_0 \) (this applies to the measured lifetimes of granules and supergranules in the Sun). This short lifetime does not prevent flux concentration at the vertices of cells, where the field can be amplified by a factor of 10 \(^4 \) in one turnover time, but it is clearly insufficient for flux to be expelled from cell interiors (Weiss 1966). Piddington (e.g., 1978) has argued, therefore, that turbulent convection should produce a tangled field unless the flux ropes are always isolated from the convecting fluid. This need not be so, for the topological relationship of the ropes to the convective eddies remains fixed. New convective cells form either by fission of preexisting cells (cf. Jones and Moore 1979), as in exploding granules (Mehltretter 1978), or by insinuating themselves between old cells at their common boundary, as suggested by Born (1974) for supergranules. In either case, flux at the boundary remains at the boundary. The detailed pattern of motion around a flux rope may change, but the rope remains in a region of converging flow. Within the cells, the field gets tangled until diffusion allows lines of force to reconnect.

The velocity structure within the turbulent convection zone of a star can only be derived from observations of motion at the surface of the Sun. At any instant, convection forms a fairly ordered, cellular pattern in the solar photosphere, though the individual cells are ephemeral and short-lived. We infer, therefore, that within a star the energy-carrying eddies have a similar structure with some horizontal scale that is locally preferred. Convection is driven thermally and, as the pattern changes, work done by pressure gradients must ultimately be dissipated by molecular viscosity. Since the Reynolds number is enormous, there must exist an energy cascade through an inertial subrange associated with small-scale turbulence that is roughly stationary and homogeneous. Formally, this suggests the use of a two-scale analysis. As a simple model, we separate the velocity field into an ordered motion on the scale of cellular convection and into small-scale turbulence whose effect is represented by an eddy diffusivity.
Magnetic fields are then advected by the ordered motion and diffuse owing to the small-scale motions. The latter bring opposing fields close enough together for resistive instabilities, or fast dynamical reconnection to occur, whether by laminar diffusion, or magnetic Reynolds number, $R_m$, is probably in the range $10^2 < R_m < 10^3$. If a diffuse field enters a turbulent region, therefore, flux will be confined to sheets or ropes within a few turnover times.

The preceding discussion applies even to weak fields, where amplification is purely kinematic. If, however, the flux through each cell exceeds $F_m$, motion will be excluded from the flux ropes and the dynamical effect of the field will also help to maintain the topological relationship between the ropes and ambient convection. Since the pattern of convection is highly degenerate, individual cells can be stabilized and persist for longer lifetimes if they are attached to dynamically active flux tubes. Pukas (Livingston and Orrall 1974) and moat cells (Harvey and Harvey 1973) provide examples of such long-lived supergranules in the Sun.

So far, we have considered only the development of the magnetic field from an initially diffuse state, as in Figure 1. If the field is created in isolated flux tubes, or if it enters the turbulent region in preconcentrated form, flux expulsion is no longer a problem. The above arguments show that the magnetic flux will remain in isolated tubes throughout the turbulent region, with a field strength determined, kinematically or dynamically, by the vigor of the motion. Moreover, any initial concentration of flux would gradually spread out, owing to turbulent diffusion, if it were not maintained in a discrete rope by the effects of convection.

Magnetic fields can be confined either to sheets, between cells, or to isolated tubes at corners where several cells meet. We have suggested that sheets should be unstable and fragment into tubes. Most of the flux, therefore, should be contained in ropes (cf. Galloway, Proctor, and Weiss 1978). As the pattern of convection changes, flux is transferred from corner to corner along the intervening surfaces. The numerical experiments described at the end of § II imply that, for fluxes significantly greater than $F_m$, separate tubes may be assembled into a few grand flux ropes, thereby facilitating heat transport. Although there is no direct experimental evidence for the amalgamation of flux tubes, there is a close analogy with the effect of turbulence on the distribution of vorticity in rotating fluids, which can be investigated in the laboratory. The experiments of McEwan (1976) show the formation of a few strong isolated vortices in the rotating system.

We believe, therefore, that turbulent magnetic fields in a highly conducting fluid are intermittent. The field strength depends on the energy of the motion, and on the magnetic diffusivity $\eta$, in the kinematic regime; in the dynamic regime, the magnitude of the amplified field is independent of $\eta$ (Weiss 1981a), though the current sheets become thinner as the electrical conductivity increases. This intermittent structure appears in two-dimensional numerical experiments (Orszag and Tang 1979). In three dimensions, magnetic flux tubes should form a flexible framework, running between convection cells, which changes as flux is transferred along surfaces from one tube to another.

IV. ORIGIN OF DISCRETE FLUX TUBES IN THE SUN

Magnetic flux emerges through the surface of the Sun in discrete tubes that may be associated in bundles (Zwaan 1978). X-ray observations from space, as well as ground-based magnetographic measurements, suggest that there are fundamental differences between the behavior of large-scale magnetic fields ($F \geq 10^{22} \text{ Mx}$) and smaller features ($F \leq 10^{19} \text{ Mx}$), though it is hard to make the distinction precise (Golub et al. 1981). The solar dynamo is apparently located near the base of the convective zone, where the large-scale fields originate. In this region, magnetic fields are predominantly toroidal and must vary with the solar cycle. The total (unsigned) flux at the base of the convective zone at sunspot maximum, $\Phi$, can be crudely estimated as follows. A large sunspot may contain $3 \times 10^{22} \text{ Mx}$, and the total flux emerging in a “complex of activity” (Bumba and Howard 1965; Svestka 1977) may be as much as $10^{23} \text{ Mx}$. This figure provides a lower bound for $\Phi$. However, magnetic flux emerges in active regions at an average rate of $5 \times 10^{21} \text{ Mx day}^{-1}$ (Golub et al. 1974), giving a total of $2 \times 10^{23} \text{ Mx}$ over the solar cycle, and even that is not an upper bound since there could be a reservoir of flux that stays below the surface. To deduce $\Phi$ more precisely, one must know how many times the same flux does duty as an active region. If, for instance, we suppose that each tube emerges 10 times as a bipolar region, at different longitudes, during the solar cycle, then we find that $\Phi \sim (2 \times 10^{25/20}) = 10^{24} \text{ Mx}$. The above arguments suggest that $\Phi$ lies between $10^{23} \text{ Mx}$ and $10^{25} \text{ Mx}$; we shall adopt $10^{24} \text{ Mx}$ as, on balance, the most likely value.

Apart from granulation and supergranulation, the structure of motion in the convection zone is largely unknown. Nevertheless all available evidence from theory and laboratory experiments suggests that in the lower half of the convection zone heat is transported primarily by eddies with a scale of at least 100,000 km, comparable to the depth of the whole zone. Whether motion associated with these giant cells extends right up to the photosphere remains unclear. In order to describe the interaction between magnetic fields and convection in the lower part of the convection zone, we assume the existence of giant cells, with parasitic eddies driven by shear instabilities, as described in § III. The turnover time for giant cells is comparable to the solar day; thus they are strongly influenced by rotation and should occur as cells elongated parallel to the Sun’s axis (cf. Busse 1977), so
that the orientation of the velocity relative to the magnetic field resembles that in the two-dimensional calculations.

In the sunspot zones, extending between latitudes of \( \pm 30^\circ \), the flux per unit length along a meridian is about \( 2 \times 10^{13} \text{ Mx cm}^{-1} \); this is sufficient to be in the dynamical regime. Is this flux confined to isolated ropes from which the motion is excluded? Such ropes must float upward owing to magnetic buoyancy (Parker 1979a), unless the local motion drags them down. Consider a longitude where the radial velocity is downward and suppose that the total flux fills a homogeneous layer extending between \( \pm 30^\circ \) in latitude. For a flux of \( 10^{24} \text{ Mx} \) and a field of \( 10^4 \text{ gauss} \), this layer must be 20,000 km thick; if the field were as low as 100 gauss, the flux from a single complex of activity would fill the whole convective zone. Alternatively, suppose that the field is confined to isolated tubes, each containing \( 10^{22} \text{ Mx} \) (corresponding to a typical active region); then the total flux \( \Phi = 10^{24} \text{ Mx} \) cannot be fitted into the lower part of the convective zone unless the filling factor is around \( 20\% \), which would severely hamper heat transport. (Interference with convection does, of course, become less serious if \( \Phi \) approaches its minimum value of \( 10^{23} \text{ Mx} \).)

From these considerations it seems most likely that the toroidal flux at sunspot maximum is contained in a shallow layer situated at the interface between the convective zone and the radiative zone below. The amount of flux in this layer varies with the solar cycle and the toroidal field reverses around sunspot minimum: These changes can produce fluctuations of order \( 0.1\% \) in solar luminosity (Spiegel and Weiss 1980). Generation of toroidal flux is part of the dynamo process that maintains the solar cycle; it is natural to suppose that the dynamo itself is also located in this magnetic layer. Then magnetic flux is generated with strong fields (around \( 10^4 \text{ gauss} \)) and there is no need to introduce weak (100 gauss) fields in the bulk of the convective zone at any stage of the solar cycle.

More work is needed in order to establish how such a layer is maintained. As we have seen, it may be unstable and develop corrugations; in addition, it is liable to instabilities driven by magnetic buoyancy (Parker 1979a) so that narrow loops of flux rise upward to the surface, guided by the pattern of convection. These loops can then emerge as active regions, which show the systematic pattern of behavior that defines the solar cycle. Provided that the flux is confined to a layer beneath the convective zone, buoyancy-driven instabilities may be partially inhibited by rotation (Acheson 1979a, b), so enabling the field to survive for an appreciable fraction of the solar cycle. If, on the other hand, the flux is within the convection zone, it will rise at least as fast as the local gas and so be lost within a month (Parker 1979a).

The smaller flux tubes that emerge as ephemeral active regions or X-ray bright points (Golub et al. 1977) show a different pattern of behavior, which suggests that they are formed higher in the convective zone (Golub et al. 1981). These features, which are prevalent at all latitudes, seem to correspond to the intermittent magnetic fields that should be expected in the convective zone. Larger flux tubes are shredded by turbulent eddies, forming smaller ropes which can be maintained by the convective motion. Thus the small-scale, turbulent magnetic fields are only weakly related to the dynamo that drives the solar cycle. Turbulent shredding proceeds most rapidly when flux ropes are too weak to withstand convection; conversely, an isolated flux tube can survive longer if it contains enough flux to exclude the motion. The critical flux can be estimated from equation (7) once the appropriate diffusivities are known. Within the flux rope there is still some motion, corresponding to overstable hydro-magnetic waves, and turbulence is more effective than laminar diffusion. The viscous and magnetic eddy diffusivities should be roughly equal, so equation (8) reduces to:

\[
F_c \sim (4\pi\eta)^{1/2} L \eta \ ,
\]

where \( \eta \) is the turbulent diffusivity, since the logarithmic factor is of order unity. The value of \( \eta \) can be estimated from the decay of long-lived sunspots (Meyer et al. 1974), from microturbulent velocities in sunspots (Beckers 1976), or from downdrafts in slender flux tubes (see below): All these estimates yield a value around \( 2 \times 10^{11} \text{ cm}^2 \text{ s}^{-1} \), less than \( 1\% \) of the eddy diffusivities outside the flux ropes (cf. Spruit 1977), and we adopt this value for \( \eta \) throughout the convective zone. From equation (7) the corresponding field is approximately the equipartition field:

\[
B_c \sim (4\pi\rho)^{1/2} U \ .
\]

Galloway, Proctor, and Weiss (1977) obtained an upper bound for \( B_m \) by equating the rate of work done in containing the magnetic field to the rate of turbulent dissipation in the whole convective cell, so that \( (v/\eta) \), the ratio of the turbulent diffusivities, was around 100; the arguments of Galloway, Proctor, and Weiss (1978) imply that \( v \) and \( \eta \) should both be evaluated at the edge of the flux rope, so that \( (v/\eta) \) is of order unity, and, therefore, give a more stringent upper limit to the fields that can be maintained by external convection.

Table 1 gives estimates of the critical flux \( F_c \) and the maximum field \( B_m \), derived from equations (9) and (10), at levels corresponding to granules, supergranules, and giant cells; the values of \( \rho \) and \( U \) are taken from a model convective zone, computed by Spruit (1977). The magnetic pressure balances the external gas pressure \( P \) when the field reaches a strength \( B_r = (8\pi p)^{1/2} \) and \( B_r \) is much greater than \( B_c \) except near the photosphere. Thus the estimates of \( B_m \) given in Table 1 should be valid for fields between supergranules and giant cells but not for intergranular magnetic fields. We see, moreover, that ropes with fluxes greater than about \( 10^{21} \text{ Mx} \) (corresponding to a small active region) can rise through the convective zone without being penetrated by external motions, while smaller flux tubes are more likely to be shredded. Similarly, ropes with fluxes greater than \( 10^{20} \text{ Mx} \) can resist supergranular motions while flux tubes with more than \( 10^{17} \text{ Mx} \) may stand up to granules. Small flux ropes, starting deep in the convective zone, therefore, can supply the fields that emerge as ephemeral active regions, while
larger ropes can remain coherent and emerge to form sunspots. The arguments developed here therefore support the traditional picture of the origin of active regions (e.g., Babcock 1961) rather than the rival views that Parker (1979a, b) has recently developed.

V. PHOTOSPHERIC MAGNETIC FIELDS

All observations necessarily record magnetic fields in or above the photosphere, where the magnetic energy density becomes comparable with the thermal energy of the gas. The Boussinesq approximation therefore ceases to be valid: A correct treatment of the interaction between magnetic fields and convection requires a fully compressible model. Until such a calculation is carried out, we have to rely either on bold extrapolation from the Boussinesq results or on some parametric representation of turbulent convection. We shall see that these oversimplified models are inadequate.

Consider first the inhibition of normal photospheric convection by a large-scale magnetic field, extending over a region whose horizontal dimensions are much greater than the diameter of a typical convection cell. In the solar context this corresponds to the suppression of granular convection in a pore or sunspot. The nonlinear studies of Boussinesq magnetoconvection imply that the transition from normal convection to oscillatory motion occurs when \( B = B_{\text{max}} \approx (g v f d)^{1/2} \), from equation (5). According to mixing length theory, the velocity is just the reduced free-fall speed, so that \( U \approx (g v f d)^{1/2} \); hence \( B_{\text{max}} \) is comparable to the equipartition field \( B_e \), and normal convection is indeed suppressed when the magnetic energy density exceeds the kinetic energy density of the motion (Cowling 1976). At the photosphere, \( B_e \) and \( B_{\text{max}} \) become comparable to the pressure-balancing field \( B_p \), and the Boussinesq approximation is no longer valid. Moreover, a reduction in convective transport leads to a drop in photospheric temperature, which increases the superadiabatic gradient. Thus a proper treatment of this problem requires a calculation for a compressible gas with a radiative boundary condition at the photosphere. However, the critical field \( B_{\text{max}} \) cannot exceed \( B_p \); In the Sun, the critical field, at the edge of a pore, is around 1500 gauss, and approximately equal to \( B_p \).

Similar difficulties arise in attempting to estimate the strength of intergranular magnetic fields. The Boussinesq calculations suggest a value of order \( B_e \), which is lower than the field strengths (around 1500 gauss) that have been observed (Harvey 1977). To be sure, this is only a rough estimate and \( B_e \) might exceed \( B_e \) by some factor of order unity. Nevertheless, it is clear that Boussinesq models are inadequate: If the field is confined to a rope, and if the flux exceeds \( F_e \), so that convection is excluded from the rope, then a further thermal instability occurs (Galloway, Proctor, and Weiss 1977). The flux tube collapses until it reaches a stable equilibrium in which it is partially evacuated, allowing the field to approach \( B_p \) (Parker 1979a).

The linear theory of this collapse has been studied by several authors in the thin tube approximation (Webb and Roberts 1978; Spruit and Zweibel 1979; Unno and Ando 1979), and nonlinear aspects have been investigated by Spruit (1979). He assumes, however, that convection can be parametrized according to mixing length theory and does not attempt to represent the interaction between magnetic fields and individual convection cells.

Small-scale solar magnetic fields are intermittent, and observations indicate that the flux has already been concentrated into ropes when it emerges through the photosphere. There are strong reasons for supposing that these ropes run between granules in regions of converging, downward flow (Weiss 1978). The solar filigree (Dunn and Zirker 1974) is made up of bright points and sheetlike crinkles in the dark intergranular lanes, and these individual elements are shifted about and buffeted by granules. Moreover, in moments of exceptional seeing the crinkles can be resolved into separate features that outline small micropores (R. B. Dunn, private communication) as though a flux sheet at the edge of a convection cell had separated into several isolated tubes. These observations are all consistent with the pattern of turbulent magnetic fields described in § III above, and they suggest that there is an intimate relationship between small-scale magnetic fields and individual convection cells.

For the structure of an isolated flux tube to be unaffected by the surrounding pattern of convection, it is necessary that there should be no significant flow across the field. The systematic downdrafts observed in small flux elements at the solar surface (Giovannelli and Slaughter 1978) are, however, difficult to explain theoretically unless there is substantial motion across the lines of force. Spruit (1979) has suggested that they could result from overstable oscillations with a velocity-brightness correlation in the observable layers, but preliminary calculations by Spruit and Galloway have so far failed to demonstrate this effect. Both the density and the downward velocity increase rapidly with depth, with scale-heights around 250 km at the photosphere. The radius of the flux tube cannot change much, since the magnetic field shows no comparable variation with height. Hence it
is impossible to provide a steady flux of matter from above: Continuity demands a radial inflow across the field lines (Giovanelli 1977). This motion can be described locally by an axisymmetric, solenoidal velocity

\[ u = U(-r/H, 0, z/H) \]

with \( U = 1.6 \text{ km s}^{-1} \) and \( H = 250 \text{ km} \), referred to cylindrical polar coordinates \((r, \phi, z)\). The characteristic time for flow across a flux tube is short (about 200 s), so there must be some form of diffusion. Now it can be shown (e.g., Proctor and Weiss 1978) that the velocity in equation (11) is compatible with a vertical field,

\[ B = B^* \exp (-Ur^2/2\eta H), \]

so the field strength drops by a factor \( e \) in a radius \( r = (2\eta H/U)^{1/2} \). For \( r_1 = 100 \text{ km} \), it then follows that \( \eta = 3 \times 10^{11} \text{ cm}^2 \text{ s}^{-1} \), giving a diffusion time \( \tau_{\eta} = r_1^2/4\eta = 100 \text{ s} \). Thus the survival of flux tubes for periods of 10 or 20 minutes cannot be explained without discussing motion across the field and the effective diffusivity \( \eta \). The source of this diffusivity remains mysterious (Giovanelli 1977), for even in a turbulent medium, flow across the field lines becomes possible only when laminar diffusion is effective at scale lengths of 1 km or less.

In this section we have shown that photospheric magnetic fields cannot be described adequately in either the Boussinesq or the thin flux tube approximation. What is needed is a fully compressible calculation that can describe both the thermodynamic structure of the flux tube and its relation to external convection, coupled with greater insight into turbulent diffusion and the appropriate boundary conditions. This is still a tall order.

VI. STELLAR MAGNETIC FIELDS

We have argued that magnetic flux emerging through the surface of a late-type star will be concentrated into isolated tubes. This intermittent structure is produced by turbulent eddies: Without convection there would be no flux ropes. Hence any description of solar or stellar magnetic fields must take into account the interaction of those fields with convection.

Turbulent convection concentrates magnetic flux until the field strength reaches a value \( B_p \), such that the kinetic and magnetic energy densities are comparable. Within a star, \( B_p \) is much less than \( B_p \), the field strength for which the thermal and magnetic energy densities are equal. At the photosphere, however, \( B_p/B_p \) is no longer negligible; the reduction of convective transport within a flux tube leads to a collapse and the field approaches \( B_p \). The field strength increases toward the axis of a large flux concentration, owing to geometric effects and the depression of surfaces of constant optical depth, but surface fields should not greatly exceed \( B_p(0) \), the photospheric value of \( B_p \). Since the surface pressure changes slowly in late-type main sequence stars, field strengths should be similar to those observed in the Sun, though total fluxes may be quite different. The observations of fields of 2500 gauss in a G8 V star are consistent with the fields in spots and active regions on the Sun (Robinson, Worden, and Harvey 1980). Thus it seems that average field strengths lie between \( B_p(0) \), in small flux elements, and \( 2B_p(0) \) in large magnetic features. This would allow fields of 5000 gauss in dM stars. On the other hand, red giants have such distended atmospheres that \( B_p(0) \sim 100 \text{ gauss} \) and surface magnetic fields are likely to be small.

A magnetic star may possess a large-scale global magnetic field if there is no vigorous convection in its outer layers. Otherwise magnetic flux will be confined to ropes that run through the convective zone or to a layer at the interface between the convective and radiative zones. The dynamo that is responsible for the solar cycle is probably located at the base of the convective zone and is maintained by the combined effects of thermal and magnetic buoyancy, interacting with the Coriolis forces. Similar dynamos should operate in other G and K stars, whose activity is related to their rotation rate. In M stars convection extends deeper, until the whole star is fully convective. If the inner radius of the convective zone is less than a scale height, the field can scarcely be confined to a spherical shell; stars with \( \delta < 0.5 \delta_0 \) probably have a different type of dynamo, depending on flux ropes that cross the whole convective zone. The appearance of spots in RS CVn and BY Dra stars, the incidence of activity in flare stars, and the enhanced coronal X-ray emissions all suggest that M dwarfs have strong large-scale magnetic fields. Flux rope dynamos are apparently more efficient than the mechanism operating in the Sun.

Ap stars have strong magnetic fields which can suppress convection locally and, therefore, may allow abundance anomalies to persist (e.g., Michaud 1976). The convection layers are relatively thin and ineffectual, and the fields are probably primordial (Mestel 1976). It is, however, to be expected that fields are generated in their convective cores, though the magnetic flux is trapped at the outer boundary of the core. Finally, we note that interstellar magnetic fields are observed together with turbulent motion. The magnetic and kinetic energies are similar (Spitzer 1978), and it seems inevitable that the field structure must be very intermittent.

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