Observational Requirements for Measurements
of Solar Rotation Inward to the Base of
the Convection Zone

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As we have already heard earlier in this symposium, the acquisition and
analysis of observational data concerning the dynamical state of the solar
interior will be crucial for improving our understanding of the solar activity
cycle. Among these data will be measurements of the rate of rotation at various
depths in the solar interior and of temporal changes in the rotation.

This afternoon I hope to convey the following three points: first, it is now
possible to measure the absolute rate of the sun's rotation (in meters per second)
below its visible surface over the outer 3% of its radius with ground-based equip-
ment; second, the theory of this technique has now been developed to the point
where it is potentially possible to measure rotation inward to the base of the
solar convection zone; and third, such deeper rotational measurements, extending
from 3% inward to 25-30% of the sun's radius can only be obtained from a space-
borne instrument which is not subject to the normal earth-based day-night observ-
ing cycle.

To begin, it is the existence of the non-radial acoustic (or p-mode) oscilla-
tions of the sun which permits the measurement of sub-photospheric solar rotation.
The first figure shows the most recent observational confirmation of the existence
of the p-mode oscillations. This figure is a grey-level display of a diagnostic
diagram, or two-dimensional \( k_\nu - \omega \) power spectrum. This spectrum was obtained
from a continuous 11-hour observing run on the McMath Telescope at the Kitt Peak
National Observatory. It was obtained on 15 April 1979 with the assistance of
Drs. Jack Harvey and Tom Duvall. It shows the concentration of observed power
into many distinct ridges. These ridges are distinct functions of the horizontal
wavenumber (running across the spectrum) and temporal frequency (running vertically).

Theoretical eigenmode analysis shows that these ridges of observed power
are due to non-radial p-mode oscillations. It is these power ridges which permit
the measurement of solar rotation with the oscillations. This is accomplished by
guiding the telescope being used on the limbs of the sun and then letting the
sun's rotation carry the pattern of oscillating motions through the telescope's
field of view. When observations are made in this manner, the p-mode ridges are
systematically shifted in frequency. For example, in this figure the upper set of
ridges can be seen at higher absolute frequencies than the lower set of ridges,
which in turn have been shifted to lower absolute frequencies. (Since the origin
of the spectrum in the figure is in the middle of the left-hand axis, positive
frequencies increase upward at the top while negative frequencies increase downward
at the bottom.)

Rhodes, Deubner, and Ulrich (1979) have recently shown that the amount of
frequency splitting which is introduced in corresponding ridges in the two parts
of the spectrum by the rotation is proportional to both the horizontal wavenumber,
k_\nu, and the rotational velocity, \( V_{\text{rot}} \). By measuring both the frequency splitting
and the horizontal wavenumber at various places in the spectrum, one can calculate
the rotational velocity (or velocities) directly.
The exciting aspect of making these measurements at different depths below the photosphere comes about because the individual p-mode waves are sensitive to rotational influences at different depths. This fact is illustrated in the second figure. In this figure I show an expanded portion of the theoretically-predicted p-mode spectrum for long wavelengths (small $k_h$). In this figure the theoretical p-mode ridges are shown as the light solid curves which run diagonally across the spectrum and which are labeled $P_0$ through $P_{14}$. The heavier solid lines which are running more vertically are contour lines which show the locations of those wavemodes which have the same effective depths. The scale of the depth contours is shown adjacent to them and is labeled in fractions of the sun's radius. Clearly, much of the spectrum is sensitive to rotation at relatively shallow depths (i.e., depths near $1.0 \, R_\odot$), while the deeper-going portions of the spectrum are crowded together at low wavenumbers. As of this time, rotational observations have only been obtained with the p-modes for the outer 3% of the solar radius. The results of these ground-based observations are contained in Deubner, Ulrich, and Rhodes (1979).

The interesting, deeply-penetrating modes are precisely those which are the most crowded together in the spectrum and hence are the most difficult to separate observationally. In addition, the expected frequency splitting due to rotation is much smaller for those modes since $k_h$ is so much smaller than for the photospheric modes. These reasons indicate that the deeper-going modes must be observed from space. This conclusion will be demonstrated in the next few figures.

In Figure 3 I have expanded the theoretical power spectrum even further in order to demonstrate the fact that the p-mode ridges are not really a set of
Continuous ridges as indicated in Figure 2. Rather, the p-modes are spherical harmonics, one for each value of $\ell$, $m$, and $n$, and the observed ridges are merely collections of these discrete modes. To measure the rate of rotation for the deepest modes, the individual $\ell$ values must be resolved. However, from any observing location on the earth the effective horizontal wavenumber resolution is limited by projection effects and is insufficient to spatially resolve these modes. In particular, for the longest practical baseline on the sun of one solar radius in extent, each horizontal wavenumber bin in the resulting power spectra will contain six different sets of $\ell$ modes. This degeneracy is illustrated in Figure 3 where the vertical lines denote the boundaries of the individual wavenumber bins for such a run. The complete set of p-modes which would lie in bin #4 is shown at the right-hand side of the figure as a vertical series of spikes.

Clearly, there is a problem in the resolution of the individual p-modes in this part of the spectrum. Since the measurement of the rotationally-induced frequency shifts is based upon the identification of matching modes in the positive- and negative-frequency portions of the power spectrum, such confusion in the identification of corresponding modes makes the measurement of this splitting very difficult. The splitting can only be measured with an observing run of sufficient duration that the individual $\ell$ values are resolved in temporal frequency within each wavenumber bin. Such individual mode resolution is necessary in this part of the power spectrum because the magnitude of the induced frequency shifts will be less than one frequency resolution bin width.

To illustrate the observational requirements necessary for resolving individual modes, I have computed a series of synthetic power spectra for the six $p_5$ and six
modes located between \( \ell \) values of 19 and 24 in Figure 3. Figure 4 shows part of the power spectral slice for wavenumber bin \#4 which resulted from a simulated continuous solar viewing run consisting of 2048 measurement frames each lasting 90 seconds and having a total duration of 51.2 hours. For such a two-day run you can see that the individual \( \ell \) values were resolved. On the other hand, when I simulated a ground-based data string consisting of a 12-hour observing day followed by a 12-hour dark night, a second 12-hour day, a second 12-hour dark night, and a short 3.2-hour data segment (so that the total length was 51.2 hours), the spectrum shown in Figure 5 resulted. This ground-based simulation shows clearly that the sidelobes introduced into the power spectrum by the day-night observing cycle make the identification and measurement of these \( p \)-modes impossible from the ground. The situation which resulted for a more typical 8-hour observing day followed by a 16-hour dark night is illustrated in Figure 6. This figure shows that such a situation is even worse.

Finally, Figure 7 is a simulation of a 51.2-hour observing run obtained from an equatorial-orbiting satellite having roughly a 67% duty cycle. Again, the individual \( p \)-modes are visible; however, in this case the sidelobes introduced by the observing window are hidden within several of the simulated solar spikes. The presence of sidelobes within the \( p \)-mode spikes will alter the frequencies that are determined for those spikes and will alter their estimated amplitudes. Such frequency errors will in principle introduce errors into the rotational-velocity determinations. The details of how much these satellite data gaps affect the rotation measurements are currently being investigated numerically. Certainly, such satellite data will be a vast improvement over what is available from the ground even if continuous solar viewing is not available in the near future.
In summary, I believe I have demonstrated that measurements of solar rotation inward to the base of the convective zone will require the acquisition of several day-long observing runs from a space-borne instrument.

REFERENCES


Figure 1. An observational two-dimensional \( (k_n - \omega) \) power spectrum. Horizontal wavenumber increases from left to right, while temporal frequency runs along the vertical axis. The frequency is zero at the midpoint of the frequency axis and increases upward for positive frequencies (i.e., the top half of the spectrum) and increases downward for negative frequencies (the bottom half). The p-mode oscillations are the curved ridges of concentrated power. The spectrum shown is the result of an 11-hour observing run at the Kitt Peak National Observatory.
Figure 2. The long-wavelength (low $k_h$) portion of the theoretical p-mode eigen-frequency spectrum. The light curves are p-modes $p_0$ through $p_{14}$. The diagonal solid lines are contour lines showing those modes having particular effective depths to rotation. The scale of the effective depths is shown in fractions of the solar radius. The deepest-penetrating modes are those in the upper-left portion of the figure.
Figure 3. The extreme low-wavenumber portion of the eigenfrequency spectrum. The discrete nature of the p-modes is illustrated here, with one p-mode for each value of spherical harmonic degree, \( \ell \). The horizontal-wavenumber bins attainable from a single observing location are shown as the solid vertical lines. This shows that six discrete sets of p-modes lie within each resolution bin. The set of p-modes within bin #4 is plotted as the vertical strip of spikes at the right side. These modes can only be resolvable in temporal frequency with a long-duration set of observations.
Figure 4. A synthetic one-dimensional ($\omega$) spectrum generated from a simulated satellite experiment located in a sun-synchronous (continuous solar viewing) orbit. The spectrum contains the six p$_5$ and six p$_6$ modes illustrated in $k_h$ bin #4 of Figure 3. A continuous data set consisting of 2048 90-second velocity samples was used to generate this spectrum. Thus, a run having a duration of 51.2 hours is needed to resolve these p-modes in $\omega$. 
Figure 5. A synthetic power spectrum generated from a simulated ground-based observing sequence in which several days' worth of data have been stacked together. The data string used here consisted of one 12-hour (480-sample) observing day followed by a 12-hour dark night containing no data, a second 12-hour day, a second 12-hour night, and a 3.2-hour segment for the total of 2048 measurements. The total duration of the data string was identical to that used in Figure 4. The same assumed p-mode amplitudes were used as input to both spectra.
Figure 6. A synthetic power spectrum generated from a simulated ground-based observing sequence consisting of an 8-hour (320-sample) observing day followed by a 16-hour (640-sample) night, a second 8-hour day, a second 8-hour night, and a 3.2-hour segment. This spectrum was computed since the acquisition of continuous ground-based data for 8 hours is more likely than it is for 12 hours. The individual p-modes are completely unresolved in this spectrum.
Figure 7. A synthetic power spectrum which resulted from a 51.2-hour observing run obtained with a simulated equatorial-orbiting earth satellite. The duty cycle used here was a series of orbits each containing 62 minutes of measurements followed by 32 minutes of darkness. The observing window had roughly 67 percent coverage. This observing window introduced sidelobes in the power spectrum which were mostly located within the same frequency bins containing other p-mode spikes. The effect of these sidelobes is to introduce errors in frequency determination for the p-mode peaks. The actual amount of frequency error introduced in this way is currently being investigated.
QUESTIONS AND COMMENTS

QUESTION BY: N. Sheeley, NRL

DIRECTED TO: E. Rhodes

QUESTION: Exactly what physical picture of the surface velocity field do you associate with the observed ridge pattern in the \((K, \omega)\) plane? For example, how does your mental picture differ from the one that would be obtained from other hypothetical patterns such as concentric circles about the point

\[[1000 \text{ km}]^{-1}, 2\pi/300 \text{ sec}]？\]

ANSWER: An instantaneous velocity snapshot of the solar p-mode, or pressure mode, oscillations in the photosphere is contained in Rhodes, Ulrich, and Simon (1977). This velocity picture has the appearance of a grid of randomly-distributed upwelling and downflowing cells. This pattern is the result of the superposition of an enormous number of spherical harmonics, each having a different horizontal cell size. The oscillatory gas velocity at every location across the solar surface is predominantly radial and not horizontal. That is, the term "non-radial p-mode oscillations" refers to the horizontal pattern of upward-and downward-flowing cells and not to the direction of the velocity itself. Each individual p-mode wave has associated with it a characteristic frequency and horizontal cell size. Some are zonal harmonics, others are sectorial harmonics, and still others are tesseral harmonics. Graphical illustrations of tesseral harmonics are contained in Kraut (1967) and in my thesis (Rhodes, 1977). The alternating pattern of radial velocities for such a mode is apparent in these figures. It is the superposition of patterns such as these which results in the instantaneous photospheric velocity field.

The reasons there were ridges in the \(k_h - \omega\) spectra I displayed rather than circles were the following: first, in the two-dimensional power spectra the velocity observations had been filtered so that information was only shown for eastward-and-westward-propagating sectorial harmonics. In contrast to tesseral harmonics, sectorial harmonics look like orange slices or the alternating stripes on a beach ball. The width of each of these stripes (i.e., the horizontal wavelength of the pattern) is inversely proportional to the horizontal wavenumber, \(k_h\). Second, the operation of the resonant cavity which traps the p-mode oscillations in the sun is like an organ pipe (i.e., there is a fundamental mode for each wavenumber and a series of higher-frequency harmonics). The depth of the inner reflecting boundary of this cavity varies with \(k_h\) and \(\omega\) in just such a way as to produce a diagonal ridge for the fundamental mode (since a deeper
pipe implies a lower frequency and vice versa. This cavity also produces parallel ridges for the overtones. A schematic cross-section of the resonant cavity is shown in Ulrich (1970) for a few modes.

It is the mathematical form of the dispersion equation for the p-mode waves which combines with the radial stratification of the solar atmosphere to result in the series of ridges in the $k_r - \omega$ diagrams. Circles would not be expected because they would not be compatible with the acoustic wave equations. On the other hand, if the velocity data had not been averaged to reveal only sectorial harmonics, a three-dimensional ($k_x, k_y, \text{and } \omega$) power spectrum would have resulted. In such a spectrum the p-mode tesseral harmonics would result in a series of concentric circles at each single temporal frequency, $\omega$. Thus, circles would result from a 3-dimensional analysis, but not from the 2-dimensional analysis discussed above.

REFERENCES:


