THE POLARIZATION AND DIRECTIVITY OF SOLAR-FLARE HARD X-RAY BREMSSTRAHLUNG FROM A THERMAL SOURCE

A. Gordon Emslie
Harvard-Smithsonian Center for Astrophysics

AND

John C. Brown
Department of Astronomy, University of Glasgow
Received 1979 June 14; accepted 1979 October 19

ABSTRACT

We evaluate the polarization and directivity of hard X-ray bremsstrahlung from a thermal source consisting of a region in which a thermal flux drives a pair of steep collisionless conduction fronts symmetrically away from a central region, heated by the flare energy dissipation process. By comparing results with similar calculations based on a nonthermal thick-target electron beam model of the source, we thus aim to determine the degree to which the observed polarization and directivity of solar flare hard X-rays favor either model.

Using Maxwellian electron phase-space distribution functions modified to take into account a directional heat flux and a steady direct current in the X-ray source, and a fully relativistic treatment of the bremsstrahlung emission process, we arrive at results which indeed exhibit significant polarization and directivity of the hard X-ray radiation emitted by the source. If the energy release is assumed to occur at the top of a semitoroidal loop, then, in contrast to the thick-target beam models, we predict a slight limb darkening of sources (although if the energy release occurs elsewhere, no simple statement such as this can be made). Further, the polarization vector need not (as in thick-target models with vertical guiding magnetic fields) be oriented along the solar disk radius to the source. The results obtained are fully consistent with solar hard X-ray anisotropy and polarization observations to date, although these observations are at present too crude to be at all conclusive.

Subject headings: polarization — Sun: flares — Sun: X-rays

I. INTRODUCTION

One of the most basic controversies regarding hard X-ray bursts in solar flares is whether they are thermal or nonthermal in character (see, e.g., Brown 1974; Kahler 1975; Švestka 1976; Emslie and Rust 1979). A large amount of literature has been based on the interpretation of these hard X-ray bursts in terms of bremsstrahlung radiation from a beamed nonthermal electron flux (e.g., Brown 1971, 1972; Syrovatskii and Shmeleva 1972; Petrosian 1973; Lin and Hudson 1971, 1976; Hoyng, Brown, and van Beek 1976). This interpretation, whether based on a thick (Brown 1972) or a thin (Datlowe and Lin 1973) target model, requires a large flux of electrons to be accelerated for long periods of time, thus placing great demands on the primary energy release mechanism and also raising questions as to the stability of the beam and its associated reverse current (Datlowe, Elcan, and Hudson 1974; Smith 1975; Hoyng, Brown, and van Beek 1976; Lin and Hudson 1976; Brown and Melrose 1977; Colgate 1978; Hoyng, Knight, and Spicer 1978; Shapiro and Knight 1978; Emslie 1980).

Consequently, there has recently been a renewed interest in thermal models of the hard X-ray burst (e.g., Crannell et al. 1978; Mätzler et al. 1978). Earlier objections to this interpretation, based on the short conductive source cooling times and the long collision mean free paths associated with deka-keV temperatures (Kahler 1971a, b, 1975) have recently been shown to be overcome by models which invoke an extremely localized energy release (Brown, Melrose, and Spicer 1979; Smith and Lillyquist 1979; Vlahos and Papadopoulos 1979). In these models, ion-acoustic turbulence, created by the instability of the reverse current associated with the outward streaming of hot electrons from the energy release site, inhibits this free streaming and results in a higher degree of plasma confinement than is found in models which assume classical conduction to be valid. For a discussion of the merits of this model over nonthermal X-ray burst models, the reader is referred to Brown, Melrose, and Spicer (1979) and to Smith and Lillyquist (1979); the model is further discussed, with regard to its radiation signatures at a variety of wavelengths and comparison with observations, by Emslie and Vlahos (1980).

This type of thermal model is characterized by the appearance of two collisionless conduction fronts, which

1 On leave from Department of Astronomy, University of Glasgow.
propagate symmetrically away from the energy release site at about the local ion-sound speed (Brown, Melrose, and Spicer 1979; Smith and Lilliequist 1979). The anisotropy of the velocity distribution of the source electrons in the nonisothermal (see § II) region between these fronts will result in X-ray bursts exhibiting both polarization and directivity, qualities not normally associated with radiation from a thermal source. While observations of the directivity of solar hard X-ray sources are at present inconclusive (Datlowe et al. 1977), measurements of the polarization of such bursts (Tindo et al. 1970, 1972a, b; Tindo, Mandel'stam, and Shurygin 1973; Tindo, Shurygin, and Steffen 1976) show that although there exists considerable uncertainty in the results, substantial (a few percent) polarization is present in most events. Brown (1972) has evaluated the polarization and directivity of hard X-ray bursts from a thick-target nonthermal beam guided by a vertical magnetic field, extending the original work of Korchak (1967), Elwert and Haug (1970, 1971), and Haug (1972) (see also Petrosian [1973]) to include the variation of electron energy and pitch angle (to the magnetic field) distributions with depth in the source. Results taking into account photospheric backscatter of photons (see Hénoux 1975; also this paper § IV) have been given by Langer and Petrosian (1977) and Bai and Ramaty (1978), and indicate good agreement with early polarization results of Tindo et al. (1970, 1972a, b) and Tindo, Mandel'stam, and Shurygin (1973). Polarizations of comparable magnitude are also predicted (McClymont 1976) for nonthermal trap models (e.g., Takakura and Kai 1966; Bai and Ramaty 1979). However, the more recent, background-corrected (see discussion in § IV), observations of Tindo, Shurygin, and Steffen (1976) have substantially smaller polarizations than predicted by the above nonthermal models.

In the light of the above mentioned anisotropy of the electron velocity distribution in a thermal source, this paper seeks to evaluate quantitatively the resulting X-ray polarization and directivity, thus testing whether or not a thermal source such as that envisaged by Brown, Melrose, and Spicer (1979) can also be reconciled with the observations. In § II we discuss in greater detail the source model and the constraints imposed on its parameters by the Brown, Melrose, and Spicer (1979) model. In § III expressions for the polarization and directivity of such a source are derived, while the results of the (numerical) evaluation of these expressions are presented in § IV. In § V we briefly discuss the significance of these results.

II. THE SOURCE MODEL

In the Brown, Melrose, and Spicer (1979) thermal model, the hard X-ray source is impulsively heated by an unspecified form of magnetic field dissipation and cools by propagation of heat into the surroundings across two symmetrically placed collisionless conduction fronts which each propagate away from the energy release site with the local ion-sound speed \( v_s = (k T_e/m_e)^{1/2} \), where \( T_e \) is the electron temperature behind the front (i.e., in the source), \( k \) is Boltzmann's constant, and \( m_e \), the proton mass. (Note that in the Brown, Melrose, and Spicer [1979] model, \( T_e \gg T_i \), where \( T_i \) is the ion temperature.) Since they were only concerned with the crude energetics of the problem, and not with the detailed source structure, Brown, Melrose, and Spicer (1979) idealized the source as isothermal and did not specify the geometry of the surrounding magnetic field. This description is not, however, sufficient for our purposes, for the following reasons.

First, as the source expands, the geometry of the surrounding magnetic field will clearly have an effect on the electron velocity distribution and thus on the polarization and directivity of X-ray bremsstrahlung produced by the source. We note that the detailed field geometry \( B \) near reconnecting regions is unknown; however, a simple parametrical source model will serve our purpose here.

The geometry of the adopted source model is shown in Figure 1. The source, of length \( 2L \) (cm), symmetrical about the z-axis, forms part of a semitoroidal loop of toroidal radius \( R \) and poloidal radius \( \ll R \) (so that the source may be considered one-dimensional), lying in the x-z plane. The electron density \( n_e \) within the source is taken to be constant (see Smith and Lilliequist 1979). We shall also permit a current to drift with velocity \( u \) (cm s\(^{-1}\)) in the positive x-direction in the source, although this generalizing feature, it turns out, is not important to the results. The (terrestrial) observer is located at a distance \( A = 1 \) AU from the source along a line with polar coordinates \( (i, \Phi) \) (i = heliocentric angular distance of the source from the center of the solar disk, \( \Phi = \) angle between the plane of arch and radius vector from disk center to source). The quantity \( \alpha = S/R \) measures angular distance along the arch. The plasma \( \beta = (\approx 8\pi n_e k T_e/B^2) \), is assumed to be small enough for the plasma to be effectively confined by the magnetic field. Note that the source geometry is thus parametrically represented by \( L \) and \( R \); the limit \( L/R \rightarrow 0 \) will correspond to the maximum polarization and directivity possible (for a horizontal \( 1-D \) source).

Second, it is clear that a strictly isothermal Maxwellian source (such as assumed by Brown, Melrose, and Spicer 1979) cannot in fact create finite bremsstrahlung polarization and directivity at all, except near the conduction fronts bounding the source. These fronts, however, are so thin (Brown, Melrose, and Spicer 1979) that they contribute a negligible amount to the source bremsstrahlung. In order to obtain nonzero values for the polarization and directivity, therefore, we must allow for the nonisothermality within the source itself. It is clear that, in practice, the finite heat flux \( Q \) (ergs cm\(^{-2}\) s\(^{-1}\)) required at the source boundaries to drive the conduction fronts, and the symmetry requirement that \( Q = 0 \) at \( s = 0 \), leads to such nonisothermality. The detailed temperature distribution in the source will depend on the extent to which the heat flux \( Q \) is dominated by collisions or by the propagation of waves outward from the fronts (see Smith and Auer 1980)—a problem as yet unsolved—and so, to quantify...
Fig. 1.—Geometry of the source model. The bremsstrahlung-emitting material of density $n_e$ (cm$^{-3}$) is bounded by two collisionless conduction fronts (Brown, Melrose, and Spicer 1979) at $s = \pm L$, and lies within a semitoroidal loop of radius $R$. A current with associated drift velocity $u$ flows as shown. Relative to the Cartesian coordinate system shown, the (terrestrial) observer is at a distance $A = 1$ AU and has polar coordinates $(\theta, \phi)$. The quantity $a = s/R$ measures angular distance along the arch.

The temperature distribution in the source, we again resort to a parametric representation. Bearing in mind the symmetry requirement at $s = 0$, we represent the anisotropic effects of the heat flux $Q(s)$ by that produced, under collision-dominated conduction, by a temperature profile of the form

$$T_e(s) = T_0 \exp \left( -\frac{(s/s_0)^2}{2} \right),$$  \hspace{1cm} (1)

where $s_0$ is thus a parametric representation of the heat flux gradient. Then, adopting the Spitzer (1962) conductivity

$$\kappa = \kappa_0 T^{3/2},$$

where $\kappa_0 = 10^{-6}$ ergs cm$^{-1}$ s$^{-1}$ K$^{-3/2}$, we derive

$$Q(s) = 2\kappa_0 \frac{s}{s_0^2} T_0^{7/2} \exp \left[ -\frac{1}{2} (s/s_0)^2 \right].$$  \hspace{1cm} (2)

The effect of this heat flux $Q$ and the drift velocity $u$, is to distort the zero-order Maxwellian velocity distribution within the source; it becomes (Spicer 1977a; see also Manheimer 1977), in a nonrelativistic approximation,

$$f_e(v) = \frac{n_e}{2\pi v_e^3} \left\{ \exp \left[ -\frac{(v \parallel - u)^2 + v \perp^2}{2 v_e^2} \right] \right\} \left[ 1 + \frac{Q(v \parallel - u)}{3n_e v_e^4} \left( \frac{v}{v_e} \right)^2 - 3 \right],$$  \hspace{1cm} (3)

where $m_e$ is the electron mass and $v_e$, the mean electron thermal velocity, is equal to $(kT_e/m_e)^{1/2}$, $k$ being Boltzmann’s constant. The quantities $v \parallel$ and $v \perp$, representing flow parallel and perpendicular to the direction of the heat flux $Q$ (i.e., along the arch), are given by $v \parallel = -v \cos \theta |s|/s$, $v \perp = v \sin \theta$, where $v = |\mathbf{v}|$ and $\theta = \pi - \cos^{-1}(v, \mathbf{I})$, where in turn $\mathbf{I}$ is the unit vector in the direction of increasing $|s|$ (see Fig. 2). Note that the definition of $v \parallel$ means that a flow parallel to the heat flux implies that both $v \parallel$ and $Q$ (eq. [2]) have the same sign. The anisotropy of $f_e(v)$ in the above expression is responsible for the resulting polarization and directivity of the thermal hard X-ray bremsstrahlung from the source.

The model parameters above cannot in fact be chosen independently. In particular, the condition (Brown, Melrose, and Spicer 1979) that the conduction fronts be at marginal stability for generation of ion-acoustic

---

$^a$ Note that although the assumption of classical conductivity is not necessarily justified (see above), the resulting expression (2) for $Q(s)$ is only intended as a parametric ($T_0, s_0$) one to indicate the degree of anisotropy associated with the heat flux; other modes of thermal conduction merely change the physical significance of these parameters from that given in equation (1).
turbulence, and that the heat flux $Q$ be a maximum at the fronts, must be satisfied. Following the analysis presented by Spicer (1977a), again based on work by Manheimer (1977), we find that the heat flux at marginal stability is given by

$$Q_{\text{MS}} \sim n_e m_e v_e^2 v_s,$$

where we have assumed (see Brown, Melrose, and Spicer 1979) that $T_e \gg T_i$ and that the electron drift is slower than the ion-sound speed, viz.,

$$|u| < v_s.$$

Equations (2) and (4) together give

$$\frac{T_0^2 L}{n_s s_0^2} \exp \left[-2(L/s_0)^2\right] = \frac{k^{3/2}}{2\kappa \rho s_0^{3/2}} = 6.3 \times 10^{-7} \text{(cgs)};$$

this together with the requirement that $L$ be below its value corresponding to the maximum of the left-hand side of (6) (so that $Q$ is a maximum at the fronts), represents a constraint on $n_e, T_0, s_0,$ and $L,$ in addition to the more basic constraint (5).

A suitable set of parameters, satisfying (5) and (6) and at the same time having $T_0$ high enough for the source to emit substantially in the deca-keV photon energy range, is

$$n_e = 2 \times 10^{11} \text{ cm}^{-3}, \quad T_0 = 3 \times 10^8 \text{ K}; \quad R = 10^{19} \text{ cm};$$

$$L = 3 \times 10^9 \text{ cm}; \quad s_0 = 5 \times 10^{10} \text{ cm}; \quad u = 0.$$

(Note that we have set $u = 0$; we shall see later [§ IV] that finite [but acceptable, see eq. (5)] $u$ does not significantly alter our results. Further, setting $u = 0$ makes the source symmetrical about $s = 0$; thus we need only consider $i$ and $\Phi$ values in the first quadrant.)

This completes our discussion of the source model. In the next section we shall derive expressions for the polarization and directivity of X-ray bremsstrahlung to be expected from such a model.

### III. POLARIZATION AND DIRECTIVITY OF BREMSSTRAHLUNG

The plane of emission of a photon in a given bremsstrahlung collision is represented by two cross sections, one ($\sigma_1$) representing emission perpendicular to the plane containing the electron precollision unit velocity vector $\hat{v}$ and the unit vector $\hat{e}$ toward the observer, and the other ($\sigma_2$) representing an emission vector lying in this plane. These cross sections are functions of the electrons initial velocity $v$, the energy $E$ of the emitted photon, and the angle $\chi$ between the vectors $\hat{v}$ and $\hat{e}$; their expressions can be found in Gluckstern and Hull (1953) and will be adopted here with the appropriate Coulomb correction factor (Elwert 1939; Elwert and Haug 1971).

The geometry of the emission process is shown in Figure 2. The electron's unit velocity vector $\hat{v}$ is in the direction $(\theta, \phi)$ referred to the toroidal axis of the loop at the point in question, with $\theta = \pi$ corresponding to increasing $|s|$. It is clear from the figure that the angle $\chi$ is given by

$$\cos \chi = \cos \theta \cos \beta - \sin \theta \sin \beta \cos (\delta - \Phi),$$

where $\beta$ and $\delta$ readily follow from consideration of the spherical triangles $ZS'V$.

Due to the continual change of the electron's velocity vector as a result of spiralling along the guiding magnetic field lines, it is necessary, in order to obtain the spatially integrated polarization and directivity, to define the emission cross sections relative to a fixed plane, which is here taken as the ($\hat{e}, \hat{e}$) plane. We thus require the introduction of two effective polarizing cross sections $\sigma_{p1}$ and $\sigma_{p2}$ given by (see Haug 1972)

$$\sigma_{p1} = (\sigma_1 - \sigma_2) \cos 2\phi;$$

$$\sigma_{p2} = (\sigma_1 - \sigma_2) \sin 2\phi,$$

where $\phi$ is the angle between the ($\hat{e}, \hat{e}$) and ($\hat{z}, \hat{e}$) planes, as shown in Figure 2 (note that the total bremsstrahlung cross section $\sigma_t = \sigma_1 + \sigma_2$ is an invariant). Figure 2 shows that

$$\cos \phi = \frac{\cos \beta - \cos \theta \cos \chi}{\sin \theta \sin \chi};$$

note that expressions (9) and (11) reduce, in the case of a purely vertical geometry ($\alpha = \pi/2$), to those of Brown (1972).

We are now in a position to evaluate the polarization and directivity of the X-ray bremsstrahlung from our model. The (optically thin) radiation intensity (photons cm$^{-2}$ s$^{-1}$ sr$^{-1}$ at distance $A$) at photon energy $E$ due to
Fig. 2.—Geometry of the bremsstrahlung collision. O is the point of the collision (not to be confused with the O of Fig. 1) and is the center of the sphere shown. The line OS' represents the tangent to the loop, in its toroidal plane, at the point (coordinate a) in question; the Z-axis corresponds with that in Figure 1, and the unit vector $\hat{f}$ shows the direction of increasing $|\mu|$ (i.e., of the heat flux $Q$); thus $\overline{SZ} = \pi/2 \pm a$ (see Fig. 1). The electron, with unit velocity $\hat{v}$ as shown, produces a photon which travels toward the observer along the unit vector $\hat{f}$, at angle $\theta$ to $\hat{f}$. E and $V$ are the intersections of $\hat{f}$ and $\hat{t}$ with the sphere. Quantities $i$ and $\Phi$ are as in Figure 1; $\theta$ and $\phi$ are the polar coordinates of the electron's velocity vector relative to $\hat{S}'$. $\beta$ and $\delta$ are auxiliary quantities, while $\psi$ (the angle between the $(\hat{f}, \hat{t})$ and $(\hat{f}, \hat{e})$ planes) is used in evaluating the effective polarizing cross-sections $\sigma_{p1}$ and $\sigma_{p2}$ (see eqs. [9] and [10]).

electrons within a length $ds$ of a 1 cm$^2$ toroidal column of the loop, with velocity vector having magnitude in the range $v$ to $v + dv$ and direction with solid angle $d\Omega$ centered on polar coordinates ($\theta, \phi$) is

$$d^3I_{t,p1,p2}(\epsilon, i, \Phi) = \frac{n_e^2}{A^3} f_d(v, \theta, s; T_0, s_0, u)\alpha_{t,p1,p2}(v, \epsilon, \chi)\delta^3d\Omega dv ds,$$

where the subscripts $t$, $p1$, and $p2$ refer to total intensity, and intensities polarized in, and perpendicular to, the $(\hat{f}, \hat{e})$ plane respectively. Setting $d\Omega = \sin \theta d\theta d\phi = -\mu d\mu d\phi$ (with $\mu = \cos \theta$) and integrating over all variables (note that the $s$ integration is performed over both sides of the arch: the different heat flux directions in each side reduces the net polarization compared to that produced by one side alone) gives

$$I_{t,p1,p2}(\epsilon, i, \Phi) = \frac{n_e^2}{A^3} \int_{-L}^{L} \int_{v_0(\epsilon)}^{v_1} f_d(v, \mu, s)\delta^3\int_{\phi=0}^{2\pi} \alpha_{t,p1,p2}(v, \epsilon, \chi)\delta\phi d\mu dv ds.$$

In the above expression the limits in the $v$ integral are the electron velocity corresponding to energy $\epsilon$, viz.,

$$v_0(\epsilon) = c \left(1 - \left(1 + \frac{\epsilon}{m_e c^2}\right)^{-1}\right)^{1/2},$$

where $c$ is the velocity of light, and the maximum velocity for which the electrons can be considered confined by the conduction fronts, viz.,

$$v_1 = 2.6v_0$$


In adopting this upper limit on $v$, we are making two assumptions. First, that on escaping, electrons of $v > v_1$ emit bremsstrahlung in the cool surroundings so inefficiently that it can be neglected compared to the thermal emission. Unless the thermal source density is very low, this will be true for $\epsilon < \frac{1}{4}m_e v_1^2$, at which typical polarization and directivity measurements are made, but the precipitating tail will dominate at higher $\epsilon$ (Brown, Craig, and Karpen 1980; Emslie and Vlahos 1980). Second, the Maxwellian tail at $v > v_1$ is entirely absent inside the thermal source. Trial computations showed us that letting $v_1 \rightarrow c$ did not in any case significantly modify the polarization and directivity results, due to the steep falloff of the Maxwellian tail. Thus our use of $v_1 \approx 2.6v_0$ is justified unless some mechanism exists to repopulate the $v > v_1$ regime so fast, compared to its rate of depopulation by escape, that it becomes overpopulated compared to a Maxwellian distribution. Such a repopulation process would in effect amount to an acceleration mechanism with a time scale that is short compared to an electron transit time, which seems unlikely. If this situation did exist, however, the emission might well be dominated by the escaping nonthermal...
tail, and we would in effect return to a thick target situation. What we have calculated here is thus an estimate of the polarization from the purely thermal part of the source.

The polarization vector $P$ and directivity $D$ of the emitted bremsstrahlung can now be evaluated from the expressions

$$
|P|(\epsilon, i, \Phi) = \left\{ \frac{[I_{p1}(\epsilon, i, \Phi)]^2 + [I_{p2}(\epsilon, i, \Phi)]^2}{I_{l}(\epsilon, i, \Phi)} \right\}^{1/2},
$$

$$
\Psi(\epsilon, i, \Phi) = \frac{1}{2} \tan^{-1}\left[ \frac{I_{p2}(\epsilon, i, \Phi)}{I_{p1}(\epsilon, i, \Phi)} \right],
$$

and

$$
D(\epsilon, i, \Phi) = \frac{I_{l}(\epsilon, i, \Phi)}{I_{l}(\epsilon, 0, 0)}
$$

where $\Psi$ is the angle (measured counterclockwise) of the polarization vector relative to the $(\xi, \dot{e})$ plane, i.e., relative to the line joining the X-ray source to the center when projected on the solar disk. These expressions were evaluated numerically, using equation (13), with the results discussed in the next section.

### IV. RESULTS

The polarization vector $P$ and directivity $D$, given by equations (16) through (18), were first evaluated for the source parameter set of equations (7), with the results summarized in Figures 3 and 4.

Figure 3 shows the magnitude and direction of the polarization vector $P$ for a range of $i$- and $\Phi$-values (degrees), for photon energy $\epsilon = 50$ keV. The corresponding diagrams for different $\epsilon$ are similar, except that the scale of the diagram is altered: the degree of polarization corresponding to the reference vector ($5\%$ in Fig. 3) is $\sim 3.8\%$ for $\epsilon = 25$ keV and $\sim 6.0\%$ for $\epsilon = 100$ keV. Note that the polarization angle $\Psi$ is either $0^\circ$ or $90^\circ$ for all $i$ in the case $\Phi = 90^\circ$; this is because of the longitudinal symmetry of the source when viewed at right angles to its length. Note further that $P$ in Figure 3 is the intrinsic polarization of the emitted bremsstrahlung; photospheric backscatter (Santangelo, Horstman, and Horstman-Moretti 1973) is neglected in the calculations. Comparisons are thus more realistically made with the nonthermal models of Brown (1972), Petrosian (1973), and McClymont (1976), which also neglect this effect. A rough idea of how backscatter affects the results in Figure 3 is provided by noting that the source polarization (Fig. 3) and directivity (Fig. 4) are small; thus the excess polarization introduced by the photospheric backscatter can be taken from the results of Hénoux (1975) and Bai and Ramaty (1978) for an isotropic source. This introduces a further polarization of some $3\%$ (for the relevant energy range), oriented along the line

\[ \Phi = 90^\circ. \]

![Fig. 3.—Variation of the polarization vector $P$ with aspect angle $(i, \Phi)$, for photon energy $\epsilon = 50$ keV, toroidal source geometry, and source model parameters given by equation (7). The results are presented at $15^\circ$ intervals; due to the symmetry in the source when $u = 0$, only first quadrant values of $i$ and $\Phi$ are necessary. The magnitude of the polarization is obtained by comparison with the $5\%$ reference vector at the top center, while the orientation angle $\Psi$ is measured with respect to the $i$-axis, as also shown top center. For the variation of $P(i, \Phi)$ with different photon energies, source geometries and source parameters, refer to the text (§ IV).](image-url)
Fig. 4.—Isophotes in the \((i, \Phi)\) plane for photon energy \(\epsilon = 50\) keV, toroidal geometry, and source model parameters given by equations (7). The contours are normalized to the \(i = 0\) case.

\(\Psi = 0\); this, when added vectorially to the values shown in Figure 3 [noting that, due to the almost cylindrical nature of the source (see Fig. 1, eqs. [7]), \(P(\pi - i, \Phi) \approx P(i, \Phi)\)] gives the approximate net polarization vector.

Figure 4 shows the isophotes in the \((i, \Phi)\) plane; a slight darkening of sources with increasing viewing angle to the vertical is indicated. In the case where Figure 1 is a true indication of the source geometry (i.e., the source is at the top of a coronal loop), this corresponds to a limb darkening of sources. This is in qualitative agreement with the results of Brown (1972) and Petrosian (1973) for the nonthermal thick-target model, since the source in our model is almost horizontal (note [eq. (7)] that \(L \ll R\)), whereas in thick-target models it is almost vertical, since the densest part of the source (and hence the region of maximum bremsstrahlung production) is near the base of the loop. However, if the energy release is not at the top of the arch, or if there are a number of such releases occurring simultaneously (see Spicer 1977), then the angles \(i\) and \(\Phi\) no longer have a simple significance, and care must therefore be taken in interpreting results in these cases.

Both Figures 3 and 4 refer to the semitoroidal geometry of Figure 1. Since, as mentioned in § II, the geometry of the energy release site is not entirely clear, a few remarks on the effect of varying source geometry are appropriate. In particular, for a straight cylindrical source (\(L/R \rightarrow 0\)), we found the following differences to our results:

(i) The polarization increased somewhat, due to the greater anisotropy of the electron velocity distribution. Using \(a = 0\) (as opposed to \(a = s/R\)) in the geometrical calculations of § III yielded polarizations higher by some 15-20\% (relative percentage; the amount depends slightly on \(i\) and \(\Phi\)) than shown in Figure 3.

(ii) The polarization angle \(\Psi\) was not significantly altered, except near \(\Phi = 90^\circ\), where it remained constant (at \(0^\circ\)) for all values of \(i\); this is due to the axial (in addition to the aforementioned longitudinal) symmetry of such a source viewed from that particular aspect angle.

(iii) The directivity \(D\) (Fig. 4) was almost unaffected.

In addition, we investigated the effects of varying the source parameter set (7). The following trends were found:

1. An increase in \(T_0\), or decrease in \(s_0\), increased \(|P|\) and caused greater variation in \(D\). This is entirely reasonable, based on the dependence of the electron phase-space distribution anisotropy (eq. [3]) on \(Q\) and thus on \(T_0\) and \(s_0\) (eq. [2]).

2. Decreasing the dimensionless ratio \(L/R\) (for the case of toroidal geometry) similarly increased \(|P|\) and slightly increased the variation in \(D\). This is caused by the source more clearly approaching the purely horizontal geometry discussed earlier.

3. The results were almost independent of \(n_e\). This is clear upon inspection of equation (3): the dependence of \(f_e(v)\) on \(n_e\) is important only when \(Q \approx Q_{MB}\) (eq. [4]), i.e., near the ends of the source; thus the dependence of the total emission on \(n_e\) is small.

4. Increasing the drift velocity \(|u|\) also increased \(|P|\) and \(\Delta D\). However, bearing in mind the requirement (5), it was found that any acceptable \(u\) did not change the results measurably (see remarks in § II). This is because it is the higher energy electrons which produce most noticeable polarization and for such electrons, their velocity \(v \gg u\) (\(\leq v_{th}\) [43]).

In Figure 5 the total bremsstrahlung \(I\) is plotted as a function of photon energy \(\epsilon\) and for a number of aspect angles. The results clearly demonstrate that the source X-ray spectrum is insensitive to the precise \((i, \Phi)\) location of the observer (and therefore of the source location on the solar disk). This follows directly from the fact that \(D \approx 1\) for all \(i, \Phi\) (Fig. 4). It is therefore evident that directional effects are unimportant in evaluating the total bremsstrahlung emitted by such sources (see a similar result for the nonthermal case, also neglecting photospheric
Fig. 5.—Bremsstrahlung energy spectrum at various aspect angles (i, \Phi) (see Fig. 1) for the source with parameters given by equations (7) (see also Fig. 3). The best power law (index \gamma) and isothermal (temperature T) fits to the data are shown dashed.

backscatter [Brown 1972]). The energy spectrum has a best fit power law of I \sim \varepsilon^{-2.7} at lower (~25 keV) energies, steepening to \varepsilon^{-3.3} at higher (~100 keV) ones; it can be reasonably well fitted with a power law with spectral index \gamma = 2.92, in agreement with a number of observations (Hooy, Brown, and van Beek 1976; note that, of course, the exact value of \gamma depends on T_0 and s_0); but it is also adequately represented, within a factor of 2, by an isothermal source fit with T = 2.2 \times 10^{8} K (see Elcan 1978). These fits are superposed on the results of Figure 5.

This completes our discussion of the calculation results. Their significance for solar flare modeling will be discussed in the next section.

V. DISCUSSION

Considering firstly the directivity D (Fig. 4), we see that the results show a variation of intensity \Delta D = \Delta I / I_0 of some 10% over the range of possible aspect angles. Drake (1971), Kane (1974), and Pizzichini, Spizzichino, and Vespignani (1974) have found that when burst occurrences are compensated for the central meridian distance (CMD) dependence of the associated H\alpha flare intensities, no significant evidence for substantial directivity of hard X-rays exists. (This is in contrast to the earlier work of Ohki [1969] and Pintér [1969] who, however, neglected to compensate for this H\alpha CMD dependence.) Most recently, Datlowe et al. (1977), in a statistical survey of 222 OSO 7 20 keV bursts, quote center to limb variations in burst intensity of less than 1%, although this figure is subject to considerable uncertainty at high confidence levels (at the 95% confidence level, the degree of variation quoted is \approx [1 \pm 40]%). Even allowing for photospheric backscatter (Bai and Ramaty 1978), the limb darkening still falls within this large range of observational error. It is thus clear that in order to quantitatively test the predictions of the modeling herein with those of nonthermal models (e.g., Brown 1972; Petrosian 1973; Langer and Petrosian 1977; Bai and Ramaty 1978), a much larger sample of events (or, even better, a set of stereoscopic observations) is needed.

Now turning our attention to the polarization vector P, there are two significant features of Figure 3. (i) There is a spread in orientation angles \Psi (see nonthermal electron beam models which predict [Brown 1972; Petrosian 1973; Langer and Petrosian 1977; Bai and Ramaty 1978] \Psi = 0 for energies \varepsilon < 250 keV, such as are being considered here). The results for a nonthermal trap (McClymont 1976) are, however, more comparable with ours. (ii) The polarization |P| can indeed attain several percent for suitable orientations of the source on the solar disk [note that |P| (i = 0) = 0, since no unique (z, \varepsilon) reference plane exists for i = 0].

There is some doubt as to the reliability of observations of the polarization of solar hard X-ray bursts. Early measurements (Tindo et al. 1970, 1972a, b; Tindo, Mandel'stam, and Shurygin 1973) indicated substantial polarization in excess of 10% in some events, with the polarization vector apparently oriented along the solar disk radius (\Psi = 0) in all relevant cases (Tindo et al. 1972a). Subsequently, Brown, McClymont, and McLean (1974) pointed out that Tindo et al. had not taken into account the photospheric backscatter and had unjustifiably assumed a zero intrinsic X-ray polarization in the decay phase of the flare. However (see Hénoux 1975; Mandel'stam, Beigman, and Tindo 1975; Langer and Petrosian 1977; Bai and Ramaty 1978), Brown, McClymont, and McLean (1974) had overestimated this problem, and later measurements by the Intercosmos 7 satellite (Tindo, Shurygin, and Steffen 1976) corrected for this effect by having detectors mounted on a rotatable drum, and arrived at net (intrinsic) |P| values of under 2%, ± 1% for the flares studied. We thus see that the model results of Figure 3 appear to be consistent at least with the most recently published observations of polarization in hard X-ray events.
In conclusion, our modeling, although rather simplistic, has clearly demonstrated that current observations of the polarization and directivity of hard X-ray bursts in solar flares are quite compatible with a thermal source. Such a model clearly requires further development, taking into account the spatial location of the energy release (or releases), and the temporal dependence of the source geometry (see the motion of the conduction fronts; Brown, Melrose, and Spicer 1979). However, it is also clear that more accurate observations of P and D are necessary before a convincing discrimination between thermal and nonthermal hard X-ray models of solar flare hard X-ray bursts can be made on the basis at such observations.

We thank R. W. Noyes, R. Rosner, and D. S. Spicer for comments on the manuscript. The paper has also benefited considerably from comments by the referee. During the course of this work, A. G. E. was supported by NAS 5-3949.

REFERENCES


John C. Brown: Department of Astronomy, University of Glasgow, Glasgow G12 8QQ, Scotland, England
A. Gordon Emslie: Institute for Plasma Research, Stanford University, Via Crespi, Stanford, CA 94305

© American Astronomical Society • Provided by the NASA Astrophysics Data System