A Fourier method for determining stellar rotational velocities

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Power-spectrum analysis of a stellar spectrum is a natural way to extract all the information on the rotation of the star that its spectrum conveys. As an illustration, the method is implemented for stars whose rotation is known.

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1. INTRODUCTION

The classical method of determining the axial rotational velocity \( v \sin i \) of a star involves analysis of an individual line profile. This technique has the shortcoming that the information on the rotation conveyed by all the other lines and blends as well as other features of the spectrum remains unused. Not long ago Danziger and Faber proposed that \( v \sin i \) be determined from the distortion of a whole section of the spectrum. In this letter we wish to demonstrate that by passing from a discussion of the spectrum itself to an analysis of its Fourier transform, one can spatially combine all the distortions introduced by the rotation, thereby facilitating the analysis and the determination of \( v \sin i \).

2. BASIC RELATIONS

The equation relating the profile \( \tilde{F} \) of a line with central frequency \( \nu_0 \) in the spectrum of a rotating star and the profile \( R \) of the same line in the spectrum of a non-rotating star has the form

\[
\tilde{F}(\nu - \nu_0) = \frac{1}{4} R(\nu - \nu_0 - \frac{v \sin i}{c} x) A(x) \ dx.
\]

where \( v \) is the equatorial rotational velocity of the star, \( i \) is the angle between the rotation axis and the line of sight, and \( A \) is known as the rotational broadening function. If we adopt a limb-darkening law of the form \( I(\theta) = L_0(1 + \beta \cos \theta) \), where \( \theta \) is the angular distance from the center of the disk and \( \beta \) is the limb-darkening coefficient, then

\[
A(x) = \left[ 1 + \frac{2}{3} \sqrt{1 - x^2} + \frac{\beta}{3}(1 - x^2) \right] (1 + \frac{2}{3} \beta)^{-1}.
\]

Equation (1) holds not only for an individual line but also for a small segment of the spectrum in the vicinity of \( \nu_0 \).

One can also convert from frequencies to wavelengths, and from profiles to intensities. In this fashion we obtain

\[
\tilde{I}(\lambda - \lambda_0) = \frac{1}{4} \int \tilde{I}(\lambda - \lambda_0 - \frac{v \sin i}{c} x) A(x) \ dx.
\]

The right-hand member is a convolution of the two functions \( I \) and \( A \). Hence by applying the Fourier transformation we have

\[
\tilde{I} (\tilde{\nu}) = \tilde{I} (\tilde{\nu}) \tilde{A} (\tilde{\nu}_0 \frac{v \sin i}{c}),
\]

where \( \tilde{I}, \tilde{I}, \tilde{A} \) are the corresponding Fourier transforms. If Eq. (2) holds, then

\[
A(x) = 2 \left( 1 + \frac{2}{3} \beta \right)^{-1} \left[ \frac{\Delta f(\nu)}{\nu^2} - \beta \left( \cos \nu \sin \nu \right) \right],
\]

where \( \Delta f \) is the Bessel function. In Eq. (4), all the information on the rotational distortion of the spectrum is contained in the factor \( \tilde{A} \). The rotation acts as a low-frequency filter, with high frequencies being suppressed more strongly as \( v \sin i \) increases.

Upon multiplying Eq. (4) by its complex conjugate and taking logarithms, we obtain

\[
\log \tilde{F} (\tilde{\nu}) = \log P (\tilde{\nu}) + \log \tilde{A}^2 (\tilde{\nu}_0 \frac{v \sin i}{c}),
\]

where \( P = \frac{\tilde{F} \tilde{A}}{\tilde{A}^2} \) and \( P = \tilde{I} \tilde{I}^* \) represent the power spectra of \( \tilde{F} \) and \( \tilde{I} \), respectively. Equation (6) is the working equation for our proposed method. It enables one to determine the rotational velocity by comparing the power spectrum of a segment of the spectrum of a star whose rotation is unknown with a family of power spectra \( P \) computed for

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various $v \sin i$ on the basis of the power spectrum of a nonrotating star having the same spectral type.

We would emphasize that the amplitude $I$ of the signal undistorted by rotation and its power spectrum $P$ convey both information on the stellar spectrum proper and various types of noise: instrumental profile effects, photographic emulsion noise, and so on. However, if the spectra are secured under standard conditions — with an identical spectrograph, with one particular emulsion, and the like — then these distortions will all enter in the same way and need not be specially eliminated.

3. OBSERVATIONS AND DIGITAL REDUCTION

We have tested our method on stars having known rotational velocities. Since the advantages of the method are enhanced when numerous lines occur in the part of the spectrum investigated, we have selected from Fukuda and Uesugi's catalog\(^4\) two stars of the same comparatively late spectral type: HR 7955 ($v \sin i = 0$, spectrum F8 IV-V) and $\delta$ Pegasi ($v \sin i = 79$ km/sec, spectrum F8 IV).

Three spectrograms of each star in the 4200–4600 Å wavelength range were obtained on 103a-O plates at 15-Å/mm dispersion with the 60-inch reflector of the Crimean Astrophysical Observatory in the autumn of 1976.

The spectra were discretized with the digital microphotometer of the Tartu Observatory. Values of the photographic density at intervals of 4 $\mu$ (or 0.06 Å) on the plate were punched on tape. Subsequent digital data processing was carried out at the Leningrad University Computation Center by Brault and White's technique.\(^5\) The basic steps in the reduction are: a) conversion of density to intensity; b) removal of the mean and trends, with the observational data approximated by a polynomial of degree $\leq 8$ and the corresponding values of this polynomial subtracted from the data; c) smoothing at the edges, with the data sequence being multiplied by a weighting function equal to 1 nearly everywhere but falling off smoothly to 0 at the ends; d) calculation of the power spectrum, using the rapid Fourier transformation; e) smoothing of the resulting power spectra by convolving them with a rectangular window, that is, taking a sliding mean.

4. RESULTS

Figure 1 displays one of the intermediate results: the smoothed power spectrum of a star (HR 7955) without appreciable rotation, as determined from the spectral interval 4360–4600 Å. In the frequency band from 10 to 30 Å$^{-1}$ the power spectrum represents plate noise, and above 30 Å$^{-1}$ frequency only white noise is present. Thus on our plates the working frequency band extends from 0 to 10 Å$^{-1}$. Power spectra of the same character have been obtained for late-type stars by Kipper and Sitkina.\(^6\)

In Fig. 2 we show power spectra for identical wavelength regions each 240 Å long in the nonrotating star HR 7955 (curve 1) and the comparatively rapidly rotating $\delta$ Peg (curves 2). Notice that rotation actually does cut off some of the signal power at high frequencies ($f \geq 3$ Å$^{-1}$). Also included in the figure are power spectra computed from Eq. (6) on the basis of the power spectrum of HR 7955 for values of $v \sin i$ equal to 70 and 90 km/sec (curves 3 and 4, respectively).

Although it may seem rather laborious, our method has proved to be quite effective. It is indeed a cumbersome task to compile and debug the digital computer programs, but once this is done we have a simple and practical tool for quickly determining stellar rotational velocities. An analogous Fourier method has in fact been applied successfully to determine the velocity dispersion of globular-cluster stars from the broadening of their spectral lines.\(^7\)

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Solar-wind velocity measurement by radio observations of phenomena propagated through the interplanetary plasma

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The velocity of the solar wind can be measured with a single radio telescope by observing the interplanetary scintillation of numerous radio sources. The mean solar-wind velocity in August–October 1976 is found to have been ≈400 km/sec at low (φ<10°) heliographic latitudes and ≈850 km/sec at high (φ=30–70°) latitudes. The wind velocity and the interplanetary plasma density have a reciprocal relationship.

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The velocities at which small-scale electron-density irregularities travel through the interplanetary plasma have now been measured for 10 years by synchronized monitoring of scintillations with several radio telescopes spaced 100–200 km apart. As at least three radio telescopes are needed for this purpose, comparatively simple antennas have been employed, and as a consequence scintillations only of the strongest radio sources have been observed.

For detailed analysis of the inhomogeneous interplanetary plasma, one should observe the scintillation of tens or even hundreds of sources. Measurements of this kind have been made with the BSA radio telescope, the large aperture-synthesis antenna array of the Lebedev Physics Institute, at a wavelength λ = 2.95 m. Because of the high sensitivity of the BSA and the capability of rapidly adjusting its beam, scintillations of about 150 radio sources can be observed in a single day.

From data acquired in 1975–1976 we have compiled maps or radio images of the interplanetary plasma. Analysis of the maps has shown that substantial changes in the large-scale structure of the interplanetary plasma occur on time scales comparable with, or much shorter than, a day. This type of variability in the large-scale structure arises primarily from disturbances propagated through the interplanetary plasma.

Estimates have also been made of the velocity at which individual disturbances are propagated, and good agreement has been found with the results of direct measurements. The velocity can be measured by continuous observation of the general state of the interplanetary plasma. For this purpose one can "photograph" maps of the plasma pattern so that the intervals between successive "pictures" is ≈1 h. Such a program can be carried out with a radio telescope having a multiple antenna beam (in two dimensions). To determine the velocity V0 of the solar wind, it would be sufficient to track details in the interplanetary plasma structure and to establish the distances d covered by them and the corresponding time intervals t. Unfortunately, we are not yet capable of making such continuous observations. Maps of this kind are now built up once per day. With measurements as widely spaced as this, some of the information concerning the changes in the interplanetary plasma structure is naturally lost.

Let us consider how the solar-wind velocity can be measured on the basis of actual maps obtained during a two-month period, Aug 16–Oct 17. The maps show the distortion of the index of the plane of projection, corresponding to the space in the vicinity of the sun through which the scintillating sources are observed. To determine the solar-wind velocity vector one must select three closely spaced local regions in appropriate places on the map. We shall lighten the task by making some simplifying assumptions.

![Diagram](image)

**FIG. 1.** a) Position of scintillating radio sources in an equatorial coordinate system. Solid curve, segment of the ecliptic corresponding to the positions of the sun during the period of measurement. b) Schematic diagram for determining the shift t in the maximum of the mutual correlation function.

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