THE GYROSYNCHROTRON EMISSION FROM QUASI-THERMAL ELECTRONS
AND APPLICATIONS TO SOLAR FLARES

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ABSTRACT

We present theoretical results on the gyrosynchrotron radiation from electrons with a
Maxwellian energy distribution. We review the analytical expressions for the gyromagnetic
absorption coefficient and find two which cover the range of interest for microwave emission
from solar flares, i.e., frequencies $\omega \sim 1000\omega_e$ and temperatures $T_e \sim 10^9$ K. Numerical
calculations are used to check the analytic expressions and to derive simplified empirical
formulae which relate the observable characteristics of the radiation to the temperature and
magnetic field in the source.

We apply the results to the sources of impulsive microwave and hard X-ray bursts from
solar flares. For an isothermal source the theory predicts a microwave spectrum where the
flux density rises as $\omega^2$ at low frequencies, maximizes as some frequency $f_{\text{peak}}$, and falls very
rapidly thereafter; this shape fits the observed spectra qualitatively. The optical depth $\tau$ of
the source varies rapidly with $\omega$, with $\tau = 1$ at $f \approx f_{\text{peak}}$. For $T_e \gtrsim 10^8$ K we derive the relation
$f_{\text{peak}} \sim T_e^{0.7} B$, which allows a direct estimate of the magnetic field $B$ in the impulsive burst
source if the temperature is known—for instance, from hard X-ray observations. For the
impulsive burst of 1972 May 18, reported by Hoyng and Stevens, we find that the microwave
and hard X-ray data are well fitted by a model source with $T_e \approx 2.3 \times 10^8$ K, $B \approx 370$ gauss,
$n_e \approx 2 \times 10^{10}$ cm$^{-3}$, and scale length $L \approx 8600$ km.

Subject headings: Sun: flares — Sun: radio radiation — Sun: X-rays — synchrotron radiation

I. INTRODUCTION

Our purpose in this paper is to present theoretical results on the gyrosynchrotron radiation from electrons
with a Maxwellian energy distribution. It is well known that gyromagnetic emission from nonrelativistic electrons
is concentrated at the electron gyrofrequency and its first few harmonics (e.g., Bekefi 1966, p. 203), and that
gyromagnetic emission from ultrarelativistic particles ($\gamma \sin \alpha \gg 1$, $\gamma$ = Lorentz factor, $\alpha$ = pitch angle) is concentrated
at very high harmonics [$s \approx (\gamma \sin \alpha)^0$] (e.g., Ginzburg and Syrovatskii 1965). In this paper we are
concerned with an application in which emission occurs at harmonics between about 10 and 100, and in which
we assume the electron distribution to be a high-temperature Maxwellian ($10^8-10^9$ K). A general analytic expression
for gyromagnetic emission and absorption by a relativistic Maxwellian distribution was derived by Trubnikov
(1958) (see also Drummond and Rosenbluth 1963 and Shkarovsky 1966). Trubnikov's general result is too
cumbersome to be of direct use in many applications. Here, in § II, we derive an approximation to the absorption
coefficient in the nonrelativistic limit and explore two approximations due to Trubnikov. Although these approximations
are useful for checking numerical calculations, they remain too cumbersome for many purposes.

Consequently, we have used our numerical results to derive simple empirical relations; these are presented in
§ III.

Our work is motivated by the problem of explaining impulsive microwave bursts from the Sun. Because of
the detailed correspondence in the time variations of impulsive microwave and hard X-ray bursts, it is believed
that both result from the same electron distribution, with the microwaves and X-rays emitted mainly by electrons
with $E \gtrsim 100$ keV and $E \lesssim 100$ keV, respectively (Holt and Ramaty 1969; Takakura 1972; Ramaty and
Petrosian 1972). In existing treatments the common assumptions are that the electrons are nonthermal, with a
power-law energy distribution, and that the microwaves are due to gyrosynchrotron emission by mildly relativistic
electrons (e.g., Takakura 1960a, b, 1967); the emission and absorption coefficients are calculated numerically
(e.g., Ramaty 1969). The hard X-rays are commonly thought to be due to bremsstrahlung when the electrons
penetrate a thick target. Characteristically, the models require a total of $10^{34}-10^{39}$ electrons with $E > 10$ keV
with a power-law index $\alpha \approx 3-5$. 

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Our interest in a thermal interpretation of impulsive microwave bursts follows from the renewed interest in the idea (Chubb, Kreplin, and Friedman 1966) that the hard X-rays may result from electrons which are heated in bulk rather than being accelerated to form a nonthermal tail. Crannell et al. (1978) have provided recent evidence for this interpretation, and Mätzler et al. (1978) have argued for it (see also Ramaty 1979). Such “bulk-energization” of electrons to form a quasi-thermal distribution with $T \gtrsim 10^8$ K has several attractive features from the point of view of both theory and observations:

1) The amount of energy associated with the impulsive bursts need not be a substantial fraction of the total flare energy, contrary to the implications of the usual interpretation. In thermal models the efficiency of X-ray production can be increased by a factor $\gtrsim 10$ compared with nonthermal models (Smith and Lilliequist 1979) because the major mechanism of energy loss in nonthermal, thick-target models—namely, heating of cold, ambient electrons by fast electrons—is absent.

2) The acceleration of large numbers of electrons to form a strongly nonthermal distribution is not required. Few or no electrons need be accelerated into a high-energy tail; at most, a small number ($<1\%$) may be needed as “seeds” for second-stage acceleration and to provide interplanetary electron bursts (Lin 1974). Heating seems to be more favorable than systematic acceleration; Smith and Lilliequist (1979) estimate that under coronal conditions only $\sim 10\%$ of energy released goes into systematic acceleration.

3) The sources can easily be small ($\lesssim 5''$) and dense ($\sim 10^{10}$ cm$^{-3}$), consistent with the inferences from some X-ray data (e.g., Kahler, Petrasso, and Kane 1976), radio data (Crannell et al. 1978), and the few radio observations of very small sources (Kundu, Velusamy, and Becker 1974; Alissandrakis and Kundu 1978; Marsh, Zirin, and Hurford 1979).

4) The observed time scales of impulsive bursts, $\sim 1$–10 s (cf. Hoyng, Brown, and van Beek 1976), can be reasonably explained by postulating that the hot regions are bounded by ion-acoustic conduction fronts which move apart at about the ion sound speed (Brown, Melrose, and Spicer 1979). Thus the most serious difficulty of thermal models, viz., the excessively rapid heat conduction—and excessively small time scales—is avoided. (The implications of the fine structure ($\sim 10$ ms) in decimeter bursts [e.g., Slottje 1978] remain to be investigated; they may give information on the basic energy release processes.)

An important feature of the spectra of some impulsive microwave bursts is the steep, sometimes exponential, decrease in flux density at high frequencies, i.e., at $f > f_{\text{peak}}$. It is difficult to assess the prevalence of such cutoffs because the number of published spectra of impulsive bursts is very limited (most published spectra seem to be for great outbursts, i.e., microwave type IV’s). Examples of impulsive burst spectra have been given by Hachenberg and Wallis (1961), Takakura and Kai (1966), Holt and Ramaty (1969), and Hoyng and Stevens (1973) (see also Hoyng 1975). Commonly the flux density increases with frequency from $\sim 1$ GHz to a maximum at $f_{\text{peak}} \sim 10$ GHz and then decreases. Early in the burst, when the X-ray spectrum is hardest, the microwave peak is at its highest frequency, i.e., $f_{\text{peak}} \gtrsim 10$ GHz, and the high-frequency cutoff is steepest (e.g., Fig. 1b).

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**Fig. 1.**—(a) X-ray spectra at four different times during the impulsive burst of 1972 May 18 obtained with the ESRO TD1A spacecraft (reproduced from Hoyng and Stevens 1973). (b) Microwave spectra at different times during the event. The data are mostly from the Sagamore Hill Radio Observatory (reproduced from Hoyng and Stevens 1973).
This steep decrease in flux is explained in the usual (nonthermal) models only by invoking a sharp high-energy cutoff in the electron energy spectrum which mimics a Maxwellian. Electron bursts in interplanetary space show such an exponential cutoff at high energies (Lin 1970).

As noted originally by Hachenberg and Wallis (1961), impulsive microwave spectra with their positive slopes (often of index \( a \approx 2 \)) at \( f < f_{\text{peak}} \) and steep negative slopes at \( f > f_{\text{peak}} \) are suggestive of sources where the electrons have a quasi-thermal distribution, and which are optically thick at \( f < f_{\text{peak}} \) and optically thin at \( f > f_{\text{peak}} \). Hachenberg and Wallis investigated whether bremsstrahlung from a hot plasma could produce the observed flux densities \( S \) of \( \sim 10 \) to \( \sim 1000 \) solar flux units (SFU) (1 SFU = \( 10^{-22} \) W m\(^{-2}\) Hz\(^{-1}\)).

In this paper we investigate the possibility that impulsive bursts result from gyrosynchrotron emission from quasi-thermal electrons. We assume that the flare energy is manifested mainly in very rapid, bulk heating of all electrons rather than in systematic acceleration of only some of them. We consider the radio emission from a source which is small (a few thousand kilometers in extent), dense \( (n_e \approx 10^6 \) cm\(^{-3}\)\), and hot \( (T_e \approx 10^8 \) K\). We emphasize that our "thermal" interpretation of impulsive bursts does not imply thermal equilibrium. We do not believe that the electron energy distribution will be strictly Maxwellian or that the electron and ion temperatures will be equal. The important characteristic of our thermal model is that there is a single component of electrons with a high mean energy and an exponential high-energy tail.

The reason that bremsstrahlung from a small thermal source cannot produce the observed flux densities and the implied brightness temperatures \( T_b \approx 10^8 \) K is that the opacity varies as \( \tau \sim T_e^{-3/2} \), resulting in optical depths \( \tau \ll 1 \) and \( T_b \ll T_e \) unless the density is extremely high \( \sim 10^{13} \) cm\(^{-3}\). However, in the presence of a magnetic field, magnetobremsstrahlung (gyrosynchrotron emission) can be much more efficient than field-free bremsstrahlung, and the required opacity can be attained. At the high harmonics \( (\omega/\Omega_e \sim 10 \) to \( 100) \) of interest here, gyromagnetic emission and absorption are dominated by higher-energy particles, and any distribution which varies exponentially at high energies, i.e., \( N(E) \propto \exp \left(-E/E_0\right) \), should be equivalent to a Maxwellian at temperature \( T_e = E_0/k \) \( (E_0 = 8.6 \) keV corresponds to \( T_e = 10^8 \) K\).

In § II we give two analytic expressions for the gyroabsorption coefficient for electrons with a Maxwellian distribution and compare them with numerical computations. In § III we state some useful formulae, compare them with observations of microwave bursts, and derive the parameters of a quasi-thermal source from the combined radio and X-ray data of the burst of 1972 May 18.

II. GYROSYNCHROTRON RADIATION FROM QUASI-THERMAL ELECTRONS

The general characteristics of the electron distribution are as follows:
1. For a Maxwellian distribution with \( T_e \approx 10^8 \) K, the average electron energy is \( \gtrsim 10 \) keV and there are large numbers of electrons with energies up to about 100 keV.
2. Above about 100 keV, the distribution \( N(E) \) decreases exponentially.
3. For \( B \gtrsim 100 \) gauss the gyromagnetic frequency is \( f_\theta = \Omega_e/2\pi \approx 300 \) MHz. Thus the range of interest for microwave emission is \( \omega \sim 100 \), to \( \sim 1000 \) GHz.

Because most of the electrons are not highly relativistic, the Airy integral approximation, used to obtain the synchrotron formulae, is not valid. Thus the formulae for the emission and absorption coefficients involve sums over Bessel functions and their derivatives. However, as will now be shown, for a nonrelativistic Maxwellian distribution the power series expansion of the Bessel functions converges rapidly and only the leading terms need be retained. The sums may then be performed analytically.

a) Analytic Expressions for the Gyromagnetic Absorption Coefficient

Useful analytic expressions for the gyromagnetic absorption are available for (i) a nonrelativistic thermal plasma in which the wave properties are assumed to be given by the magnetoionic theory, and (ii) a mildly relativistic plasma for emission perpendicular to the field lines and with the wave properties those of transverse waves in vacuo.

i) Nonrelativistic Electrons in a Plasma

The gyromagnetic absorption coefficient, \( \gamma \), at the \( s \)th harmonic for nonrelativistic Maxwellian electrons at temperature \( T_e \) is (Sitenko and Stepanov 1956; Ginzburg and Zheleznyakov 1959; Melrose 1979b, p. 274)

\[
\gamma^s(s, \omega, \theta) = \left( \frac{\pi}{2} \right)^{1/2} \frac{\omega^2 A^s(s, \omega, \theta)}{\omega n_e^2 \beta_0 \cos \theta \sigma(\omega n_e)/\sigma \omega} \exp \left[ -\frac{(\omega - c\Omega_e)^2}{2\omega^2 n_e^2 \beta_0^2 \cos^2 \theta} \right],
\]

(1)

where \( \sigma \) denotes either of the two magnetoionic modes (\( \sigma = +1 \) for the \( \alpha \)-mode, \( \sigma = -1 \) for the \( \beta \)-mode), \( n_e(\omega) \) is the refractive index of that mode, \( \theta \) is the angle between the wave-normal direction and the magnetic field, \( \Omega_e \) and \( \omega_\theta \) are respectively the electron gyrofrequency and plasma frequency, and \( \beta_0 = (kT_e/m_e)^{1/2} \) with \( k = \) Boltzmann's constant. The polarization of the modes appears in \( A^s \) as described by Melrose and Sy (1972) and Melrose (1979a, p. 43).
The absorption coefficient (1) is sharply peaked around integral values of \( \omega/O_e \), a characteristic feature of the nonrelativistic approximation. It is convenient to define an averaged absorption coefficient at the \( s \)th harmonic by writing

\[
\gamma^a(s, \theta) = \int_{-\infty}^{\infty} \frac{d\omega}{\Omega_e} \gamma^a(s, \omega, \theta).
\]

The simplest useful approximation to \( \gamma^a(s, \theta) \) is one given by Zheleznyakov (1970, p. 454). It may be derived by inserting (1) in (2), retaining only the leading term in the power series expansion of the modified Bessel functions which appear in \( A^a \), making the quasi-circular approximation to the magnetoionic wave properties and evaluating the \( \omega \)-integral by the method of steepest descents. We find

\[
\gamma^a(s, \theta) = \frac{\pi \omega_p^2}{\Omega_e} \left[ 1 - 2\beta_0^2 \cos^2 \theta \left( 1 - \frac{9}{4} \frac{\omega_p^2}{s^2 \Omega_e^2} \right) \right] A^a(s, \omega_0, \theta),
\]

\[
A^a(s, \omega_0, \theta) = \frac{e^{-\lambda}}{4} \frac{s^2}{s!} \left( \frac{\lambda}{2} \right)^{s-1} \left( 1 - \frac{\sigma \sin^2 \theta}{2s \cos \theta} \right) \times \left( (1 - \frac{\sigma \cos \theta}{2})^2 - (1 - \frac{\sigma \cos \theta}{2}) \sin^2 \theta \left[ \frac{1}{s} + 2\beta_0^2 n^4 \cos \theta \right] \right),
\]

with

\[
\lambda = s^2 \beta_0^2 n^2 \sin^2 \theta (1 - 2\beta_0^2 n^4 \cos^2 \theta) \quad \text{and} \quad n = \left( 1 - \frac{\omega_p^2}{s^2 \Omega_e^2} \right)^{1/2}.
\]

To lowest order in \( \beta_0^2 \) and in \( 1/s \), equation (3) with (4) reduces to

\[
\gamma^a(s, \theta) = \frac{\pi \omega_p^2 s^2}{4 \Omega_e s!} \left( \frac{\beta_0^2 n^2 \sin^2 \theta}{s} \right)^{s-1} (1 - \sigma \cos \theta)^2,
\]

which is essentially Zheleznyakov's result. For \( \theta \) sufficiently close to \( \pi/2 \) the quasi-circular approximation breaks down and (5) does not apply. For \( \theta = \pi/2 \) the counterpart to (5) is

\[
\gamma^a(s, \pi/2) = \frac{\pi \omega_p^2 s^2}{2 \Omega_e s!} \left( \frac{\beta_0^2 n^2 \sin^2 \theta}{s} \right)^{s-1},
\]

\[
\gamma^a(s, \pi/2) = 0.
\]

The major limitations on the use of (5) and (6a, b) arise from the neglect of relativistic effects and from the range of validity of the expansion of the Bessel functions. The power series expansion, and hence (5), is expected to break down for \( s^2 \beta_0^2 > 2 \). Comparison of analytic and numerical results below confirms that (5) breaks down for \( s^2 \beta_0^2 \) between one and two.

ii) "Mildly Relativistic Electrons in Vacuo Emitting at \( \theta = \pi/2 \"

Relativistic effects were included for \( \theta = \pi/2 \) by Trubnikov (1958); subsequent discussions (e.g., Bekefi 1966, pp. 200–208) have been largely based on Trubnikov's results. For \( \theta = \pi/2 \), equation (1) implies zero line width in the nonrelativistic approximation. One of the important relativistic effects is the spread in gyrofrequencies \( \Omega_e/\gamma \) due to the range of the Lorentz factor \( \gamma \). Another is the fact that the peak in the absorption at the \( s \)th harmonic occurs at \( \omega/O_e < s \), with the value of \( \omega/O_e \) at the peak decreasing with increasing \( s \) (e.g., Bekefi 1966, p. 207). Trubnikov (1958, eqs. [3.14] and [3.16]) gives the following two approximate expressions for a mildly relativistic plasma and for \( \theta = \pi/2 \):

For

\[
\rho = \frac{9 \omega}{2 \Omega_e} \beta_0^2 \gg 1
\]

he finds

\[
\gamma^a(\omega, \pi/2) = \frac{\omega}{\Omega_e} \frac{3 \pi^{1/2}}{e \rho \rho} \exp \left\{ -\frac{1}{\beta_0^2} \left( \frac{\rho^{1/3}}{2} - \frac{9}{20\rho^{1/3}} \right) \right\},
\]

\[
\gamma^0(\omega, \pi/2) = \frac{\beta_0^2}{\rho^{1/3}} \gamma^a(\omega, \pi/2).
\]
and for $(\omega/\Omega_e)^2 \beta_0^2 \ll 1$ he finds

$$\gamma^s(\omega, \pi/2) = \omega_1^2 \left(\frac{\pi \Omega_e}{2 \omega}\right)^{1/2} \left(\frac{\epsilon \beta_0^2}{2 \Omega_e}\right)^{\omega/\Omega_e},$$

(9a)

$$\gamma^s(\omega, \pi/2) = \beta_0^2 \gamma^s(\omega, \pi/2).$$

(9b)

### iii) Comparison and Generalization

On making Stirling's approximation to $\sin^{1/2}$, (6a) becomes identical to (9) provided one rewrites $s$ as $\omega/\Omega_e$. Although $s$ is equal to $\omega/\Omega_e$ in the nonrelativistic approximation, this equality does not hold in the relativistic case because of the relativistic frequency shift. The equivalence of (6a) and (9) leads to the important conclusion that relativistic effects may be included in (6a) simply by writing $\omega/\Omega_e$ in place of $s$ and disregarding the nonrelativistic equality $s = \omega/\Omega_e$. It is reasonable to apply the same procedure for $\beta_0^2$ and to generalize (5) to

$$\gamma^s(s, \theta) = \omega_1^2 \left(\frac{\pi \Omega_e}{2 \omega}\right)^{1/2} \frac{1}{\beta_0^2 \sin^2 \theta} \left(\frac{\epsilon \beta_0^2 \omega \sin^2 \theta}{2 \Omega_e}\right)^{\omega/\Omega_e} (1 - \sigma |\cos \theta|)^2,$$

(10)

which should be valid for $(\omega/\Omega_e)^2 \beta_0^2 \sin^2 \theta \ll 2$.

As far as we are aware, the approximations (8) and (10) have not been compared with numerical results; we do so in the next section and find them to be accurate approximations in their stated ranges of validity. It would be desirable to generalize (8a, b) to arbitrary angles $\theta$, as (9a, b) are generalized by (10), but we have no theoretical basis on which to make such a generalization.

Empirically, replacing $\beta_0^2$ by $\beta_0^2 \sin \theta$ in (8a) seems to give a reasonably good fit over most of the range of validity.

### b) Comparison with Numerical Calculations

In this section we compare the foregoing analytical results with numerical computations. The numerical work is based on the equations for the gyrosynchrotron absorption coefficient of Eidman (1958) as given by Ramaty (1969), but including the correction pointed out by Trulsen and Fejer (1970). The sums over Bessel functions are simplified by use of the formulae of Wild and Hill (1971). Relativistic effects and the influence of the plasma (medium suppression, i.e., the Razin effect) are taken into account. The electron energy distribution $w(e, \Omega_e)$, where $\gamma$ is the Lorentz factor, is taken to be the Maxwellian

$$w(\gamma) = \frac{\pi}{2} \frac{2mc^2}{\pi kT_e} \gamma^{3/2} \exp \left[-\frac{(\gamma - 1)mc^2}{kT_e}\right].$$

Except for the normalizing factor, this equation is valid for all $T_e$; the form of the normalization used here is accurate for $kT_e \ll mc^2$, or $T_e \ll 6 \times 10^9$ K.

In Figure 2 we compare values of the absorption coefficient, $\kappa$, from the analytical results (8) and (10) with the numerical computations. (Note that $\kappa = \gamma/\epsilon$.) For (10) (Fig. 2a), the agreement is very good for $10^7 \leq T_e \leq 10^9$ K and $\omega/\Omega_e \lesssim 15$, i.e., for $(\omega/\Omega_e)^2 \beta_0^2 \sin^2 \theta < 2$. For (8) (Fig. 2b), the agreement is very good for $10^6 < T_e < 10^9$ K and $\omega/\Omega_e \gtrsim 10$, i.e., for $(9/2)\omega/\Omega_e \beta_0^2 < 1$. Note that in drawing our numerical curves we have averaged the absorption coefficient at low harmonics by eye whenever individual harmonics appeared. A definite averaging procedure, defined by (3), has been adopted for the analytic results. Therefore, some disagreement at low harmonics may be attributed to the different averaging procedures.

In Figure 3 we show the results of numerical computations of the absorption coefficient for temperatures up to $10^9$ K and frequencies $\omega$ up to $10^5 \Omega_e$. The reason for multiplying $\kappa$ by $B/n_e$ to make the results very nearly independent of $B$ or $n_e$; only for calculating the index of refraction were specific values of $B$ and $n_e$ required (we used $B = 300$ gauss and $n_e = 10^{16}$ cm$^{-3}$). Additional computations have shown that for fixed $n_e$, there are negligible changes in the quantity $k B/n_e$ when $B$ is in the range $100 < B < 300$ gauss; however, when $B$ is fixed and $n_e$ is lowered, say to $10^6$ cm$^{-3}$, $k B/n_e$ changes little for $\omega/\Omega_e \gtrsim 15$ but rises by up to a factor of 10 at $\omega/\Omega_e < 10$. This is because medium suppression, which mainly affects low frequencies, has little effect when the density is low.

Figure 3 shows curves for both $\theta = 45^\circ$ and $\theta = 75^\circ$. At low $\omega/\Omega_e$, the analytical result (eq. [10]) predicts that changing $\theta$ from $45^\circ$ to $75^\circ$ (an 87% increase in $\sin^2 \theta$) will be equivalent to an 87% increase of temperature. The numerical calculations are in qualitative but not quantitative agreement with this prediction, with the increase being less than predicted, going approximately as $\sin \theta$ (i.e., similar to what we find for eq. [8]).

### III. APPLICATIONS TO IMPULSIVE BURSTS

We now apply the results just derived to the sources of impulsive hard X-ray and microwave bursts from solar flares. These are taken to be small regions, possibly portions of magnetic loops. We assume that these have characteristics similar to those postulated at the Skylab Workshop on Solar Flares (Ramaty 1979): a scale size
Fig. 2.—The gyromagnetic absorption coefficient for the $x$-mode $\kappa_x$ (units: cm$^{-1}$) derived from the analytical results, eq. (10) (Fig. 2a) and eq. (8) (Fig. 2b), compared with numerical computations. The magnetic field was assumed to be inclined at $\theta = 45^\circ$ from the line of sight for Fig. 2a and $75^\circ$ for Fig. 2b. In the numerical work, the refractive index was calculated for a source with a plasma frequency $f_p = 90$ MHz ($n_e = 10^8$ cm$^{-3}$) and a gyromagnetic frequency $f_B = 280$ MHz ($B = 100$ gauss). The right and top scales show optical depth $\tau_x$ and microwave frequency respectively for such a source with a scale length $L = 2000$ km.

$L$ of a few thousand kilometers, a density $n_e \sim 10^{10}$ cm$^{-3}$, a magnetic field $B \sim 100$ gauss and an electron temperature $T_e \gtrsim 10^8$ K. In this paper our purpose is to test whether this model, in its simplest form, can fit the microwave spectra. We take a single, isothermal source with the same scale length in depth as in projected area. We consider only $x$-mode radiation, reserving a discussion of polarization to the end. We consider only the early part of an impulsive burst, when the X-ray spectrum is hardest and the microwave peak is at its highest frequency; we do not consider the subsequent (presumed) expansion of the sources. We assume that the electrons have an isotropic velocity distribution.

Fig. 3.—The gyromagnetic absorption coefficient for the $x$-mode from numerical computations for two values of $\theta$ and seven values of $T_e$. The shortened dashes indicate the general trend in regions where there is fine structure because the influence of individual harmonics is not entirely wiped out by Doppler shifts. When calculating the refractive index, $f_p = 900$ MHz ($n_e = 10^8$ cm$^{-3}$) and $f_B = 840$ MHz ($B = 300$ gauss) were assumed. The right and top scales show the optical depth $\tau_x$ and microwave frequency, respectively, for such a source with scale length $L = 2000$ km.
The right-hand scales on Figures 2 and 3 show the optical depth \( \tau_x = \kappa_x L \) of such sources; it is important to note the small range of \( \omega/\Omega_e \) over which sources of given temperature change from optically thick to optically thin. The scales at the top of the figures show the microwave frequencies corresponding to the various values of \( \omega/\Omega_e \). For a source at a given temperature, the frequency of largest flux density, \( f_{\text{peak}} \), is that for which \( \tau \approx 1 \); at lower frequencies \( \tau \gg 1, T_e \approx T_b \), and the flux density \( S \propto f^{-2} \); while at higher frequencies \( \tau \ll 1, T_b \ll T_e \), and \( S \) decreases very rapidly with \( f \).

In Figure 4 we plot the value of \( \omega/\Omega_e \) for which \( \tau_x = 1 \), denoted \( s^* \), as a function of temperature. Because \( s^* \) varies with \( \theta \), the observed value of \( f_{\text{peak}} \) depends on the orientation of the magnetic field relative to the observer; thus if magnetic fields in impulsive burst regions have a preferred orientation (say horizontal or vertical), a center-to-limb effect is to be expected. (If this is the cause of the slight [20–50\%] decrease in burst intensity near the limb found by Kakinuma, Yamashita, and Enome (1969), it would imply that the magnetic fields tend to be horizontal so that, on average, \( \theta \) is larger near disk center.)

Figure 4 also shows that \( s^* \) is insensitive to a change in source density (or equivalently scale length, since \( \tau \propto n_e L \)); a change of two orders of magnitude in \( n_e \) produces less than a 50\% change in \( s^* \).

We are now able to write down some relationships which are of use in interpreting the data. For the temperature range of about \( 10^8 - 10^9 \) K, the results of Figures 3 and 4 can be approximated by the simplest useful relations

\[
\begin{align*}
\tau_x & \approx 3 \times 10^9 T_8^7 \sin^6 \theta (\omega/\Omega_e)^{-10}, \\
\theta_0 & \approx 0.2 \tau_x, \\
& \approx [(1 - |\cos \theta|)/(1 + |\cos \theta|)]^2 \tau_x \quad (|\pi/2 - \theta| \gg \Omega_e/2f), \\
s^* & = 9T_8^{0.7},
\end{align*}
\]

where \( T_8 \) is the electron temperature in units of \( 10^8 \) K. On taking the (small) density, angle, and scale-length dependences into account and changing \( \omega \) to \( f \), \( \Omega_e \) to \( B \), equations (11a, c) are replaced by slightly more general expressions:

\[
\begin{align*}
\tau_x & \approx 5 \times 10^9 T_8^7 \sin^6 \theta (n_{10}L_9/B_2) B_2^{10} f_9^{-10}, \\
s^* & \approx 10 (n_{10}L_9/B_2)^{0.1} \sin^{0.6} \theta T_8^{0.7},
\end{align*}
\]

where \( n_{10} \) is the electron density in units of \( 10^{10} \) cm\(^{-3} \), \( B_2 \) is the magnetic field strength in units of \( 10^2 \) gauss, \( L_9 \) is the scale length in units of \( 10^9 \) cm = 1000 km, and \( f_9 \) is the frequency in GHz. Changing from \( s^* \) to \( f_{\text{peak}} \), which occurs at \( \tau \approx 2.5 \) and \( \omega/\Omega_e \approx 0.9s^* \), we have

\[
f_{\text{peak}} \approx 2.3T_8^{0.7} B_2,
\]
or, using (12b) in place of (11c),

\[ f_{\text{peak}} \approx 2.4(n_{\text{10}}L_0/B_0)^{0.1} \sin^{0.6} \theta T_8^{0.7} B_2, \]  

(13b)

where \( f_{\text{peak}} \) is in GHz.\(^1\)

The peak frequency \( f_{\text{peak}} \) is an observed quantity in a microwave burst, and \( T_8 \) can be derived from an X-ray spectrum. Consequently (13a) can be used to estimate the magnetic field in the source region.

Another useful relation comes from the microwave flux density \( S \), which, for \( f \leq f_{\text{peak}} \), is given by the Rayleigh-Jeans law for an optically thick source. In terms of the quantities introduced above, \( S \) (in units of SFU) can be written

\[ S \approx 1.36 \times 10^{-2} T_8 f_{\text{peak}}^2 L_0^2. \]  

(14)

Thus given \( T_8 \) and the flux density at any \( f < f_{\text{peak}} \), this equation can be used to estimate the scale size of the source region.

The two other parameters of most interest, the temperature and density of the source, can be derived with the help of a hard X-ray spectrum. The photon spectrum \( I(E) \) can be written as (e.g., Crannell et al. 1978)

\[ I(E) = 1.3 \times 10^3 EM_{45} E^{-1.4} T^{-0.1} \exp(-E/T), \]  

(15)

where \( I(E) \) is in units of photons cm\(^{-2}\)s\(^{-1}\) keV\(^{-1}\), the photon energy \( E \) and the temperature \( T \) are in keV, and the emission measure \( EM_{45} \) (in units of \( 10^{45} \) cm\(^{-3}\)) is given by

\[ EM_{45} = 0.1n_{\text{10}}^2 L_0^3. \]  

(16)

Thus both the temperature and emission measure of the source of an impulsive burst can be estimated if the X-ray intensity is measured at two or more photon energies, \( E_1 \) and \( E_2 \) say.

In summary, given the hard X-ray intensity \( I(E_1) \) and \( I(E_2) \) and the microwave \( f_{\text{peak}} \) and \( S_{\text{peak}} \), then the temperature and emission measure can be derived from equation (15), the magnetic field from equation (17), the scale length from equation (14), and the source density from equation (16).

(a) Impulsive Burst of 1972 May 18

In this section we illustrate the application of the foregoing results by considering the impulsive burst of 1972 May 18, 1406 UT, reported by Hoyng and Stevens (1973), Hoyng (1975), and Hoyng, Brown, and van Beek (1976). We choose this event because both the X-ray and microwave observations are of high quality. It is, perhaps, not an ideal event for illustration because it was relatively strong and complex, with several peaks; thus multiple source regions or particles produced by second-phase acceleration may play some role. However, it is of particular interest because of its high value of \( f_{\text{peak}} \approx 15 \) GHz.

Figure 1 shows the X-ray and microwave spectra at different times during the burst. We will concentrate on the spectra taken near the X-ray and microwave peaks, 1406:09 to 1406:15 UT. From the X-ray spectrum and equation (15), we derive

\[ T_8 = 2.3 \times 10^8 \text{ K} \quad \text{and} \quad EM = 2.0 \times 10^{45} \text{ cm}^{-3}. \]

Using these values, the radio spectrum, and equations (13), (14), and (16), we derive

\[ L = 8600 \text{ km}, \quad B = 370 \text{ gauss}, \quad \text{and} \quad n_e = 2 \times 10^9 \text{ cm}^{-3}. \]

As seen on Figure 5, these values give good agreement with the X-ray intensity at the three higher energies but predict a lower intensity than observed at the lowest energy, 30 keV. Figure 5 also shows that values of \( T_8 \) in the range of about 1.5–3.0 give poorer but still acceptable fits. On Figure 6 we compare the observed radio spectrum with one calculated from the foregoing parameters and equation (12a). The agreement is good at frequencies near \( f_{\text{peak}} \), but the observed points at several of the lower frequencies are higher than the calculated ones —i.e., the observed spectrum rises more slowly than \( f^2 \).

In sum, both the radio and X-ray data indicate that a single isothermal source is a surprisingly good representation, especially with respect to the high radio frequencies and high X-ray energies which come from the hottest, densest part of the flare. However, they both indicate the need for additional material at a lower temperature to

\(^1\) We should remark on the exponent of \( nL/B \) in (12b): under somewhat similar conditions Bekefi (1966, p. 207) found an exponent \( \frac{4}{3} \) rather than our 0.1. Bekefi’s estimate was for \( T = 50 \) keV, and for \( n_e = 8 \times 10^{11} \text{ cm}^{-3}, L = 10^8 \text{ cm}, B = 5 \times 10^4 \text{ gauss} \), i.e., a value of \( nL/B \) some three orders of magnitude less than ours. On using our curve for \( T = 5 \times 10^8 \text{ K} \) in Figure 3, noting that Bekefi’s exponent is for fixed \( T \), and using Bekefi’s value of \( nL/B \), we find that \( S \propto (nL/B)^{1.0} \) fits quite well. This comparison emphasizes that the exponents in (11), (12), and (13) apply around the chosen values \( n_{\text{10}} = 1, T_8 = 3, L_0 = 2, B_0 = 3 \), and that somewhat different exponents would be obtained for substantially different values of any of these parameters. For example, changing \( T_8 \) from 8 to 9 and leaving the other parameters the same, the exponent 10 in (11a) changes from about 14 to about 8.
GYROSYNCHROTRON EMISSION AND SOLAR FLARES

1145

Fig. 5.—X-ray spectrum of the impulsive burst of 1972 May 18, 1406:09 UT. The data points are from Hoyng and Stevens (1973). Solid curve, spectrum for a source with the parameters derived in the text. Dashed curves, for other values of temperature and emission measure as indicated.

Fig. 6.—Microwave spectrum of the event at 1406:15 UT. The data points are from Hoyng and Stevens (1973). Solid curve, a hand-drawn fit to the data points. Dashed line is computed using the parameters derived in the text and the curves of Fig. 3.

Give the extra low-frequency/low-energy flux. As demonstrated by Milkey (1971) and Brown (1974), most X-ray spectra can be fitted by the radiation from a thermal plasma of varying temperature; a similar situation may apply to the radio spectrum (Piddington 1950; see also Pawsey and Smerd 1953).

IV. DISCUSSION AND CONCLUSION

The results of this paper are relevant to thermal models for impulsive microwave sources. The useful results are the approximate formulae (13) and (14), which relate the peak frequency and peak flux density in a thermal, self-absorbed, gyrosynchrotron source to its temperature, size, and magnetic field strength. The temperature and density may be estimated from the associated hard X-ray burst. Thus, if both the microwave and X-ray spectra of impulsive bursts result from electrons which have been energized in bulk to a near-Maxwellian distribution, it is possible to derive most of the interesting flare parameters using the simple formulae summarized here. The example we have considered is the flare of 1972 May 18, and the derived parameters are \( T_e = 2.3 \times 10^8 \) K, \( B = 370 \) gauss, \( L = 8600 \) km, \( n_e = 2 \times 10^9 \text{ cm}^{-3} \), and \( EM = 2.0 \times 10^{45} \text{ cm}^{-3} \).

Crannell et al. (1978) studied a series of impulsive bursts, most of which were simpler and weaker than the one we considered, and concluded that a thermal interpretation is compatible with the X-ray data. The temperatures they derived ranged from about 1.7 to \( 7 \times 10^9 \) K and the emission measures from about \( 0.1 \) to \( 2 \times 10^{16} \) cm\(^{-3}\), so that our example has a comparatively low temperature and high emission measure. From microwave data they deduced scale lengths of \( 1700 \) to \( 25,000 \) km for the different bursts.

Mätzler et al. (1978) analyzed two of the Crannell et al. (1978) events in more detail, in a fashion similar to ours. Using some calculations of gyromagnetic absorption coefficients by Drummond and Rosenbluth (1963), they estimated the magnetic field in the source region to be about 100 gauss. They then suggested that the magnetic energy density was only about 3 times the kinetic energy density, implying that the adiabatic heating postulated by them produced a relatively high beta plasma, i.e., \( \beta \approx 1 \). In contrast, for our event we find \( \beta \approx 0.01 \), showing a pronounced dominance of magnetic energy.

Böhme et al. (1977) have also studied the burst of 1972 May 18 which we have analyzed. They considered a two-component model, with the microwave radiation at \( f \geq 10 \) GHz coming from a small core containing electrons with a power-law energy distribution and the radiation at \( f \leq 5 \) GHz coming from a larger halo. They found that the X-ray and microwave observations could be satisfied with such a model if the core diameter was about \( 15,000 \) km and, at the center, \( n_e \approx 10^{10} \text{ cm}^{-3} \) and \( B \approx 1500 \) gauss. These values are larger than ours by factors of 2, 5, and 4, respectively.
As suggested by an anonymous referee, we compare the efficiency of hard X-ray production in thermal and nonthermal models at the same value of electron density. We find that the production efficiency of the thermal model is about 4 times greater than the nonthermal model. This is, of course, not very large factor; it is significantly less than the factor > 10 estimated by Smith and Lilliequist (1979) for a source of higher density. However, we are not convinced that this comparison is very meaningful because of the thick-target assumption and because the parameters chosen may not be the optimum for a nonthermal model. None of our results prove that a thermal model is the correct one, only that it is feasible.

Finally we make a few remarks about the polarization of the microwave emission. From equation (11b) we see that the gyromagnetic absorption coefficient for the o-mode should be smaller than the x-mode by a factor of about 0.03 for $\theta = 45^\circ$ and 0.3 for $\theta = 75^\circ$. Thus, at low frequencies, where the source is optically thick for both modes, the net polarization is calculated to be zero. At higher frequencies, where $\tau \lesssim 1$, the degree of circular polarization is calculated to be high, up to 90% for $\theta = 45^\circ$. However, we do not expect the observed polarization to agree in detail with these calculations, since they were done for an idealized, homogeneous source with sharp boundaries. In real sources, the density may be nonuniform, the magnetic field both nonuniform and anisotropic, and edge effects may occur (e.g., Melrose 1978). While these factors will have only a minor effect on the microwave spectrum, they could produce major changes in the polarization. Thus, the degree of polarization could be nonzero at low frequencies and only moderate at high frequencies. Observations of impulsive bursts at 3.7 and 9.4 GHz by Kakinuma, Yamashita, and Enome (1969) are consistent with these general expectations; there are larger numbers of highly polarized ($\geq 50\%$) bursts at the higher frequency, and bursts with polarization $\geq 20\%$ are not uncommon at either frequency.

After this paper was submitted, a very interesting paper by Mätzler (1978) appeared which addressed similar problems. While Mätzler did not give any analytical results, his numerical calculations were similar to ours; in his Figure 1 the curves (for $\theta = 90^\circ$) are nearly identical to those for $\theta = 75^\circ$ in our Figure 3. In his discussion Mätzler concentrated on the spectrum and polarization expected from model sources, while we have concentrated on the application of the results to a particular hard X-ray and microwave event. In the area of overlap his conclusions agree with ours, especially in regard to the feasibility of a thermal interpretation.

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