THERMAL INSTABILITIES IN MAGNETICALLY CONFINED PLASMAS:  
SOLAR CORONAL LOOPS  
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Received 1979 April 20; accepted 1979 July 6  
ABSTRACT  
The thermal stability of confined solar coronal structures ("loops") is investigated, following both normal mode and a new, global instability analysis. We demonstrate that: (a) normal mode analysis shows modes with size scales comparable to that of loops to be unstable, but to be strongly affected by the loop boundary conditions; (b) a global analysis, based upon variation of the total loop energy losses and gains, yields loop stability conditions for global modes dependent upon the coronal loop heating process, with magnetically coupled heating processes giving marginal stability. The connection between the present analysis and the minimum flux corona of Hearn is also discussed.  

Subject headings: hydromagnetics — instabilities — plasmas — Sun: corona  

I. INTRODUCTION  
The thermal instability of the solar corona has been the subject of many investigations, the most extensive due to Field (1965) and Defouw (1970; see also Alikhanov 1967; Parker 1953; and references cited therein). Many of those earlier calculations were primarily concerned with the problem of forming cool condensations (viz., prominences) at pressure equilibrium with an ambient hot, optically thin coronal plasma; in any case, the ambient plasma was assumed to be structureless, and hence not to be associated with any intrinsic length scales (other than its pressure scale height).  
The recent Skylab ATM observations of the solar corona, which show in X-rays an abundance of structuring ("loops") on all observable scales (cf. Withbroe and Noyes 1977; Vaiana and Rosner 1978), have rekindled interest in the thermal instability problem. The context in which this problem is cast is now somewhat different: given a magnetically confined hot, optically thin plasma, is it thermally stable? Furthermore, the interest is no longer focused only upon accounting for prominence-like coronal features, but also upon reproducing the highly variable emission of coronal loop structures. A number of authors have recently discussed this problem qualitatively (see Rosner, Tucker, and Vaiana 1978 and Pye et al. 1978; see, however, Priest 1978 for a more detailed discussion); it is the aim of this paper to provide a quantitative basis for the discussion of loop thermal instability, including both the effects of thermal conduction and boundary conditions.  

Before entering into the detailed discussion, we briefly outline the physical basis for thermal instability, and show that the simplest argument—though suggestive—can be very misleading. The radiative loss for an optically thin plasma is given by the expression  
$$E_{\text{rad}} = n_e n_i P(T) \text{ ergs s}^{-1} \text{ cm}^{-3},$$  
where $n_i \text{ cm}^{-3}$ is the number density of the $i$th species and $P(T) \text{ ergs}^{-1} \text{ cm}^3$ is a function of the plasma temperature $T$ (cf. Tucker and Koren 1971). It is common to assume $n_e \sim O(n_i)$, and we shall for convenience set them equal: $E_{\text{rad}} = n^2 P(T)$. Theoretical calculations for free-free, free-bound, and bound-bound radiative losses show that for $T \gtrsim 2 \times 10^6$ K, $dP/dT < 0$ (e.g., Raymond and Smith 1977); this has the consequence that if radiation is the only local energy loss mechanism, an increase in temperature leads to a reduction in local losses, and a decrease in temperature to an increase in the local losses, if the density remains constant. This is in essence the argument presented by Pye et al. (1978), and represents the simplest version of the radiative thermal instability. Note, however, that if we use the equation of state $p = 2nk_BT$ (the pressure), then the radiative losses are given by  
$$p^2/4k_BT^{-2} P(T).$$  
If the perturbation is isobaric (so that the density does not remain constant), then equation (1.2) shows that the sign of $dP/dT$ is insufficient to establish instability; instead, one must consider the slope of $T^{-2} P(T)$, which is equal to or less than zero, over a far larger range of temperatures than is the slope of $P(T)$. In any case, the simple argument above is flawed by the fact that radiation is not the only local loss mechanism. The existence of these obvious—and realistic—complications suggests that thermal stability analysis should be approached with caution;
significant local energy loss and gain mechanisms must be specified; and mechanisms likely to be responsible for perturbing loop atmospheres must be determined. In the former category we include, in addition to radiation, the contributions of thermal conduction, enthalpy, and local nonradiative heating; in the latter category we must entertain the physically distinct possibilities of perturbations originating in the corona (viz., fluctuations in the coronal heating rate) and perturbations originating at chromospheric levels, below the nominal coronal inner boundary (viz., fluctuations in base pressure).

In the following we shall address two issues: first, we discuss linearized normal mode instability analysis in the context of a finite (loop) geometry (§ II); second, we develop a new global instability analysis procedure, and discuss the effects of various heating mechanisms upon loop stability. In the final section (§ IV) we discuss the implications of our results.

II. CORONAL LOOP ATMOSPHERE INSTABILITY: NORMAL MODE ANALYSIS

We consider the thermal stability of a topologically-closed coronal plasma loop (Fig. 1) subject to the time-dependent energy balance equation

$$\frac{3}{2} \frac{dp}{dt} = \frac{5}{2} \frac{p}{n} \frac{dn}{dt} = Q(n, T) - n^2 P(T) - \text{div} F_c \tag{2.1}$$

for given boundary conditions $T(s_{\text{min}}) = T_a$, $p(s_{\text{min}}) = p_a$, and $dT/ds|_{s_{\text{min}}} = T'_a$, where $p = 2nk_B T$ is the plasma pressure, $n$ (cm$^{-3}$) the number density, $T$ (K) the electron plasma temperature, $Q$ (ergs s$^{-1}$ cm$^{-3}$) the local heating function, $P(T)$ (ergs s$^{-1}$ cm$^{-3}$) the radiative loss function for an optically thin plasma, and $F_c$ the thermal conductive flux:

$$F_c = -\kappa T^{\delta/2} \text{ grad } T. \tag{2.2}$$

For computational simplicity (and no other reason) we restrict our attention to symmetric loops with uniform cross section. We shall assume all plasma transport to occur along the local magnetic field, so that the equations of motions are spatially one-dimensional. Although we expect, in general, a perturbation of a loop atmosphere to involve changes in both its pressure and its temperature, we shall examine the two extremes—isobaric and isothermal perturbations.

a) Isobaric Case: Coronal Heating Perturbations

We begin our analysis by assuming the local thermal fluctuation within the loop to be caused by a coronal fluctuation in, e.g., the local heating rate; that is, we assume the base boundary conditions (which are presumably fixed by conditions in the underlying atmosphere) to remain fixed. In that case, because within the loop the sound travel time is substantially smaller than typical radiative and conductive cooling times, perturbations to the coronal loop atmosphere are essentially isobaric; we therefore set $dp/dt = 0$ in equation (2.1). In the following, we shall furthermore take advantage of the differential relation between density and temperature,

$$\frac{dn}{n} = -dT/T; \tag{2.3}$$

in addition, we approximate the radiative loss function $P(T)$ by a power law of the form

$$P(T) = \alpha T^\delta, \tag{2.4}$$

where $\alpha$ and $\delta$ are constant within well-defined temperature regimes (Rosner, Tucker, and Vaiana 1978).

![Fig. 1.—Schematic representation of loop atmosphere geometry. For simplicity, only the symmetric case is examined. Note that the atmosphere base is defined by a fixed temperature, whose physical position (height) $s_{\text{min}}$ must be regarded as a time-dependent quantity.](image-url)
In hydrostatic equilibrium, the loop atmosphere consists of the solutions \( T_0 = r_0(s) \) and \( n_0 = n_0(s) \) obtained from equation (2.1) by setting \( d/ds = 0 \):

\[
Q(n_0, T_0) - n_0^2 P(T_0) - dF/|ds = 0,
\]

as well as from the time-independent momentum balance equation; the coordinate \( s \) is assumed to lie along the magnetic field defining the loop. Now, following Field (1965), we consider isobaric perturbations of the equilibrium solution

\[
T = T_0 + \delta T,
\]

with \( |\delta T| \ll T_0 \), so that we may linearize the problem. With the aid of equations (2.1)-(2.4), we then obtain

\[
\frac{d}{dt} \frac{dT}{5p} \approx T_0 \frac{dT}{ds} k - T_0 \frac{5p}{5p} \text{Im} [B/A],
\]

\[
\omega_1 \approx \frac{T_0}{5p} \left[ \text{Re} (B/A) - \frac{\alpha(\delta - 2) p^2 T_0^{-1} Q - \kappa^2 T_0^{5/4} \delta T + k^2 T_0^{5/4} \delta T} \right].
\]

Stability demands that \( \omega_1 \leq 0 \); using equations (2.5) and (2.9), we obtain after some algebra the condition for stability:

\[
k^2 > k_c^2 \approx \frac{5}{2} \left( \frac{2}{5} \text{Re} (B/A) - T_0^{-1} Q + \kappa T_0^{5/4} (dT_0/|ds|)^2 \right)^2 + \frac{\alpha p^2}{10k_b^3} \left( \frac{9}{2} - \delta \right) T_0^{-3}.
\]

Thus, perturbations whose wavelength \( \lambda \) is larger than the critical \( \lambda_c = 2\pi/k_c \) are unstable. We note that:

1. The local heating function \( Q \) can be either stabilizing or destabilizing, depending upon the relative sign of the perturbation amplitudes; thus, if \( \text{Re} (B/A) \leq 0 \) (viz., local heating depends inversely upon the temperature to some power), the heating process is stabilizing (see § III below).
2. Heat conduction is always stabilizing.
3. Radiative losses may be either stabilizing or destabilizing; however, for \( T \geq 10^{10} \) K, \( \delta < 4.5 \) (cf. Rosner, Tucker, and Vaiana 1978). Therefore, radiative losses destabilize above \( 10^{10} \) K.

The smallest \( \lambda_c \) is obtained by setting all the stabilizing terms in equation (2.10) to zero. In that case we obtain an upper bound upon the wavelength \( \lambda \) which guarantees thermal stability; for \( T_0 > 10^6 \) K, we find (using \( \log \alpha = -17.73 \) and \( \delta = -\frac{3}{2} \))

\[
\lambda_c \approx 1.4 \times 10^{-14} p^{-1} T_0^{10/12} \text{ cm}.
\]

Thus, for \( \lambda \leq \lambda_c \), all perturbations are thermally stable, independent of the value of the stabilizing terms in equation (2.10). There is a range of wavelengths larger than \( \lambda_c \) which are also stable; this range, however, depends upon the values of the terms \( -T_0^{-1} Q \) and \( -\kappa T_0^{1/4} (dT_0/|ds|)^2 \) in equation (2.10). Note that upon solving for \( T_0 \) in equation (2.11), we obtain

\[
T_0 \approx 3.3 \times 10^9 (\rho \lambda_c \min^{12/37}) \text{ K },
\]

a scaling law similar to that first found by Rosner, Tucker, and Vaiana (1978); it is therefore tempting to identify the observed quasi-stationary loops with the thermally stable regime derived here (cf. Priest 1978). We return to this question below.

### b) Isothermal Pressure Perturbation: Coronal Fluctuations Induced by Chromospheric Changes

A second example of an analytically tractable normal mode perturbation analysis is a perturbation in pressure \( \delta p \), which is, in the linearized regime, associated with temperature perturbations of lower order, e.g., \( |\delta p|/p \gg |\delta T|/T; \)
in this regime, the perturbation can be regarded as locally isothermal \((dT/dt \approx 0)\). Our analysis follows exactly along the lines of § IIa above, with perturbed quantities given by

\[ p = p_0 + \delta p, \quad Q = Q_0 + \delta Q, \quad (2.13) \]

where \(p_0\) and \(Q_0\) are the unperturbed pressure and heating function, respectively. The linearized equation governing the evolution of the system is obtained with the use of equations (2.1), (2.4), (2.5), and (2.13), and is given by

\[ \frac{d}{dt} \delta p = -\delta Q + \frac{2p_0 T^{3/2}}{4k_b^2}. \quad (2.14) \]

Note that in this case thermal conduction plays no (stabilizing) role because we have assumed the temperature perturbation to be of lower order than the imposed pressure perturbation. The appropriate dispersion relations for \(\omega_R\) and \(\omega_I\) are then found as in § IIa above; the presently relevant result for \(\omega_I\) is

\[ \omega_I = -\text{Re} \left( \frac{B}{A} \right) + \cdot (2.15) \]

Stability again requires \(\omega_I \leq 0\), or

\[ \text{Re} \left( \frac{B}{A} \right) \gtrsim 2p_0 T^{3/2}/4k_b^2. \quad (2.16) \]

Note that the left-hand side of equation (2.16) is the real part of the ratio of the heating and pressure perturbations, whereas the right-hand side is just twice the ratio of the local radiative loss \(E_R = n^2 P(T)\) and the local pressure. The significance of this result can be obtained by evaluating equation (2.16) in the vicinity of the initial, unperturbed state. In that case, we have \(\text{div} F \sim E_R\) throughout most of the loop, so that we can estimate \(Q_0\) by

\[ Q_0 \approx 2E_R. \quad (2.17) \]

Equation (2.16) can then be simply rewritten in the form \(Q_0/p_0 \lesssim \text{Re} (\delta Q/\delta p)\), or

\[ \frac{\delta Q}{Q_0} \gtrsim \frac{\delta p}{p_0}, \quad (2.18) \]

for stability. In the simple case in which \(Q\) can be written as a power law in the pressure (see § III below), e.g., if \(Q \sim p_0^m\), then our stability requirement is that

\[ m \gtrsim 1. \quad (2.19) \]

The significance of this result in light of specific simplified coronal heating models is discussed in the next section; here we note only that, at minimum, this analysis shows the variation of local coronal heating in response to coronal pressure fluctuations to be quite relevant to the question of stability.

c) Discussion of Normal-Mode Instability Analysis

The two cases we have analyzed—temperature perturbations at constant pressure and pressure variations with fixed temperature conditions—represent two extremes which, we expect, bound the actual behavior of coronal fluctuations. That is, it is our expectation that pressure variations at loop footpoints in fact do cause coronal temperature perturbations (unlike what we assumed in § IIb), whose evolution in turn is governed by an analysis similar to that of § IIa above. In this connection we wish to strongly emphasize that the observed brightness variations of X-ray emitting coronal structures must reflect this very process. The argument is as follows.

i) The soft X-ray observations indicate that loop brightness variations are primarily accounted for by changes in loop plasma density, not in loop temperature (cf. Davis et al. 1975); as discussed by Rosner, Tucker, and Vaiana (1978), brightness fluctuations are well correlated with loop pressure variations.

ii) Observational and theoretical analysis of the atmospheric layers underlying loop footpoints—e.g., the photospheric and chromospheric portions—show that, for temperatures above \(\sim 7500\) K, the gas pressure drops only slightly with height (see Fig. 2); that is, the pressure at \(T \sim 7500\) K must be essentially the same as the coronal gas pressure (for coronal structures, such as active region loops, whose scale size is less than or comparable to the coronal pressure scale height). For example, a comparison between the chromosphere models of Basri et al. (1979) and evaluations of coronal pressures derived from soft X-ray observations shows immediately that the range of chromospheric pressures at \(T \sim 7500\) K coincides closely with the observed range of coronal \((T > 10^6\) K) pressures. This correspondence extends in detail to the pressure variation with level of activity. Thus Basri et al. derive pressures of \(\sim 4\) dyn cm\(^{-2}\) at \(T \sim 7500\) K for bright plage regions, and pressures at least an order of magnitude lower in quiet regions; in comparison, pressures derived from soft X-ray observations of bright active region loops cluster in the range 2–5 dyn cm\(^{-2}\), while quiet Sun ("large-scale structure") loops have pressures again an order of magnitude lower (see Table 1, adapted from Vaiana and Rosner 1978).
iii) On energetic grounds, it is extremely improbable that chromospheric pressures at the $T \sim 7500$ K level in quiescent (nonflaring) regions are determined by coronal processes. That is, excluding the extreme case of flares, coronal energy deposition is not expected to influence chromospheric structure (cf. Athay 1976). The observed correlation between chromospheric and coronal quiescent activity levels (and pressures) is far more easily accounted for if one assumes that the processes heating the chromosphere and corona are physically related, so that increased chromospheric energy deposition occurs together with increased coronal energy deposition. This physical correlation may be a simple consequence of a common heating process for the chromosphere and corona, or may result from distinct chromospheric and coronal heating processes coupled through their common dependence upon the ambient gas pressure; for example, increased heating of chromospheric layers yields pressure increases at coronal levels which may in turn increase the efficiency of coronal energy deposition (viz., decreases the damping length of Alfvén or fast MHD modes).

If the above argument prevails, it is then clear that the concept of coronal instabilities—in isolation—is not meaningful. Instead, one must regard the chromospheric boundary layers of coronal flux tubes as defining the locus of initial loop atmospheres whose coronal portions may or may not be subject to thermal or acoustic instabilities; in this sense, the chromosphere must be regarded as the coronal perturber via its role in defining the coronal loop pressure.

### TABLE 1

**Coronal X-Ray Parameters**

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>CH $^a$</th>
<th>LSS $^c$</th>
<th>BP</th>
<th>AR</th>
<th>Small (C) Flare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature, $T_e (10^6 \text{ K})$</td>
<td>$1.3 \times 10^6$</td>
<td>$2.1 \times 10^6$</td>
<td>$3 \times 10^6$</td>
<td>$6 \times 10^6$</td>
<td>$6 \times 10^6$</td>
</tr>
<tr>
<td>Characteristic vertical size, $L (\text{cm})$</td>
<td>$1.8 \times 10^6$</td>
<td>$2.0 \times 10^6$</td>
<td>$4 \times 10^6$</td>
<td>$140 \times 10^6$</td>
<td>$140 \times 10^6$</td>
</tr>
<tr>
<td>Electron density, $N_e (\text{cm}^{-3})$</td>
<td>$1.3 \times 10^9$</td>
<td>$2 \times 10^9$</td>
<td>$4 \times 10^9$</td>
<td>$5 \times 10^9$</td>
<td>$5 \times 10^9$</td>
</tr>
<tr>
<td>Pressure = $2N_e k T \rho (\text{dyn cm}^{-2})$</td>
<td>$0.05$</td>
<td>$0.11$</td>
<td>$2.0$</td>
<td>$3.5$</td>
<td>$140$</td>
</tr>
</tbody>
</table>

**Note.**—Abbreviations are as follows: AR, active region; QS, quiet Sun; CH, coronal hole; LSS, large-scale structure; BP, bright points.

$^a$ The S-054 data permit a range of allowed temperatures. $T_e = 1.3$ is assumed in the calculations (Maxson and Vaiana 1977).

$^b$ The pressure scale height is used as the characteristic size.

$^c$ Large-scale structures (LSS) are identified with the quiet Sun (QS).

$^d$ X-ray spectroscopic telescope.
A further element of uncertainty regarding the above instability analysis enters because normal-mode analysis assumes either that perturbations of the form $\exp(iks - i\omega t)$ are normal modes of the finite system, or that the wavelength $2\pi/k$ of the perturbations is small relative to the system dimension(s), so that the effects of boundaries may be safely ignored (cf. Field 1965). Our result, however, indicates that isobaric perturbations within a coronal loop whose wavelength is small compared to the length of the loop are thermally stable, while perturbations whose wavelength is comparable to the loop size are sensitive to boundary effects. For this reason, we regard the analysis presented in this section not as a statement concerning the stability of coronal loops, but rather as a further demonstration that variations at the boundaries of these loop atmospheres strongly affect loop conditions (including stability), and that instabilities—if extant—must be global in character. This point is pursued further below (§ III).

An additional complication in our linearized calculations is the fact that as the perturbations enter the finite-amplitude regime, the assumption of constant amplitude (see eq. [2.8]) is not likely to hold; Kopp (1963) has in fact shown that local amplitude growth of finite-amplitude disturbances is a strong function of local temperature if thermal conduction is important—as it is here.

III. THERMAL INSTABILITY: GLOBAL ANALYSIS

The limitations of normal-mode instability analysis in the coronal context suggest that a radically different approach, one based upon the global properties of confined coronal loop atmospheres, may be more fruitful. Our line of attack is as follows. In § II we have shown that short-wavelength perturbations (e.g., $\lambda \ll$ loop length) are thermally stable; we shall therefore focus solely upon perturbations with characteristic scales comparable to that of the loop structure. Because of the nonlinearity of the equations of motion, the long-wavelength eigenmodes of the system cannot be analytically determined. However, one can pose the following stability problem: Is a given loop atmosphere stable against a functionally specified temperature (or pressure) perturbation (one not necessarily an eigenmode of the system)? If the atmosphere is shown to be stable, no conclusion can be drawn concerning the true stability of the system. However, if the atmosphere is unstable against the given perturbation, it is reasonable to conclude that the atmosphere is truly unstable; the intuitive basis for this conclusion is that if some channel for instability is available to the system, a suitable superposition of eigenmode perturbations will in fact drive the system to instability.

Because of the complexity of the general problem, we shall restrict our attention to loop atmospheres whose initial state has its temperature maximum at the apex of a symmetric loop and for which the assumption of uniform pressure applies; this restriction is a matter of computational convenience, and is not to be interpreted as signifying the initial state of all possible actually occurring loop structures. We begin with the one-dimensional form of equation (2.1):

$$\frac{5}{2} p \frac{dT}{dt} - \frac{dp}{dt} = Q(p, T) - \frac{1}{4k_B^2} p^2 T^{-2} p(T) - \frac{\partial F_e}{\partial s},$$

where $s$ is again the coordinate along the loop axis, and the boundary conditions are as previously specified. In order to take all losses and gains of the system into account, we choose a base temperature $T_b$ for which chromospheric model atmospheres (viz., Basri et al. 1979) yield a negligible thermal conductive flux over a wide range of chromospheric conditions; in this manner we ensure that the infinitesimally perturbed atmosphere retains the property of negligible thermal conductive flux at its base, as defined by the base temperature $T_b$. We note, however, that the physical location (viz., height) of the base, $s(T_b)$, is not necessarily fixed, but rather may well vary as a result of the applied perturbation. For this reason, we regard the specification of the base temperature at a fixed height in the atmosphere as an unphysical constraint, and results arising from an analysis based upon this constraint to be of questionable applicability. In order to circumvent this difficulty, we slightly modify the approach of Landini and Monsignori-Fossi (1975) and Rosner, Tucker, and Vaiana (1978) by rewriting the time-dependent equations of motion using the time-dependent solution $T(s)$ as the independent variable. By changing coordinates from $s$ to $T$, we obtain a description of the loop atmosphere which automatically accounts for the change in position of the transition region and lower boundary as a result of the temperature perturbation. From Figure 2, we see that appropriate choices for $T_b$ are $2 \times 10^4$ K or $\approx 7500$ K; the latter choice allows the study of loop atmospheres over a wider range of pressures, but involves the significant complication that the loop atmosphere may no longer be considered optically thin everywhere. As a matter of convenience, we therefore choose $T_b \approx 2 \times 10^4$ K. If the time-dependent loop temperature has the form

$$T(s, t) = \tau(s) + \delta T(s, t),$$

where $\tau(s)$ is the solution to the static problem (eq. [2.5]) and $\delta T$ is the applied perturbation, and if the perturbation has the functionally simplest form (e.g., is independent of $s$),

$$\delta T(s, t) = \delta T_b(t), \quad \delta T \ll \tau(s) \text{ everywhere},$$

where $\tau(s)$ is the solution to the static problem (eq. [2.5]) and $\delta T$ is the applied perturbation, and if the perturbation has the functionally simplest form (e.g., is independent of $s$),

$$\delta T(s, t) = \delta T_b(t), \quad \delta T \ll \tau(s) \text{ everywhere},$$

where $\tau(s)$ is the solution to the static problem (eq. [2.5]) and $\delta T$ is the applied perturbation, and if the perturbation has the functionally simplest form (e.g., is independent of $s$),
where \( T_m \) is the value of the maximum loop temperature, then, to lowest order in \( \delta T_m \), we obtain

\[
\frac{5}{2} \frac{\partial}{\partial t} \delta T_m - \frac{dp}{dt} = Q(p, T) - \frac{1}{4\kappa} T^{1/2} \delta P(T) - \frac{1}{2\kappa} T^{-5/2} \frac{\partial}{\partial T} F_e^2.
\]  

(3.4)

Although we expect, in general, the parameter perturbation \( \delta T(s, t) \) to be the consequence of an externally applied perturbation of, for example, the total nonradiative loop heating rate, we have no \textit{a priori} knowledge of the functional relationship which must connect these perturbations. In particular, our choice for the simple functional form of \( \delta T(s, t) \) may not correspond to the actual form of such perturbations; however, in the absence of a theory connecting the rate of total heating to the loop atmosphere response, our essentially arbitrary choice seems inevitable.

We now integrate equation (3.4) over the volume of the loop (e.g., from \( T = T_a \) to \( T = T_m \)), taking advantage of the fact that \( F_c(T_m) = F_c(T_a) = 0 \). Again following Rosner, Tucker, and Vaiana (1978), we express the heating term \( Q \) as a function (power law) of pressure and temperature and adopt the functional form of \( P(T) \) used in § II (eq. [2.4]). Rosner, Tucker, and Vaiana have discussed various functional forms of \( Q(p, T) \), depending upon the specific heating process considered. The crucial unknown in equation (3.4) is the relation between the base loop pressure \( p \) and the maximum coronal temperature \( T_{\text{max}} \). Following our discussion in § IIc, we shall assume the physical process responsible for coronal fluctuations to cause fluctuation in the nominal boundary conditions as well, as is suggested by the model atmospheres shown in Figure 2. Thus, in the simplest such case, we ensure that the initial state is defined by a maximum loop temperature \( T_m \) which is related to the loop pressure \( p \) for a fixed geometry (e.g., fixed loop length) via

\[
T_m = c_0 p^{1/\beta},
\]  

(3.5)

where \( c_0 \) and \( \beta \) are constants fixed by the heating process determining the loop interior and boundary conditions. The heating function can then be expressed in terms of temperature alone:

\[
Q(T_m) = c_0 T_m^\gamma,
\]  

(3.6)

where \( \gamma \) is found by combining equation (3.5) with the functional forms for \( Q(p, T) \) given by Rosner, Tucker, and Vaiana; thus \( \gamma \) is given by

\[
\begin{align*}
\gamma &= 2 & \text{acoustic heating/viscous damping} \\
&= \beta - \frac{1}{2} & \text{acoustic heating/shock dissipation} \\
&= \beta & \text{current dissipation} \\
&= \frac{7\beta}{6} & \text{Alfvén wave heating/mode conversion; semicollisionless tearing mode heating.}^1
\end{align*}
\]

(3.7)

Using equations (3.4)-(3.7) and the functional form for \( P(T) \) given by Rosner, Tucker, and Vaiana, we finally obtain

\[
\frac{\partial}{\partial t} \delta T_m = \frac{2c_0^\beta}{2 - 4\beta/7} T_m^{-\gamma}(c_1 T_m^{-\gamma+1}[1 + (\gamma + 1)\delta T_m/T_m](\gamma + \frac{1}{2})^{-1} - \alpha T_m^{2\gamma-3/2}[1 + (2\beta - \frac{3}{2})\delta T_m/T_m](4\kappa^2 c_0^2)^{-1}),
\]

(3.8)

where \( T_m \) here denotes the initial value of the maximum temperature and we have assumed \( T_m > 10^{5.11} \). In equilibrium, the term within the brackets vanishes; utilizing the equilibrium solution for \( T_m \) then yields the result

\[
\frac{\partial}{\partial t} \ln \delta T_m = 2c_0^\beta c_1 T_m^{-\gamma-\beta}[(\gamma - 2\beta + \frac{3}{2})(2 - 4\beta/7)(\gamma + \frac{1}{2})].
\]

(3.9)

Stability therefore requires that

\[
\gamma < 2\beta - \frac{3}{2}
\]  

(3.10)

for \( \beta < 3.5 \). This result can be interpreted in two distinct ways: First, given a particular heating theory, equation (3.10) specifies the necessary correlation between loop pressure and temperature to insure stability; second, given the correlation between loop pressure and temperature (possibly by observations), equation (3.10) specified the range of heating theories consistent with loop thermal stability.

1 Galeev \textit{et al.} (1979) show that for loop heating via collisionless tearing mode instability, \( T_m \propto p^{1/3} \) and \( Q \propto p^{1/6} \); therefore \( Q \propto T^{1/2} \), consistent with \( \gamma = \frac{7\beta}{6} \) for \( \beta = 3 \).

2 Recall, however, our previous caution that showing the system to be stable for a given form of the perturbation does not guarantee stability; it may be that the functional form chosen for the perturbation is not adequate to demonstrate instability.
Applying the first approach, we use equations (3.7) and (3.10) to find the condition for stability, which translates into an inequality for $\beta$ of the form

- acoustic heating/viscous damping $\beta \geq 2.25$;
- acoustic heating/shock damping $\beta \geq 2$;
- current dissipation $\beta \geq 2.5$;
- Alfvén wave heating semicollisionless tearing mode heating $\beta \geq 3$. (3.11)

In principle, equation (3.11) can be used as a first-order test of heating theories by noting the locus of observed $(p, T)$ for stable loops. Thus Rosner, Tucker, and Vaiana (1978) found $\beta \approx 3$ to give a reasonable description of the observed soft X-ray properties of coronal loops (see also Withbroe 1978 and Emslie and Machado 1978); in this sense all of the above mechanisms yield stable loops.$^3$

The second approach takes as its starting point the value of $\beta \approx 3$ derived by Rosner, Tucker, and Vaiana (1978) for quasi-stationary (presumably stable) coronal loop structures seen in soft X-rays (see also Withbroe 1978; Emslie and Machado 1978); alternatively, we may view $\beta \approx 3$ as an empirical result. In that case we obtain

$$\gamma \leq 3.5.$$ (3.12)

Comparing equation (3.12) with equation (3.7), we find that all heating theories predict stable loops, but that magnetically related theories lie at the marginally stable level defined by equality in equation (3.12). The linear decay rates, calculated with the aid of equation (3.9), are found in Table 2.

IV. SUMMARY AND DISCUSSION

Our instability analysis of confined coronal plasma structures has shown that: (a) linearized normal mode analysis predicts unstable modes whose wavelength is commensurate with the size of coronal loop structures, and stability for shorter wavelength modes; however, long-wavelength modes are not properly treated by this technique; (b) instability analysis for global modes shows that for simplified models of coronal heating, stability always obtains, but that in the case of magnetically related heating processes, stability may be at the marginal level. This result, in conjunction with the observed highly variable nature of emission from solar coronal loops, suggests—but does not prove—that solar coronal loop structures (the topologically "closed" solar corona) are maintained energetically by magnetic field–related local energy deposition.

These results must, however, be cautiously interpreted because of the schematic manner in which local coronal heating processes were treated. In particular, we have made no attempt to derive the local heating rate self-consistently with the atmospheric structure; such calculations are prohibitive if analytical solutions are sought (as we do here), and recourse to numerical integration of the equation of motion must instead be made. An example of the class of possible instabilities which can be studied in this detailed approach—but not in the present case—would be instabilities due to variations in the transition region structure resulting from coronal fluctuations. For example, changes in transition region structure imply variations in the reflection and refraction properties of the loop footpoints which in turn will modulate the level of energy flux reaching coronal levels; these effects will be most important for coronal heating processes associated with wave modes whose propagation is very sensitive to temperature and density variations, viz., acoustic modes. With these cautions in mind, we regard our calculations as suggestive of the kind of dynamics coronal loops may be subject to, and we are pursuing further analysis by means of time-dependent numerical simulation.

An interesting aspect of our calculations is the superficial resemblance between the instability calculation of § III and the formalism introduced by Hearn (1975) in his study of "minimum flux" coronae. Because of the current interest in coronal structuring as a general problem (cf. Vaiana and Rosner 1978), we briefly focus upon the consequences our results have upon the "minimum flux" concept.$^4$

As an aside, we note that the above stability analysis confirms the suggestion of Endler, Hammer, and Ulmschneider (1978) that specification of the coronal heating mechanism is required in order to properly apply the Hearn (1975) minimum flux coronal model (see § IV).

### TABLE 2

<table>
<thead>
<tr>
<th>Heating Mode</th>
<th>Stability</th>
<th>Decay Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acoustic heating/viscous damping</td>
<td>Stable</td>
<td>$10^3$</td>
</tr>
<tr>
<td>Acoustic heating/shock damping</td>
<td>Stable</td>
<td>$10^4$</td>
</tr>
<tr>
<td>Current dissipation</td>
<td>Stable</td>
<td>$3 \times 10^4$</td>
</tr>
<tr>
<td>Alfvén wave heating/mode conversion; semicollisionless tearing mode</td>
<td>Marginally stable</td>
<td>...</td>
</tr>
</tbody>
</table>

$^a$ The functional form for $Q(T)$ for the fast mode wave heating was derived for the same parameters as in eq. (3.7) using the model of Habbal, Leer, and Holzer 1979.
**a) Minimum Flux Corona**

In order to keep our discussion self-contained, we very briefly summarize the approach detailed by Hearn (1975) as is relevant to our problem. The outer extended atmosphere is assumed to consist of a hot hydrostatic single-temperature ($T = T_0$) region (the isothermal corona) in thermal contact with a temperature transition region lying between it and stellar photosphere. In the (simplifying) absence of stellar winds, the total loss from the corona is due to radiation ($F_r$ [ergs s$^{-1}$ cm$^{-2}$]) and thermal conduction to the boundary layer ($F_c$ [ergs s$^{-1}$ cm$^{-2}$]). Under the assumptions of spatially uniform pressure ($= p_0$), negligible direct heating of the boundary layer, and negligible conduction losses at the bottom (lower) boundary surface of the transition layer, the thermal conductive flux at the corona/transition layer interface can be explicitly calculated for a given gas pressure and coronal temperature; similarly, the coronal radiative loss is also explicitly obtained for given values of coronal pressure and temperature. By minimizing the total coronal losses (e.g., $F_r + F_c$), Hearn (1975) obtains a relation between $p_0$ and $T_0$,

$$p_0 = p_0(T_0),$$

which he maintains represents the (stable) equilibrium state of the coronal atmosphere. The crucial point of this analysis, in the present context, is that because of the assumption of an open atmosphere, equation (4.1) obtains only by imposing the ad hoc minimization constraint $\partial\partial T_0(F_r + F_c) = 0$, so that the variation in coronal temperature is made at fixed coronal pressure.

**b) Confined ("Closed") Corona**

As was first shown by Rosner, Tucker, and Vaiana (1978), constraining the coronal temperature distribution by the finite geometry of coronal structures also results in a relation between coronal pressure and temperature in the equilibrium state (e.g., eq. [3.5]); this state may or may not be minimally dissipative, and may or may not be stable. The analysis of § III shows that for simplified (analytically tractable) coronal heating processes, the equilibrium states are indeed thermally stable; note that the stability calculation involves the coronal geometry (viz., the structure size), and does not fix the coronal pressure. In contrast, the "minimum flux" corona does not distinguish between "open" and "closed" coronal geometries; geometry does not enter as an explicit constraint. Because of the overwhelming evidence that, at least in the solar case, nature chooses to impose the geometry upon the corona, the "minimum flux" concept in its present form therefore does not appear to be an appropriate description of coronal formation.

We would like to extend our thanks to T. Holzer, E. Leer, G. Rybicki, and G. S. Vaiana for their many useful comments regarding the instability analysis, and to E. Avrett for his aid in acquainting us with chromospheric diagnostics. This work was supported in part by NASA contracts NAS 8-31374, NAS 5-3949, and grant NSG-7497.

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