STELLAR LUMINOSITY STABILITY: LUMINOSITY VARIATIONS AND LIGHT CURVE PERIOD CHANGES IN BY DRACONIS STARS

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Received 1978 September 21; accepted 1978 December 28

ABSTRACT

We examine the implications for convection in late type stars arising from the observations of BY Draconis type variability. The possibility that the total luminosity of such a star is not constant is emphasized, and further observational tests to better define the variability are suggested. An alternative to standard spot models is explored in which the "missing" flux from "dark" spots is temporally redistributed; this model makes definite predictions about the correlation of mean light and color, and about the quiescent (i.e., unspotted) magnitudes of BY Draconis stars. The time scales of the long-period variability of these stars appear to require secular changes in convective energy transport. We also consider the evidence for period changes in the optical light curves, and conclude that the reality of such changes is far less certain than previously claimed.

Subject headings: convection — stars: atmospheres — stars: late-type

I. INTRODUCTION

Studies of the dMe stars—such as the flare star BY Draconis—have shown that their optical fluxes are modulated on a wide range of time scales. The more rapid quasi-sinusoidal variations have been interpreted as due to rotational modulation of the observed flux by inhomogeneously distributed surface features called, in the solar analog, "spots." Variations in the amplitude of these fluctuations are accounted for by changes in the distribution, areas, and temperatures of the spots (Chugainov 1966; Krzeminski 1969; Bopp and Evans 1973; Torres and Ferraz Mello 1973). Similarly, the slower variations in mean light are also thought to be the result of spot activity (Vogt 1975; Oskinan et al. 1977) may represent redistributed flux. The observed time scales of the light variability suggest that modification of convective energy transport is the most likely mechanism.

Because of the importance of understanding the behavior of BY Dra variables, we shall briefly review the observational evidence and re-examine the conclusions drawn by various authors on the basis of spot models. Our particular concern is the "missing flux" implied by the concept of surface brightness inhomogeneities, and we shall focus on the question whether these inhomogeneities signify real changes in the luminosity. We discuss how studies of mean light and color variability can address this question, and how the "bright spots" invoked at times for BY Dra (Oskinan et al. 1977) may represent redistributed flux. The observed time scales of the light variability suggest that modification of convective energy transport with latitude are presently in dispute (cf. Gilman 1976; Parker 1977a, b; Cowling 1977); the availability of analogous data for stars with drastically different convection zones may, therefore, provide significant new constraints upon available theories. An understanding of the data from which the interpretation of stellar surface differential rotation derives is crucial to such an effort.

The paper begins with a phenomenological analysis of the observations, emphasizing conclusions that are independent of the true nature of starspots, and ends with an analysis of physical mechanisms that might produce variable luminosities. We first summarize the observational evidence for the existence of "star-spots," by which we mean "surface brightness inhomogeneity" ($\Pi I\alpha$). In $\Pi I\beta$ we review spot models, emphasizing those conclusions regarding energy flux conservation which are relatively independent of the detailed physical structure of starspots, and show that the problem of "missing flux" in sunspots (cf. Parker 1974) is relevant to starspots as well. In this context observational tests of the "missing flux" picture are suggested. We next review the evidence for differential rotation, as inferred from period changes in the short
period variability, and examine in detail previous efforts to determine these period changes (§ III). In § IV we discuss in detail the various mechanisms that may lead to stellar luminosity variations, and show that there may be a very close connection between these considerations and problems of solar activity; our conclusions are summarized in § V.

II. SPOT MODELS AND VARIABILITY

a) Evidence for the Existence of Starspots

Any successful theory of BY Draconis type variability must explain the wide range in behavior observed (cf. Oskanyan et al. 1977 for recent summary). The optical amplitudes range from a few hundredths to ~0.3 mag. The light curves are generally quasi-sinusoidal in appearance, but there are cases of asymmetries (Torres and Ferraz Mello 1973) or “flattops” (Evans 1959) in the light variations. For a single star, the magnitude of the amplitudes, the mean light, and the phase of the variation are functions of time (Oskanyan et al. 1977). Only the period remains fairly constant, and this, too, is suspected to change (Oskanyan et al. 1977). The arguments supporting “spot” models, and eliminating alternative theories, can be briefly summarized as follows.

The possibility of eclipses accounting for the observations is ruled out by phase shifts and the typical quasi-sinusoidal light curves (Krzeminski 1969). Pulsational models are untenable in view of the short periods (Gabriel 1967). Evans (1971) has considered obscuration by circumstellar material, but Torres and Ferraz Mello (1973) ruled this out on dynamical grounds. In addition, this explanation will not work for BY Dra, where the orbital and photometric periods are not commensurate (Krzeminski and Kraft 1969), nor for single BY Dra variables (Bopp and Espenak 1977).

In the case of the most rapidly rotating BY Dra star, YY Gem, line broadening shows that the rotational period is close to the photometric period. For most other binary BY Dra variables (except BY Dra) the photometric period is close to the orbital period; this conforms with the idea that tidal friction enforces synchronous rotation.

The chromospheric emission from these stars is certainly a surface phenomenon. There is evidence that the Hα emission is correlated with the visible light variation (Busko, Quast, and Torres 1977) as would be expected from active regions concentrated near dark spots. Spot models can successfully reproduce the light and color variations seen (Bopp and Evans 1973; Torres and Ferraz Mello 1973; Vogt 1975). A further strong argument in support of “spot” models emerges from the observation that if surface brightness inhomogeneities exist, then in an eclipsing binary, distortions of the eclipse should occur on much shorter time scales than associated with the modulation of the light due to rotation. Such a distortion was observed in the 1948 light curve of the eclipsing binary YY Gem by Kron (1952). The asymmetry observed in primary eclipse would require a light variation of about 5%, over ~2.5 hours, whereas a smooth interpolation of the out-of-eclipse variation would indicate only a 0.8% change during this time period. Thus the conclusion of Kron (1952), that the asymmetric eclipse is due to the eclipse of spots or to some obscuring material lying between the stars, seems justified. Similar arguments have been used by Eaton and Hall (1979) to conclude that spots were present on the giant binary system RS CVn. Finally, extending the solar analogy, we recall that sunspots are correlated with solar flares and chromospheric emission; it is thus not surprising that starspots should occur on extremely active flare and emission-line stars.

b) Spot Models and Flux Redistribution

Recent spot modeling efforts (Vogt 1975; Oskanyan et al. 1977) assume that starspots are uniform and that they radiate like blackbodies (or main-sequence photospheres) with an effective temperature different from that of the ambient photosphere. The portion of the stellar surface not covered by spots—the undisturbed stellar photosphere—is assumed to be unaffected by the presence of neighboring spots. With these assumptions, spots implicitly lead to changes in the total luminosity of a star as determined by its total photospheric radiative flux, particularly on time scales far shorter than those usually associated with main-sequence stellar luminosity variation.

For illustrative purposes, consider the 1965 observations of BY Dra, for which the V amplitude was 0.23 mag. The temperature of the spot is not well determined by the observations; however, Vogt (1975) found an upper bound for the spot temperature $T_s$ of 3700 K for an assumed quiescent photospheric temperature of 4000 K. If $T_s = 3700$ K, then the fractional stellar surface area covered by the spot would be ~35%, and the “missing” luminosity (i.e., the difference between the total flux radiated by the undisturbed photosphere of area equal to that of the spot and the flux radiated by the spot itself) is ~9% of the total stellar luminosity. The spot temperature is bounded below as well: $T_s > 0$ K. At this (unphysical) extreme, the spot would cover ~12% of the surface, and the “missing” flux would be correspondingly large. Thus, for the large-amplitude light variations of such stars, spot models imply luminosity changes $\sim 10^{-1}$ of the luminosity of the unblemished star occurring on time scales of the variations in the amplitude of the quasi-sinusoidal V-magnitude light curve. Because the total outward energy flux at the stellar surface integrated over time must equal the total energy released in the nuclear-burning core, the obvious question arises as to how starspots modulate the total stellar luminosity, i.e., where does the missing energy go? To answer this question we have recourse to three possibilities:

1. The flux is not missing, but has been either redistributed in wavelength or converted into presently unobservable forms of outward energy transport (viz., Alfvén waves, etc.). In this case the assumption of the spot model that the “unblemished” photosphere
remains constant (i.e., unaffected by the presence of spots; Oskanyan et al. 1977) is satisfied.

2. The missing flux is redistributed spatially over the ambient "unblemished" photosphere. In this case the assumptions of the standard spot model are not satisfied.

3. The missing flux can be stored in the interior, and is eventually released (temporal redistribution). This case does not satisfy the constant photosphere assumption either.

Note that in the first two cases the total stellar luminosity is not subject to secular changes, but in the third case it is.

Consider first spectral redistribution: can the "missing" visible radiative flux appear in other spectral regimes? We shall approach this question primarily from a phenomenological point of view. First, on the thermal model, the "missing" flux cannot be redistributed into the infrared: line blanketing might be able to redistribute optical flux to longer wavelengths, but the substantial wavelength dependence of blanketing between 4000 Å and 6000 Å in these stars (see Allen 1973) suggests that for amplitudes greater than 10%, significant color variations (≫1%) should be observed, which is not the case. Furthermore, preliminary observations by C. Anderson and one author (L. H.) of two spot stars show no evidence of the anticorrelation of visual and infrared flux expected on this hypothesis.

Flux redistribution to shorter wavelengths has been considered by several authors. Thus, Mullan (1974), following work by Savage (1969; see also Danielson 1964; Parker 1974) has suggested that the "missing" flux is converted (with 100% efficiency) into an Alfvén wave flux; damping of the resulting wave flux then presumably results in chromospheric or coronal heating, and hence ultimately in UV, XUV, and soft X-ray emission, as well as in possible mass outflow (winds). Irrespective of the details involved in the redistribution process, observations place strong constraints upon the magnitude of this effect. The data of Phillips and Hartmann (1978) show that BY Dra has undergone a quasi-sinusoidal mean light variation, with a time scale of ~60 years and a range of ~0.3 mag. If spectral redistribution is invoked, then the amount of "missing" flux to be accounted for is of the order of 10⁻² L*.

Given that BY Dra is located at a distance of ~16 pc, we place the "missing" energy in the UV above ~10³ Â; the corresponding flux at the Earth is then ~10⁻¹² ergs s⁻¹ cm⁻² Â⁻¹, an amount presently well within the observational capability of, for example, the IUE satellite. The fact that the far brighter (in the visible) RS CVn stars observed with IUE have far less flux in the UV than this amount (Dupree 1978, private communication), even though these stars have rather strong chromospheres, argues strongly against flux redistribution into the UV. In any event, this hypothesis is now easily verifiable.

Similarly, we might place the missing flux in X-rays. If ~10% of the missing energy were steadily radiated in the 0.2–0.28 keV band, the expected flux at the Earth would be ~10⁻¹⁰ ergs s⁻¹ cm⁻². In contrast, the observed peak flux observed from a flare on YZ CMi (Heise et al. 1978) was 6 × 10⁻¹⁰ ergs cm⁻² s⁻¹; thus, such missing flux would again be easily observable, particularly if it were not transient (e.g., flarelike) in character. No such steady X-ray emission has been reported. If it is argued that the required X-ray emission is entirely transient in character, being released solely during flaring events, similar difficulties arise. Thus, Kunkel (1970) found that the largest flare ever seen on YZ CMi had a total energy ~6.6 × 10⁴ ergs; to account for 10% of the luminosity of this star, such a flare would need to occur on the order of once every 8 hours. However, Kunkel (1970) estimated that such an event occurred less than about once every 300 days, or 10⁶ times less frequently than necessary. Furthermore, Lacy, Moffett, and Evans (1976) have shown that the mean luminosity in the broad U band due to flaring for several stars is typically ~10⁻⁴ of the total luminosity of the star. Unless the energy in other wavelength regimes is 10⁶ times the U flux, which seems doubtful, flares seem unlikely to be able to carry away enough energy to make up for the missing flux. These difficulties are reminiscent of the problems encountered by the Alfvén wave model of Parker (1974) for sunspots: as discussed by Cowling (1977), the missing flux in the solar case would be sufficient to allow continuous flaring; furthermore, Cowling suggests that the Alfvén wave model is unlikely because of the extremely high efficiency of conversion of convected energy into Alfvén waves required (>80%, see, however, Parker 1977b). In any case, only flux redistribution into some as yet unobservable form (e.g., continuous wave and/or mass flux ejection without any associated heating and observable radiative emission) can sustain the first redistribution possibility.

Next, we consider the possibility that the missing flux is simply redistributed (eventually) over the ambient photosphere, rather than being converted into some other (unobservable) energy transport mode; such energy redistribution has been commonly considered in the standard model of sunspot formation (see Cowling 1977 and references therein). Because this redistributed flux must eventually emerge, spatial or temporal energy redistribution will result in, respectively, spatial or temporal inhomogeneities in the color of the star. Now, BY Dra has been observed to vary by 0.07 mag in mean B - V (Oskanyan et al. 1977), and it is therefore natural to ask whether this color might reflect flux redistributed from spot areas.

We can obtain an estimate of the effects attributable to purely spatial redistribution, ensuring temporal constancy of the luminosity, by considering the above variation of BY Dra; we assume a spot area covering ~35% of the stellar surface and a spot temperature Tₕ ≈ 3700 K (in order to minimize the "missing" luminosity). If the "missing" energy is smoothly redistributed across the rest of the stellar surface, a photospheric temperature change, from 4000 K to 4140 K, results. Using Johnson's (1966) calibration of B - V and Tₕ, we obtain Δ(B - V) ≈ ~0.05; taking into account the secondary, which contributes approximately one-third of the light (Hartmann and Anderson...
correspondingly larger. The exact result depends upon
that bright, as well as dark, spots were required to
change in \( \frac{B}{V} \) ~ ~0.03, rather smaller than the
bright regions in the course of accounting for the
connection, we recall that Oskanyan et al. (1977) found
upon the exact relation between spot activity and the
spatial flux redistribution provide these relatively
meet the observations if the unblemished photosphere
be most important whenever the largest dark spots
occur. Two distinct possibilities then arise, depending
on the exact relation between spot activity and the
observed light variability.

First, let us assume that the level of spot activity
correlates with the amplitude of the short-period (e.,
rotationally modulated) light variation. In that case,
one would expect the occurrence of large amplitudes to
correlate with periods of relatively bluer average
\( B - V \) because the ambient photosphere must com-
Talpate for the flux deficit of the spot by increasing its
effective temperature; this prediction is inconsistent
with the actual behavior of BY Dra (cf. Fig. 1 of
Oskanyan et al. 1977). Furthermore, in the course of
large-amplitude variations, essentially no change in
\( B - V \) is observed over the course of a (rotation)
period; the implication is that any existing surface
inhomogeneity must be dark (e., only dark spots may
exist), as large bright surface inhomogeneities are
expected to produce significant \( B - V \) changes (Torres
and Ferraz-Mello 1973; Vogt 1975). For example, at
the time of the 0.23 mag amplitude, if flux were re-
distributed over an area of approximately the same
size as the spot (e., 35% of the surface), then from
the previous calculation of the color change, a change
\( \sim -0.06 \) in \( B - V \) is expected. This color change,
rotationally modulated, would be easily apparent.
Even if the flux were deposited at the observed pole,
and not modulated by rotation because of the inclina-
tion of BY Dra, the change of 0.06 in mean \( B - V \)
also ought to be apparent. Uniformly redistributing
the flux should result in a mean \( B - V \) change of
\( -0.03 \), which again should be possible to detect. In
any of these cases the color should be correlated with
the amplitude; this correlation is not observed. There-
fore, we conclude that spatial flux redistribution is
inconsistent with spot activity as defined by the short-
period amplitude.

An alternative definition of spot activity is related to
the level of mean light. In this view, the fractional
stellar surface area covered by spots determines the
mean light level, while the amplitude of the rotational
modification is fixed by the degree of surface inhom-
geney of the spot distribution. The bluest color
should, therefore, occur near minimum mean light
(corresponding to the maximal total spot area). How-
ever, the data shown in Figure 1 for BY Dra indicate
that the star tends to be bluest at maximum mean light;
weaker evidence for a similar correlation in the be-
behavior of CC Eri is also indicated. The \( \langle B \rangle \) versus
\( \langle B - V \rangle \) changes are roughly consistent with a simple
change in effective temperature at constant radius, as
calculated using Johnson's (1966) calibration and \( M_e \)
from Allen (1973) for the main sequence; we return to
this point in § IV. It may be argued that, because of the
inclination of BY Dra to the line of sight, significant
flux redistribution occurs in the (permanently) ob-
sured portion of the stellar surface. Such "invisible"
redistribution is quite ad hoc and requires transport of
the missing flux from widely separated regions, which
is not easily accounted for: why should the "missing"
flux be redistributed precisely to that part of the stellar
surface which cannot be observed? In any event, this
redistribution process can be observationally tested by
studying stars seen nearly equator-on such as YY Gem.
We conclude that in spatial redistribution of the
"missing" starspot flux does not seem to be consistent
with available observations.

Next, we consider temporal flux redistribution, that
is, temporal variations in the total stellar luminosity
such that the mean luminosity integrated over time
scales larger than considered here (\( \geq 10^3 \) s) is constant.
In this case, the flux of energy "missing" from stars-
spots must be trapped or stored by some means below
the photosphere, to be released when the spots decay.
Postponing to § IV a discussion of the means by which
this temporal redistribution can be accomplished, we
note here only that the data shown in Figure 1 now
yield to a simple explanation: maximum mean light,
corresponding to minimal spot activity, is now expected to coincide with the release of "dammed-up" energy, and hence with a higher-than-average surface luminosity and effective temperature. We can similarly consider the time-dependent nature of the $B-V$ change of BY Dra (Fig. 1 in Oskanyan et al. 1977): during 1965-1966, both large surface inhomogeneities and low levels of mean light were observed, indicating large (dark) spot activity. Following this period, the mean light level increased, reaching a plateau in roughly 1970. Concurrently, the star appears to have become bluer with the rise to maximum light. This sequence of events is strongly suggestive of temporal energy redistribution, with the deficit incurred during maximum spot activity balanced by subsequent energy release during the decay phase of the dark spot(s). This process ensures that—in contrast to purely spatial redistribution—brightening of the "blemished" photosphere is temporally separated from periods of substantial spot area, making it more likely that rotational $B-V$ variations are seen at maximum light. The detailed evolution of $B-V$ depends upon the energy release rate and the area over which the excess is distributed; clearly, far more data are required to investigate this suggestion in detail than are presently available.

An immediate consequence of this scenario is that the quiescent ("immaculate") magnitude level, e.g., the star's brightness during extended periods of low spot activity, must lie below maximum mean light. Although determination of the immaculate level is quite uncertain, the results for BY Dra reported by Oskanyan et al. (1977) are in accord with this expectation.

To summarize, we may draw the following conclusions. Present data suffice to exclude spatial flux redistribution as the primary cause of variability, and to strongly constrain spectral redistribution; in the latter case, broader spectral coverage observations of BY Dra variables are likely to settle the issue, although present data suggest that this process is also unlikely to account for the "missing" radiative flux from starspots. The ad hoc proposal that spatial redistribution to unseen portions of the stellar surface (for stars with rotation axis relatively aligned with the line of sight) is responsible for balancing the observed flux deficit can be tested by studying the behavior of spot stars of different rotational inclination as a function of viewing angle (cf. § IV below). More detailed photometric studies of the short-period $B-V$ variation can be usefully employed to set tighter constraints upon the level of "unspotted" photosphere brightening concurrent with spot activity; furthermore, longer-based observations would be useful in determining whether the time-integrated emission above the (hypothesized) immaculate level equals the flux deficit incurred below this level, providing a direct test of temporal flux redistribution. Such observational efforts are central to understanding the "missing flux" problem, which figures largely in our understanding of solar (and stellar) surface phenomena. The problem of missing flux goes directly to the central assumption of previous spot models, namely, that the unblemished photosphere remains constant.

III. SPOT ROTATION PERIOD CHANGES

Analysis of the temporal behavior of the light variability exhibited by stars such as BY Dra has led a number of authors to conclude that the presumed surface features must undergo changes in their rotation period (Vogt 1975; Oskanyan et al. 1977), possibly due to differential rotation in latitude of the stellar surface layers, in further analogy with the solar prototype. As the presence of such differential rotation is an important ingredient for our understanding of both theories of stellar convection and magnetic dynamos, the observation of such effects on stars significantly different from the Sun has obvious import. Here we reexamine previous "spot" rotation period determinations to demonstrate that the claimed period changes may not be significant.

The particular data we focus upon is that of BY Dra discussed by Oskanyan et al. (1977). These authors derived spot rotation periods from photometric observations as follows: a given set of photometric observations was fitted with a sine wave of the form

$$f(t) = A \sin \left[ \frac{2\pi(t - t_0)}{P} + \phi \right] + \tilde{f} \quad (3.1)$$

where $\tilde{f}$ is the mean magnitude of the data set, $A$ its amplitude, $t_0$ an arbitrary reference time ($t_0 = JD 2,440,000$), $P$ the period, and $\phi$ the phase; the procedure involved a least-squares fitting of the parameters $\tilde{f}$, $A$, $P$, and $\phi$ (cf. Vogt 1975 for an alternative procedure substantially insensitive to the light curve shape). The full data set employed in the period analysis could not be well fitted by a single sinusoid; Oskanyan et al. therefore segregated the full data set into superficially homogeneous subsets, which were then analyzed individually. Their period determinations are given in Table 1. A remarkable aspect of these results is that the quoted uncertainties for the derived periods are one to two orders of magnitude smaller than the period "jitter" between data samples exhibited by these determinations (see also Vogt 1975). Do these errors accurately reflect the real uncertainties in our knowledge of the light curve period?

The detection of periodicities in discretely sampled data, as well as the general area of parameter estimation, is a very well studied subject; we refer the reader to recent general reviews by Lampton, Margon, and Bowyer (1976) and Rothschild (1977), who focus upon astronomical applications (primarily in the X-ray domain), as well as to standard texts on error and time series analysis (see Bloomfield 1976). Two distinct types of period analysis must be distinguished:

1. One considers discretely sampled data which are—on a priori grounds—believed to contain a fixed number of periodic components whose periods are to be determined. In the simplest case the data have the form

$$f(t) = s(t) + e(t), \quad (3.2)$$

where $s(t + T) = s(t)$ is the sought-for periodic signal
Light Curve Period Analysis for BY Draconis

### TABLE 1

<table>
<thead>
<tr>
<th>Data Sample (JD t - 2,440,000)</th>
<th>Period* (days)</th>
<th>Period† (days)</th>
<th>Period‡ (days)</th>
<th>Nyquist Period§ (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 2181-2668</td>
<td>3.739 ± 0.008</td>
<td>...</td>
<td>...</td>
<td>9.55</td>
</tr>
<tr>
<td>2. (a) 2181-2218</td>
<td>3.806 ± 0.005</td>
<td>(7)</td>
<td>...</td>
<td>7.40</td>
</tr>
<tr>
<td>(b) 2237-2281</td>
<td>3.829 ± 0.003</td>
<td>...</td>
<td>...</td>
<td>3.52</td>
</tr>
<tr>
<td>(c) 2295-2373</td>
<td>3.892 ± 0.04</td>
<td>3.8 (+1.2, -0.7)</td>
<td>4.0 ± 0.3</td>
<td>1.82</td>
</tr>
<tr>
<td>(d) 2599-2609</td>
<td>3.909 ± 0.02</td>
<td>4.2 (+0.9, -0.6)</td>
<td>3.9 (+0.3, -0.4)</td>
<td>2.00</td>
</tr>
<tr>
<td>(e) 2623-2657</td>
<td>4.140 ± 0.02</td>
<td>5.0 (+5.2, -1.7)</td>
<td>4.1 ± (7)</td>
<td>2.62</td>
</tr>
<tr>
<td>(f) 2640-2657</td>
<td>4.4 (+0.7, -0.4)</td>
<td>4.3 ± (7)</td>
<td>2.34</td>
<td></td>
</tr>
<tr>
<td>3. 2623-2657</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

* Period determinations due to Oskanyan et al. (1977), using a least-squares sinusoid fitting procedure.
† Period determination via discrete Fourier analysis; quoted uncertainties are obtained from the width at half-maximum of the dominant peak in the power spectrum.
‡ Period determination via residual analysis, with least x^2 fitting of the phase and amplitude of sinusoid trial functions; quoted uncertainties are obtained as described in the text.
§ Nyquist period is defined as the inverse of the Nyquist frequency [( = 0.5/Δt)], where Δt is the mean sampling interval for the data time series in question.
|| Inadequacy of sinusoid fitting leads to sufficiently large x^2 values that meaningful assignment of uncertainties is impossible (see text).

The superposition of a single well defined periodic component and noise, or the superposition of a number of distinct periodic components and noise—and the physical question—viz., what is the physical realization (e.g., the stellar surface morphology) which underlies the component(s) making up the observed light curve? In the absence of information other than the observed light curve, the mathematical question can be answered only by means of the second of the two analysis procedures; the result of this analysis may then help resolve the physical question. If one instead answers the physical question first (e.g., either by adopting an ad hoc hypothesis or by using new information, other than that contained in the observed light curve), then the mathematical question as posed above becomes irrelevant, and the first of the analysis procedures can be used. In this second instance, the results of light curve analysis verify not the correctness of the initial hypothesis but only its consistency; thus, the small uncertainties in the period determinations of Oskanyan et al. (1977; see also Vogt 1975) are the errors in the derived periods associated with the particular assumed surface structure, which may not be unique.

For example, consider the following alternative means of generating a light curve with well-defined period on time scales comparable to the stellar rotational period, but varying period on significantly longer time scales: (a) A single star "spot" group is dominant at any given time, is stable on the rotation period time scale, and has a temporal overlap with other "spot" groups whose duration is short when compared to the group lifetime; successive "spot" groups form at distinct stellar latitudes, which are associated with distinct rotation periods (surface differential rotation). (b) A single "spot" group is dominant at any given time, has a lifetime somewhat longer than the rotation period, and is composed of "spots" whose individual lifetime is much shorter than that of the group as a...
whole and which individually rotate at the same stellar rotational frequency; however, emergence of new “spots” occurs preferentially at larger (or smaller) stellar longitude than previously emerged “spots,” so that the “spot” group as a whole rotates faster (or slower) than the star itself. If one begins with these alternatives, the light curve data can be used to test for consistency (by testing for goodness of fit), and to derive the rotational frequency of the spot group in question; to distinguish between the alternatives is impossible on grounds other than plausibility. Note that the accuracy of the frequency determination is relatively independent of the appropriateness (goodness of fit) of the hypothesis: the uncertainty in the theoretical light curve is a poor representation of the observed light curve. These methodological distinctions are complicated by the fact that the light curve data show changes in amplitude, mean light, and apparent phase on a variety of time scales. Thus, Oskanyan et al. (1977) show that changes in mean light and amplitude from year to year are the rule rather than the exception in BY Dra, and that much shorter time-scale changes can occur. In the same vein, Vogt’s (1975) analysis assumed constancy of “spot” properties over the time span covered by his August and November 1973 observations; however, the mean light level changed by 0.015 mag during this time period, showing that “spot” properties may have changed significantly when compared with the 0.04 mag total V amplitude. In consequence, the two alternative models presented above are far too simplistic, and ad hoc model building becomes much more complex and uncertain.

The practical distinction between the two methodologies applied to “spot” star light curves is, therefore, one of interpretation. The cyclic light curves are clearly variable on many time scales; the physical question is whether these variations are due to the action of differential rotation alone, or to the time-dependent nature of “spot” areas and “spot” emergence at the stellar surface, or to both. In the absence of spatial resolution, there seems no compelling reason for a priori believing differential rotation alone to be responsible; however, if it could be shown that the observed light curve is strongly dominated by a single frequency component at any one time (suggesting spot stability), the differential rotation hypothesis gains considerable plausibility (but is not proved). To this end, we have applied the second of the two alternative methodologies to the data given by Oskanyan et al. (1977). Two distinct (but related) analysis procedures are available:

1) Discrete Fourier Analysis

As the sought-for effect is frequency jitter in the light curve data, its successful detection requires that the frequency resolution of the data exceed the magnitude of the expected frequency variation. A direct test involves the computation of the discrete power spectra for the time series in question; some examples are shown in the second column of Figure 2. Referring to Table 1, we first note that in only two of the cases is the range of allowed periods well constrained (intervals 2a and 2f), and is somewhat less constrained in two further cases (intervals 2d and intervals 2a and 2f taken together); in the remaining cases, no conclusion can be drawn, primarily because the sampling rate does not meet the Nyquist criterion (cf. Bloomfield 1976). If one assumes that the frequency spread in the power-spectra peaks—for the three cases in which evidence for a periodic component exists—is entirely due to insufficiently extended sampling, then an estimate for the range of significantly contributing frequency components can be obtained by measuring the frequency width at half-power. The results are to be found in Table 1 and can be interpreted as follows: in the absence of other, ad hoc information, the given data sets do not allow us to distinguish between a single periodic component and two (or more) distinct periodic components whose periods differ by an amount less than the period uncertainties quoted in Table 1. This does not mean that any single component whose period lies in this range fits the data as well as any other in this range; rather, this analysis shows the range of possible contributing frequency components of substantial amplitude.

We note that this analysis is substantially complicated if—as is the case here—the data are strongly anharmonic, so that sinusoids are poor fitting functions. Under such circumstances substantial power lies in the harmonics of the fundamental; in addition, the discrete Fourier transform will produce an artificially broadened peak at the fundamental frequency, thereby reducing the signal-to-noise and increasing the period determination uncertainty (cf. Bloomfield 1976). In consequence, changes in the mean light level and in the light curve shape on time scales comparable to the rotation period can mimic secular changes in the (true) fundamental frequency. Because of the low sampling rate of the present data, more sophisticated analysis (viz., involving examination of the extended harmonic spectrum) is, unfortunately, not warranted.

2) Residual Time Series Analysis

The least-squares fitting procedure employed by Oskanyan et al. (1977) can be generalized by computing the residuals

\[ r(\tau) = \frac{1}{N - n} \sum_{i=1}^{N} [f(\tau, t_i) - I(t_i)]^2 / \sigma_i^2 \]  

as a function of rotation period \( \tau \), where \( N \) is the number of observations in the observing interval, \( f(\tau, t_i) \) the trial fitting function, \( n \) = 4) the number of parameters, \( I(t_i) \) the observed magnitude at time \( t_i \), and \( \sigma_i \approx 0.01 \) the uncertainty in the \( i \)th observed magnitude measurement. As the trial fitting functions (sinusoids of the form given in eq. [3.1]) contain additional fitting parameters—the mean \( f \), phase \( \phi(\tau) \), and amplitude \( A(\tau) \)—the fit is obtained only after a least-\( \chi^2 \) fit of the mean phase and amplitude for every \( \tau \). This procedure
Fig. 2.—Time series analysis of photometric observations of BY Dra. (a) Photometric data for data samples 2(d) and 2(e) [Table 1] from Oskanyan et al. (1977); (b) Power spectra of data samples 2(d) and 2(e); (c) Least reduced $\chi^2$ period analysis of data samples 2(d) and 2(e). Methods of analysis are described in the text; the data samples displayed here are the only ones from Table 1 to which both types of analysis can be successfully applied. Note that the relatively large values of reduced $\chi^2$ here obtained for all values of the period indicate that sinusoids are inappropriate as fitting functions.

Therefore explicitly tests the hypothesis that a given periodic component, with period $\tau$, is present in the data. Because of the strong nonlinearity of the fitting procedure for the phase $\phi$, stable $\chi^2$ fitting is insured by rewriting the fitting functions in the form

$$A \sin [2\pi t/\tau + \phi] = A' \sin (2\pi t/\tau) + B' \cos (2\pi t/\tau),$$

(3.5)

where $A' = A(1 - \sin^2 \phi)^{1/2}$ and $B' = A \sin \phi$ are the new $\chi^2$ fitting parameters. The results are shown in the last column of Figure 2, in which we plot the residuals (which are in fact the reduced $\chi^2$ values for the four fitting parameters $A'$, $B'$, $f$, and $\tau$) as a function of test period for each of the observing intervals considered. We note that the residual analysis confirms the results of the discrete Fourier analysis, as expected. This technique also allows one to place quantitative limits upon the confidence levels to be associated with these period determinations. Specifically, the errors quoted in Table 1 correspond to the 0.01 probability confidence level of exceeding the given values of the reduced $\chi^2$ for cases 2d and 2e. Note that the large values of reduced $\chi^2$ obtained confirm our earlier assertion that sinusoids are inappropriate fitting functions. It is clear that—in the absence of additional information—the period determinations are inadequate to permit conclusions regarding secular period changes to be drawn.

In summary, we emphasize that previous light curve period determinations have been based upon the a priori assumption that the time-dependent nature of starspots may be ignored over the duration of given
data sample intervals. For certain subsets of the observations, this assumption may well be valid. However, in the absence of spatial resolution, it cannot be proved from the data; and in light of our results, which show the data to contain frequency components of substantial amplitude over a significant frequency range, the assumption does not appear plausible at present. Further observations, involving more extended sampling of consecutive rotational periods than performed to date, are required to establish plausibility; in view of the sampling requirements discussed above, the observational effort required is certain to be very major.

IV. MECHANISMS PRODUCING LUMINOSITY VARIATIONS

In §II we showed that BY Dra stars may have short-term variable luminosities. We examined alternatives to this conclusion, and found that (1) spectral flux redistribution is unlikely, but needs to be tested by further observations; (2) surface redistribution of flux appears to be excluded unless it is hidden near an unobservable rotational pole. In this section, we examine possible physical mechanisms which may lead to either of the two surviving viable alternatives—temporal or nonuniform spatial flux redistribution—focusing particularly upon the possibility that the bolometric luminosity of a BY Dra star is variable.

Even excluding flares, the range of observed time scales involved in flux changes is still remarkably broad. Thus, Oskanyan et al. (1977) show that variation in spot properties can occur in less than a day, although in general it appears that the light curves are stable over an observing season. Furthermore, Phillips and Hartmann (1978) have shown that the overall behavior of mean light in BY Dra appears to have a strong modulation with time scales of decades. Our inquiry into possible mechanisms for luminosity variations centers on the analysis of their respective time scales. Following Gough (1977), we shall distinguish between the characteristic time scales governed by the following:

1. Dynamical effects.—These include stellar oscillations (viz., acoustic modes), convection, and rotation. These time scales range from the convection time scale ($\tau_c \sim L/v$, $L$ eddy scale size, $v$ convective velocity, $\tau_c$, on the order of minutes, at the top of the solar convection zone) to the rotation period.

2. Thermal effects.—These reflect the efficiency of thermal diffusion in the outer stellar envelope, as well as its heat capacity. For appropriate stellar models (cf. Copeland, Jensen, and Jorgensen 1970), these time scales are of the order of $10^{14}$–$10^{15}$ s.

3. Nuclear burning.—Here the crucial determinant is whether reaction chains are in an equilibrium state, in which case stellar models yield time scales in excess of $10^{17}$ s. In any case, the time scale of observable fluctuations due to secular changes in the nuclear-burning core is bounded below by the photon diffusion rate from the core outward, which for these models is in excess of $10^{11}$ s.

In order to gain a more quantitative perspective on this issue, we have calculated various significant time scales for main-sequence stars, using the stellar structure models of Copeland, Jensen, and Jorgensen (1970); the results are plotted in Figure 3. The time scales are obtained as follows. The convective time scale is set equal to the turnover time of the largest eddy under the mixing length theory; thus, $\tau_c \sim H/\nu$, where $H$ is the maximum local pressure scale height in the convective zone, and $\nu \approx (L_\nu/4\pi r^3 \rho)^{1/3}$ the corresponding convective velocity (with $L_\nu$ the stellar luminosity, $r$ the radial position of the cell, and $\rho$ the associated density).1 Dynamic effects are characterized by the free-fall time $\tau_{ff} = [R_*/(GM_*/R_*)]^{1/2}$, where $R_*$ is the stellar radius and $M_*$ the stellar mass. The thermal time scale is estimated by calculating the Kelvin-Helmholtz time scale, given by $U_*/L_*$, where $U_*$ is the thermal energy content of the convective zone.

1 This estimate is uncertain and likely to be a least upper bound on the convective time scale. Effects such as departures from the mixing length theory (cf. Graham 1975) and corrections to the energy transport rate from detailed mixing length models will increase $\tau_c$; an estimation of the likely magnitude of this increase results by noting that solar coronal holes, which are thought to reflect deep-seated solar convection, have a lifetime approximately an order of magnitude larger than $\tau_c$, as calculated for the Sun (Fig. 3; see Timothy, Krieger, and Vaiana 1975). A more appropriate choice for the relevant length scale could therefore be the depth of the outer convection zone, which may be several times larger than $H$. 

![Figure 3](https://example.com/figure3.png)
As indicated, long-term (e.g., not associated with rotational modulation) changes in the mean stellar luminosity for "spot" stars such as BY Dra or CC Eri occur on time scales of $10^8$--$10^9$ s (Phillips and Hartmann 1978); it is striking that only the convective time scale falls close to the range associated with observed "spot" star luminosity variations. We are therefore led to ask if there exist plausible mechanisms which may account for convection-modulated luminosity variations; three candidates are considered below.

a) Spatial Redistribution and Permanent Obscuration

The simplest method of accounting for the slow secular changes in observed mean flux is to assume a persistent spatial redistribution of the surface flux (cf. § II) such that the visible portion of the stellar surface has a consistently higher (or lower) surface flux than the corresponding permanently obscured portion. This hypothesis requires the line of sight to the star to lie significantly outside its equatorial plane (so that a surface area sufficiently large to account for the magnitude of the change in mean light is permanently obscured), and can be immediately excluded if a "spot" star is found which both shows long-term mean light variations and has its rotation axis perpendicular to the line of sight. The obvious attractiveness of this mechanism is that it does away with the possibly awkward requirement of producing true changes in the total stellar luminosity. Its weakness lies in the hypothetical process which can account for the lateral heat transport necessary to ensure spatial flux redistribution; that is, it seems rather unlikely that the redistribution process manages to eliminate the observed flux excess (deficit) by decreasing (increasing) the surface luminosity precisely where one cannot observe it. Note in particular that this process must act to redistribute flux from well-separated portions of the stellar surface (as changes in mean light under this hypothesis are entirely due to contrast variations between the permanently obscured and visible surface regions). Whether turbulent convection can accomplish this task is an open question (cf. Cowling 1977; Parker 1977b). While this possibility seems very unlikely, present data are not conclusive because the best-studied examples (viz., BY Dra) have rotational axes inclined appreciably to the line of sight. Studies of stars seen equator-on, such as YY Gem, should easily settle this point.

b) Stochastic Thermal Flux Inhibition

Convection in late-type main-sequence stars is generally held to be turbulent, the number of convective eddies of a given scale size fluctuating in time; thus, under the assumption of Poisson statistics, one expects variations in the number of eddies having a lifetime $\tau$ to be of order $\sqrt{N}$, $N$ the mean number of such eddies, occurring on time scales comparable to $\tau$. Because the total instantaneous energy flux carried by the eddies is simply the product of the energy transported by one eddy and the total number of such eddies, fluctuations in luminosity may occur simply as the result of such statistical variations in total eddy number (Schwarzschild 1975; Gilman 1976). However, the results given in Figure 3, which show that the longest eddy turnover time—under the mixing length hypothesis—falls near the lower limit of the observed luminosity fluctuation time scale range, would seem to exclude this mechanism as responsible for these observations.

Before accepting this negative conclusion, we caution that its validity hinges on the suitability of mixing length calculations. Thus, Graham (1975) has shown that—assuming values of the Rayleigh number within $10^5$ of its critical value—for two-dimensional compressible convection, the dominant convective eddies are far larger than assumed under the mixing length approximation. For example, if the size scale of the largest eddies is determined by the physical dimension (depth) of the outer convection zone, which is an order of magnitude larger than the largest eddy calculated under the mixing length theory for a $0.6 M_\odot$ main-sequence star, then $\tau$, falls within the most rapid time scales of variability. Such convective cells, if subject to fluctuation in their number as discussed above, would produce luminosity variations of the order of $15\%$; this is of the same order of magnitude as the predictions of standard spot models (cf. § II) and suggests that weakly turbulent, large-scale convection cells can account for at least some of the data. However, the observed variations occurring on the scale of years to decades (Phillips and Hartmann 1978) are still unaccounted for.

c) Convective Efficiency Fluctuations

Within the context of mixing length convection theory, changes in convective efficiency can also result from variation in the parameter $\alpha = l/H$, $l$ the eddy size scale and $H$ the local pressure scale height (cf. Deinzer 1965; Ulrich 1975; Dearborn and Newman 1978). Although the notion of a mixing length is not meaningful in the nonlinear regime in which the bulk of the convective zone operates (see Graham 1975), we shall nevertheless retain $\alpha$, and treat it in the following as a measure of the mean efficiency of convective energy transport. For example, a decrease in $\alpha$ from its nominal (mean) value reduces the energy flux carried by convection and, as convection transports virtually all the energy outward to the stellar surface in late-type main-sequence stellar envelopes, results in a decreased stellar luminosity. These variations are self-limiting in the sense that, because the nuclear core's energy output is essentially constant over the time scales of interest, the "dammed-up" energy at the base of the convection zone forces a temperature increase at the base, whose ultimate effect is to restore the efficiency of convective energy transport—irrespective of the mechanism responsible for the fluctuation in $\alpha$ (cf. discussion by Moss and Tayler 1969); in this case, luminosity recovery occurs on the convective time scale ($\gtrsim 10^9$ s for $M_*=0.6 M_\odot$). In contrast, if the convective efficiency is maintained at a reduced level (by some unknown means), the stellar luminosity will also regain its
former value, but on the thermal time scale characteristic of the convective envelope (~10^{12}s); these long time scales prevail because alternative transport processes in the convection zone, such as radiative diffusion, are ineffective in transporting significant energy as a result of the low value of the radiative diffusivity in this region. Because the variation considered here is very rapid when compared with the thermal relaxation time scale, the 0.1 L* fluctuations we are concerned with should not significantly affect the physical conditions in the radiative core.

The only mechanism capable of affecting convective efficiency which has been studied in some detail involves the suppression of convection by magnetic fields (see Moss and Tayler 1969 for references prior to 1969; also Ulrich 1975; Gough 1977). In order to obtain quantitative estimates for the change in convective efficiency required in order to reproduce the observed luminosity variations, we have calculated the fractional change in luminosity δL*/L* resulting from given changes in the efficiency parameter α, using the analysis of Dearborn and Newman (1978) applied to the main-sequence stellar models of Copeland, Jensen, and Jorgensen (1970). Our analysis is based upon the following considerations. The observations indicate that variations in luminosity occur on a time scale τ of days to several years, and hence on a time scale substantially shorter than the convection zone thermal time scale τ_{th}. If a change in convective efficiency is responsible for these observed variations—as we are suggesting—then this behavior occurs in a regime in which luminosity variations closely follow changes in α (i.e., δL*/L* ∝ δα; see Dearborn and Newman 1978). That is, rapid changes in α result in rapid changes in L*; only cessation of convective efficiency fluctuations leads to a relaxation of the luminosity to its quiescent value (=radiative core energy output rate) on the thermal time scale τ_{th}. Figure 4 gives the calculated change in luminosity under the assumption of rapid fluctuations in convective efficiency, and shows that for a 0.5–0.6 M_⊙ "spot" star, 5% luminosity variations are produced by ~10% variability in α. Because the convection zone represents a large reservoir of internal energy and gravitational potential energy, the concomitant change in stellar radius on these short time scales (<<τ_{th}) is negligible; we therefore expect the star to closely follow a constant radius trajectory in the ⟨B − V⟩ versus ⟨B⟩ diagram. This expectation is consistent with the observed changes in ⟨B⟩ and ⟨B − V⟩ given in Figure 1, which also shows the constant radius trajectory.

These results impose a lower bound upon the variation in convective efficiency required to produce a given luminosity change, and an upper bound on the change in ⟨B − V⟩ for a given ⟨B⟩ change because we assume the effect to be homogeneous. If changes in α are not homogeneous—for example, if they are larger in the vicinity of "spot" regions—then δα/α will be larger locally than given in Figure 4. As an example, consider the sunspot model of Deinzer (1965), who employed mixing length theory to deduce the efficiency reduction required to reproduce the observations. The values of α obtained ranged from 0.185 (T_{spot} ≈ 3800 K) to 0.556 (T_{spot} ≈ 5000 K); we shall be conservative and adopt the value α = 0.556. Then, assuming α ~ 1.5 in the absence of magnetic fields, a star with one sixth to one third of its surface covered by such spots will have a reduced convective efficiency ⟨δα/α⟩, averaged over the surface, of 0.1–0.21. As can be seen from Figure 4, this suffices to account for the suggested luminosity variations. It is evident, however, that many more observational data on other stars are required, particularly to verify the relationship indicated in Figure 1; measurements at a wide

![Fig. 4.—Fractional change in stellar luminosity as a function of given fractional changes in convective efficiency, for various values of the stellar mass (solid lines, α = 1.0; dashed lines, α = 1.5). Calculations are based upon the main-sequence stellar interior models of Copeland et al. (1970), and follow the (solar) analysis of Dearborn and Newman (1978).](image-url)
range of wavelengths are also desirable in order to better define the expected changes in stellar effective temperature.

What is the physical process by which magnetic fields can affect the overall efficiency of the convection zone, yielding the required 10–15% fluctuations in $\alpha$? Theoretical studies to date are unfortunately inconclusive. There is little doubt that magnetic fields are effective in suppressing the onset of convection, although this inhibition is likely to be transitory (cf. Moss and Tayler 1969). The more relevant question of how magnetic fields affect the convective efficiency—once convection has been initiated—remains unclear. The numerical studies of Weiss and collaborators (see Weiss 1976) show that for laminar convection, initially uniform magnetic fields are concentrated by the flow, leading to the virtual exclusion of magnetic fields from a portion of the convecting volume; in steady state, the fluid segregates into freely convecting volumes which are virtually field-free and strong-field regions in which convective motions are suppressed. If we impose a time-independent energy flux entering the base of the convection zone, then does the presence of localized regions in which convection is inhibited affect the total rate of convective energy transport? We suggest that the answer must depend upon the fractional surface area occupied by strong field region. That is, we expect an effective reduction in convective efficiency, at minimum, during the time interval in which the still freely convecting (field-free) volume is adjusting (viz., by increasing the vigor of convection) to the increased demand for energy transport placed upon it. If “spots” occupy a small fraction of the stellar surface, this time interval may be very short (e.g., of the order of the turnover time of eddies with size scales comparable to the “spot” diameter). If the “spot” area is sufficiently large, the field-free volume may not be able to adequately respond; reduced effective convective efficiency for the convection zone as a whole would then be maintained until the temperature rise at the base is sufficient to force readjustment of the strong-field regions, possibly by destroying the flux concentrations.

The above arguments are only suggestive of processes likely to be important in modulating convective energy transport, and must remain speculative until appropriate numerical calculations can be performed. Particularly relevant are studies of (1) the response of fully developed laminar convection (including magnetic fields) to time-dependent temperature boundary conditions at the base, and (2) the evolution of such a flow under the imposition of constant energy flux at the base of the convecting volume.

In the absence of confirming numerical calculations, the above nevertheless suggests a simple resolution for the apparent paradox that the observed luminosity variability of the Sun and BY Dra stars is so strikingly different, even though the calculated sensitivity of their luminosities to changes in convective efficiency is quite similar (cf. Fig. 4); the observed difference in variability may simply result from the relatively far larger fractional surface area occupied by “spots” on BY Dra stars, and hence from the substantially greater fraction of the convective zone volume subject to convective transport inhibition.

Finally, we note that the above consideration of time scales applies even if the spatial redistribution of flux (viz., § IVA above) is important; in that case, the observed flux variation results from localized fluctuations in the surface flux which cancel when averaged over the stellar surface, so that the luminosity remains constant. Nevertheless, the required energy flux diversion must be global in character (see discussion in § IVA); in fact, this process may be thought of as a special case of convective efficiency reduction, for which convection inhibition is local but redistribution global just so that the freely convecting volume which responds so as to carry the “missing” flux lies on the permanently obscured portion of the stellar surface. Thus, the long-term variation in light from BY Draconis stars must reflect the time scale of the mechanism inhibiting convection under the assumption of either spatial or temporal flux redistribution. In the latter case, luminosity variability of BY Draconis stars may be simply a more dramatic symptom of variable magnetic activity in the stellar interior than the long-term solar cycle variability suggested by Eddy (1977).

V. CONCLUSIONS

The results of our study of luminosity variability in BY Draconis stars can be briefly summarized as follows:

1. The observed optical variability very likely represents real temporal luminosity variations, involving a temporal redistribution of flux.

2. The alternatives to temporal flux redistribution—spectral and spatial flux redistribution—are strongly constrained by present observations; further observations designed to specifically test these alternatives are suggested.

3. The available data are insufficient to permit unambiguous determination of change in the rotation period of surface inhomogeneities (“spots”); the existence of surface differential rotation on BY Draconis stars is therefore open to doubt.

4. A plausible mechanism for accounting for stellar luminosity variability is constructed, involving the (transitory) inhibition of convection by magnetic fields, and is supported by analysis of time scales and of the response of stellar luminosity to changes in convective efficiency. Further numerical experiments are suggested, and are required in order to place better constraints upon the range of time scales and the magnitude of the energy transport inhibition allowed under this hypothesis.

Because BY Draconis stars represent a state of stellar convection radically different from that of the
Sun, our analysis indicates that they represent a perhaps unequaled opportunity to study the mechanisms involved in solar activity on an exaggerated—and hence more easily observed—scale, and offer a test of convection theories quite distinct from those presently available. While a number of strong inferences can be drawn from the present data, further theoretical work will require additional observational constraints, and therefore additional observations are desirable to better determine the spectral and temporal changes in emission from BY Draconis stars.

We wish to acknowledge helpful conversations with B. Bopp and J. Vernazza, the aid of G. Harris with the computer calculations, useful suggestions of an anonymous referee, and the many enlightening conversations with N. O. Weiss, who also provided valuable comments on the manuscript. This work was partially supported by the Langley-Abbot Program of the Smithsonian Institution; L. H. was supported in part by NASA grant NSG 7176.

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