A NEW TECHNIQUE FOR MEASURING SOLAR ROTATION*

EDWARD J. RHODES, JR.†
George W. Downs Laboratory of Physics, California Institute of Technology
FRANZ-LUDWIG DEUBNER‡
Air Force Geophysics Laboratory, Sacramento Peak Observatory, Sunspot, New Mexico
AND
ROGER K. ULRICH
Department of Astronomy, University of California, Los Angeles
Received 1978 April 18; accepted 1978 August 1

ABSTRACT

A new technique for measuring solar photospheric and subphotospheric rotation rates is described. The technique utilizes the standing-wave nature of the nonradial \( p \)-mode oscillations of the whole Sun. Specifically, the technique is based upon the observed concentration of \( p \)-mode oscillatory power into well defined ridges in two-dimensional wavenumber-frequency \((k_h, \omega)\) power spectra. The frequencies of the ridges in the eastward- and westward-traveling portions of an individual spectrum are systematically shifted in opposite directions by a drift of the standing-wave pattern across the observing field of view. The magnitudes of these frequency shifts are related to the drift velocity and to the horizontal wavenumber as follows: \( |\Delta \omega| = |v|k_h \), Thus, measurement of the observed frequency shifts in a spectrum yields the drift velocity for that observing run. By guiding on the solar limbs and observing the velocity field at disk center, the observed drift velocity obtained in this way is exactly the rotational velocity of the solar \( p \)-mode pattern, and of the solar gas itself.

Subject headings: Sun: atmospheric motions — Sun: rotation

I. INTRODUCTION

The measurement of the rate of rotation of the Sun has long been a difficult and occasionally contradictory endeavor. Since the discovery of solar meridional differential rotation by Carrington in the 1860s, the surface rotational properties of the Sun have been studied by two basic techniques: (1) measurements of the Doppler shifts of solar spectral lines and (2) observations of various "tracers"—sunspots, faculae, filaments, EUV emission regions, coronal emission features, and also solar radio emission.

In his review of the state of recent knowledge of solar rotation, Gilman (1974) points out that both of these measurement methods contain significant difficulties. First, Doppler measurements suffer from the lack of an absolute zero velocity for reference; instead, only relative velocities are currently being measured. Furthermore, because the actual differences in the rotational velocities are smaller than the oscillatory, granular, and supergranular velocities, the rotational signal is very noisy. Additional Doppler problems have been summarized by Howard and Harvey (1970). Second, because we do not know how well the various "tracers" follow the fluid flow in which they are embedded, or even what atmospheric level they are located in, we cannot be sure just what records of tracer movements really mean. For example, sunspots involve mass flows and electromagnetic forces which may directly affect measurements of the rotation rate.

In spite of the above difficulties, the observations of Howard and Harvey (1970), and of Howard (1976) showed apparently real variations in the differential rotation with time and longitude, as well as with latitude. Because of these observations, Gilman (1974) pointed out that any description of the state of solar rotation in terms of average quantities is incomplete and potentially misleading. In addition, Gilman also mentioned that there have as yet been no techniques developed which can provide direct measurements of the rotation of the solar interior.

In this paper we will describe a new observational technique for measuring solar rotation which effectively bypasses many of the problems associated with both the spectroscopic and the tracer techniques. This technique will also allow measurements of subphotospheric rotation to be made for the first time. The technique to be described utilizes power spectra of the solar \( p \)-mode (5 minute) oscillations (Rhodes, Ulrich, and Simon 1977) to measure rotational velocities at right angles to the line of sight. The
technique described in this paper was developed by one of us (E. J. R.) during the course of his dissertation research, and modified by another of us (F. L. D.) for rotation measurements at different depths in the Sun. The different depths to which the observed \( p \)-mode power spectra are sensitive are calculated in Ulrich, Rhodes, and Deubner (1978, hereafter Paper II). The first results obtained with this technique are contained in Deubner, Ulrich, and Rhodes (1978, hereafter Paper III).

II. USE OF \( p \)-MODE POWER SPECTRA TO MEASURE TRANSVERSE PATTERN SHIFTS

a) Spatial Characteristics of Nonradial \( p \)-Mode Oscillations

At the photospheric and chromospheric levels where they are observed in the solar spectrum, the \( p \)-mode oscillations consist principally of radial velocity variations. That is, the "nonradial" designation refers to the horizontal spatial structure of the oscillations and not to the direction of the velocity variations themselves. Neglecting the small nonradial velocity components, the \( p \) mode oscillations may be represented as

\[
\delta r_{\text{ms}}(r, \theta, \phi, t) = U_{\text{ms}}(r) Y_{\ell m}(\theta, \phi) e^{i\omega t}. \tag{1}
\]

In this expression, \( r, \theta, \) and \( \phi \) are the usual spherical angles, \( U_{\text{ms}}(r) \) is the radial velocity eigenfunction at radius \( r \) for angular degree \( \ell \) and for \( n \) radial velocity nodes, \( Y_{\ell m}(\theta, \phi) \) is the normalized spherical harmonic for degree \( \ell \) and order \( m \), \( \ell \) is the unit vector in the radial direction, and \( \omega_{\text{ms}} \) is the real component of the eigenfrequency for degree \( \ell \) and nodal number \( n \). From this expression we see that the horizontal structure of the \( p \)-modes is given by a superposition of spherical harmonics. In particular, the horizontal wavelength, \( \lambda \), of a single harmonic component is related to the horizontal wavenumber, \( k_h \), and to the degree of the harmonics as follows:

\[
k_h^2 = \left( \frac{2\pi}{\lambda} \right)^2 = l(l + 1)/R_0^2. \tag{2}
\]

Thus, the observational study of nonradial \( p \)-modes must simultaneously determine both \( k_h \) and \( \omega_{\text{ms}} \) for horizontally propagating wavefronts in which the particle motions are nearly vertical.

Our use of equations (1) and (2) implies a global structure for the oscillations. Various perturbations may disorder the global structure and cause a smearing in the discrete eigenvalue \( l \). This question is discussed further in Paper II. The discrete nature of the oscillations implied by equations (1) and (2) plays no role in the technique described here. We do depend on discreteness in \( n \) for our technique to work. This in turn requires phase coherence in the radial direction. Since reflecting layers in this direction are separated by about 0.1 \( R_0 \), the smearing effect of any perturbation is roughly two orders of magnitude less important in destroying phase coherence in the radial direction than it is in the azimuthal direction. Present observations (Deubner 1975; Rhodes, Ulrich, and Simon 1977) have clearly established that radial phase coherence is maintained by the oscillations. It is not yet established that the oscillations have azimuthal phase coherence.

Rhodes, Ulrich, and Simon (1977) and Rhodes and Simon (1977) recently described the details of the three-dimensional \((X, Y, T)\) observations of the line-of-sight velocity field which were used to isolate \( k_h \) and \( \omega_{\text{ms}} \) for the different oscillatory nodes. In particular, these authors discussed a spatial filtering procedure in which the raw \((X, Y, T)\) observations were averaged in \(256' \times 2'\) columns and collapsed into two-dimensional \((X, T)\) arrays for the computation of the \((k_h, \omega)\) power spectra. This averaging process filtered out most of the \( p \)-mode waves which propagated along the columns. In this way the \((k_h, \omega)\) power spectra which resulted contained information about principally those \( p \)-modes which propagated orthogonally to the averaging direction.

By then rotating the scanning array on the Sun until the averaging direction was north-south, it was possible to compute power spectra which contained information about only those wave fronts which propagated to the east and west; i.e., which propagated parallel to the solar equator. It is the motion of these wave fronts which allows us to measure the solar rotation rate.

b) Properties of Two-dimensional Power Spectra

When a one-dimensional time series is converted into a power spectrum with the use of a one-dimensional Fourier transform, unique power estimates are obtained only from \( \omega = 0 \) to the Nyquist frequency, \( \omega_{\text{Ny}} \). This is due to the Hermitian properties of real numbers, which ensure that the power estimates at negative frequencies, \(-\omega\), are identical with those at the corresponding positive frequencies. However, when a two-dimensional, \((X, T)\), array is transformed into a \((k, \omega)\) power spectrum with a two-dimensional fast Fourier transform (FFT), the resulting spectrum contains four quadrants: \((+k, +\omega)\), \((+k, -\omega)\), \((-k, +\omega)\), and \((-k, -\omega)\); only two of these quadrants contain independent power spectral estimates. Specifically, the \((+k, +\omega)\) and \((+k, -\omega)\) quadrants are independent, while the \((-k, -\omega)\) quadrant is identical with the \((+k, +\omega)\), and the \((-k, \omega)\) quadrant is identical with the \((+k, -\omega)\) quadrant. Thus, in a two-dimensional power spectrum, only one half-plane contains all of the independent spectral information.

In the following discussion we will refer to the \(+k\) half-plane as the complete two-dimensional \((k, \omega)\) power spectrum, and hence we will refer to the \((+k, +\omega)\) and \((+k, -\omega)\) quadrants, respectively as the "positive" and "negative" frequency halves of the spectrum.

Next, we note that the positive and negative frequency portions of such a spectrum contain information on oppositely propagating wavefronts. This fact is illustrated schematically in Figure 1. In the upper half of this figure a single one-dimensional cosine
The spacetime diagrams in the middle show the functions in two-dimensional wavenumber-frequency power gating traveling waves and their representations as delta functions. As shown by the arrows, the top wave propagates toward smaller values of x (i.e., to \(-x\)) while the bottom wave propagates toward larger values of x (i.e., to \(+x\)).

A given phase moves to smaller values of x while time increases to the right. The phase velocity is just \(V_{ph} = -\omega/k\). In the two-dimensional power spectrum of this idealized cosine wave, it would be represented as a delta function in the positive-frequency half of the spectrum located at k and \(\omega\). This delta function would be multiplied by the root mean square of this wave is a spike in the negative-frequency portion of the spectrum which is also located at k but now at \(-\omega\). For a pair of oppositely propagating waves such as this, in which both waves have the same wavenumbers and frequencies, the resulting wave is a standing wave whose power spectral representation is a pair of delta functions located at \((k, \omega)\) and \((k, -\omega)\).

c) Standing-Wave Nature of p-Modes

We have just illustrated the spectral representation of a standing wave. We now discuss the extent to which the solar p-mode oscillations form a large-scale standing wave pattern. The eigenfrequencies calculated by Ulrich and Rhodes (1977) were computed for a nonrotating coordinate system. The influence of rotation upon the eigenfrequencies of nonradial oscillations has been discussed by Cowling and Newing (1949), Ledoux and Walraven (1958, § 82), and by Backus and Gilbert (1961) for the case of seismic waves in the earth.

In general all of these studies showed that rotation can split the degenerate, nonrotating eigenfrequencies, \(\omega_{non}\), into nondegenerate eigenfrequencies, \(\omega_{rot}\). However, in the case of the solar p-mode oscillations, the rotational frequency is a fraction of the eigenfrequencies small enough to render unobservable this induced rotational splitting (see, e.g., Rhodes 1977, pp. 99–104, for a more detailed discussion).

In the absence of any observable rotational splitting, the p-mode eigenfrequencies should be standing waves over large regions. There is no other lack of symmetry which would suggest propagating waves (except possibly at times of solar flares). In what follows we are concerned only with quiet Sun conditions and not with possible flare-generated acoustic waves. Consequently, in the rotating frame of the Sun each of the various p-mode eigenfrequency “ridges” should be composed of a superposition of individual p-mode standing waves. That is, there should be one pair of oppositely propagating waves for each value of \(n\) and \(l\).

In practice, however, it has not yet been possible to isolate the individual values of \(l\) along the p-mode ridges. Rather, for a given nodal number, \(n\), a pair of “ridges” is observed, one in the \(+\omega\) and one in the \(-\omega\) portion of the spectrum. The complete spectrum then consists of a set of these pairs of ridges. An example of an observational spectrum is shown in Figure 3.

The p-mode oscillations have been observed to cover the whole Sun (Leighton, Noyes, and Simon 1962; Deubner 1969). Thus every position on the Sun can be viewed as a source of oppositely propagating waves. The observed standing-wave pattern at any solar location is then a superposition of all of the oppositely propagating waves which reach that location from all other source locations. Spatially averaging the two-dimensional velocity pictures as mentioned earlier makes it possible to study only eastward- and westward-propagating wave fronts. In addition, by averaging over a long slit, nearly plane wave fronts are selected.

d) Sensitivity of p-Mode Power Spectra to Transverse Motions

The velocity-field observations which Rhodes, Ulrich, and Simon (1977) used to obtain p-mode spectra were obtained over an observing run of 256 minutes. During the course of that run the solar surface drifted across the field of view of the telescope. Since the image drift was principally in the direction of solar rotation, it introduced an apparent propagation of the p-mode standing-wave pattern from east to west during the run.

This apparent propagation of the whole standing-wave pattern to the west meant that the westward-propagating wave fronts appeared to be propagating with a velocity equal to the sum of the drift velocity, \(V_{drift}\), and the horizontal phase velocity, \(V_{phase}\), while the eastward-propagating waves appeared to be traveling at the difference of these two speeds.

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The apparent pattern motion resulted in frequency shifts for the propagating waves which were given by the Doppler effect. That is, the westward-propagating waves were blueshifted to larger (absolute) frequencies, while the eastward-propagating waves were redshifted to smaller (absolute) frequencies. Similar to the Doppler shift law, the frequency shift due to rotation is given by

\[ \Delta \omega / \omega = \pm V_{\text{drift}} / V_{\text{phase}}, \]

where the plus sign holds for motion of the observer toward the wave source and where the minus sign holds for motion away from the source. Now, the horizontal phase velocity of the waves is just

\[ V_{\text{phase}} = \omega / k_h. \]

Upon substituting the phase velocity into the Doppler law, we get the following relationship between \( \Delta \omega, V_{\text{drift}}, \) and \( k_h \):

\[ \Delta \omega / \omega = \pm (V_{\text{drift}} k_h) / \omega \]

or

\[ \Delta \omega = \pm V_{\text{drift}} k_h. \]

Finally, since \( \Delta \omega \) can be measured for all values of \( k_h \), the drift velocity can be obtained directly, as follows:

\[ V_{\text{drift}} = \pm \Delta \omega / k_h. \]

Equation (6) shows that the induced frequency shifts are equal in magnitude and opposite in sign in the +\( \omega \) and -\( \omega \) portions of the spectrum as qualitatively described above. This equation also shows that the size of the frequency shifts increases as \( k_h \) increases. These frequency shifts are shown schematically for a single, idealized standing-wave ridge in Figure 2. In this figure the +\( \omega \) ridge (upper solid line) is everywhere shifted to larger frequencies (dashed line) and the -\( \omega \) ridge (lower solid line) is shifted to less negative (smaller absolute) frequencies. What was a standing-wave pattern now has the appearance of being split by the rotation across our field of view into two oppositely directed traveling waves of different frequencies.

### III. Observational Demonstration

Figure 3 contains a raw (\( k_h, \omega \)) power spectrum for the 1975 January 8 run previously described in detail by Rhodes, Ulrich, and Simon (1977). For this spectrum the averages were made along the rows of the individual frames so that the averaging direction was nearly north-south (as may be seen in Fig. 1 of Rhodes, Ulrich, and Simon). With this averaging direction the spectrum was sensitive primarily to east-west drifts as described above.

The spectrum in this figure contains both of the two independent spectral portions (i.e., +\( \omega \) and -\( \omega \)). The solid curves are the \( p \)-mode standing-wave ridge frequencies calculated for the \( l/H = 3.0 \) model of Ulrich and Rhodes (1977), where \( l/H \) is the ratio of mixing length \( l \) to pressure scale height \( H \). The absolute frequencies for these theoretical ridges are identical in the two parts of the spectrum. The observed ridges, however, are seen to lie on opposite sides of the +\( \omega \) and -\( \omega \) theoretical ridges. In fact, the +\( \omega \) ridges are blueshifted, while the -\( \omega \) ridges are redshifted, just as predicted by equation (6) above. (We note here that the +\( \omega \) and -\( \omega \) frequency shifts are not quite equal in magnitude with respect to these theoretical ridges. This is because the observed \( p \)-mode standing-wave ridges do not fall exactly along these theoretical ridges, but appear instead at systematically smaller frequencies. This residual frequency shift is discussed in detail in Rhodes, Ulrich, and Simon 1977.)

Since the \( p \)-mode ridges are shifted in opposite directions in the +\( \omega \) and -\( \omega \) portions of the spectrum, the total observed frequency shift is just equal to \( 2 \Delta \omega \).

With equation (6) above we can calculate the magnitude of these frequency shifts and compare them with the intrinsic frequency resolution of the spectrum shown in Figure 3. For this 256 minute observing run the resolution, \( \delta \omega \), was equal to 4.09 \( \times \) 10\(^{-4} \) s\(^{-1}\). At a wavenumber of 0.2 Mm\(^{-1}\), a velocity of 2 km s\(^{-1}\) would introduce a total frequency shift of \( 2 \Delta \omega = 8 \times 10^{-3} \) s\(^{-1}\), or just two frequency resolution elements. Similarly, at \( k_h = 2.0 \) Mm\(^{-1}\), at the right edge of the spectrum, \( 2 \Delta \omega = 8 \times 10^{-3} \) s\(^{-1}\), or 20 \( \delta \omega \).

Thus, an accuracy of 5% (\( = 100 \) m s\(^{-1} \)) would require discrimination of frequency shifts equal to 0.1 \( \delta \omega \) at \( k_h = 2.0 \) Mm\(^{-1}\). Clearly, higher accuracy of velocity measurement is possible at larger wavenumbers.

As a test of this technique, we used equation (7) to estimate the drift velocity from the measured frequency shifts. The image drifts introduced by this drift velocity were then removed from the data, and a new power spectrum was computed from this "registered" data. The removal of the frequency-splitting introduced by the image drifts is demonstrated in Figure 4. In this figure the power estimates in the
Fig. 3.—Observational example of frequency shifts illustrated in Fig. 2. This is a raw \((k_h, \omega)\) spectrum computed from the 1975 January 8 run. The orientation of the observing raster on the Sun resulted in a westward image drift during the run. The observed \(p\)-mode ridges are seen to be blueshifted in the \(+\omega\) part of the spectrum and redshifted in the \(-\omega\) part as in Fig. 2. The exact dependence of the frequency shifts upon the drift velocity is given in the text. By measuring the observed frequency shifts, it is possible to calculate the drift velocity.
Fig. 4 — Left, unregistered absolute $\omega$ power spectrum made by adding $+\omega$ and $-\omega$ parts of Fig. 3 together. The oppositely shifted $p$-mode ridges in the two parts of the spectrum add up to a noisy spectrum with no clear $p$-mode ridges. Right, corresponding $|\omega|$ spectrum made after digital images were registered to remove effects of image drift. The $p$-mode ridges are distinct in this spectrum since the ridges now fall at the same absolute frequencies in the two halves of the raw power spectrum. The same set of theoretical $p$-mode ridges is plotted on both spectra for comparison.
Fig. 5 — *Left,* unregistered $|\omega|$ spectrum from the 1975 January 24 run. P-mode ridges are clearly doubled here. *Right,* registered $|\omega|$ spectrum from same data. The improvement in the $p$-mode ridges due to the removal of the image drift is more striking here, as this run was 6.5 hours long, rather than 4.25 hours, as with Figs. 3 and 4.
+ω and -ω portions of the spectra have been combined to show the power as a function of |ω|. The spectrum at the left is just the spectrum of Figure 3 combined in this way, while the spectrum at the right was computed from the "registered" data. In the left-hand spectrum the p-mode pattern is again present, but the ridges are considerably broader than those of Figure 3. In the right-hand spectrum, however, the improvement is evident. The poorly resolved ridges in the unregistered spectrum have been considerably sharpened in the registered spectrum.

Finally, because the observed p-mode ridges are not superpositions of pure delta functions, but instead have a finite width in frequency, many estimates of the observed frequency shifts must be numerically combined in order to obtain a single estimate of the drift velocity. One approach to doing this is the following set of operations: (1) the drift velocity is assumed to be constant during the observing run; (2) the amount of image motion in arc seconds per hour resulting from a given velocity is calculated, converted into the number of picture elements shifted per hour, and the digital images are then registered by removing this calculated image drift; (3) a power spectrum is computed from the registered digital images; (4) the +ω and -ω spectral estimates in this spectrum are cross-correlated; and (5) the above steps are repeated for different estimated drift velocities. Then, since the removal of the image motion introduced by the true drift velocity shifts the +ω and -ω p-mode ridges to identical absolute frequencies, the best estimate for the actual drift velocity is obtained by finding the maximum cross-correlation coefficient resulting from the above procedure. An example of this technique is illustrated in Figure 6.

This figure shows the dependence of the zero-lag spectral cross-correlation coefficient, ρ, on the estimated drift velocity, V, for the same 1975 January 8, data used in Figures 4 and 5. The correlation coefficient can be seen to peak near \( v = 1.15 \text{ km s}^{-1} \). This measured drift velocity is lower than the true surface rotation velocity of 2 km s\(^{-1}\). This difference, which has been confirmed with other data sets, illustrates a problem with the current guiding system of the Sacramento Peak Tower Telescope.

For the cross-correlation analysis of Figure 6 all of those power-spectral estimates having an effective depth below the photosphere of from 0 to 8000 km were included (see Paper II for a discussion of the effective depth scale). This was done since the observational results of Paper III show no discernible gradient in the rotation rate over those depths. This portion of the power spectrum contained 3746 spectral estimates.

IV. TECHNIQUES FOR MEASURING SOLAR ROTATION

In the previous sections we referred to the transverse velocity as a drift velocity. This designation was used because, by separately performing the above analysis on different segments of each observing run, the speed of the image drift was found to vary systematically.

We wish to thank Dr. George W. Simon of the Sacramento Peak Observatory for his assistance in obtaining the velocity data from which the \( (k_\lambda, \omega) \) power spectra were computed. Without Dr. Simon's assistance these data would not have been obtained. We also wish to thank Frank Hegwer, Dick Mann, and Horst Mauter of the Sacramento Peak observing staff for their assistance in modifying and operating equipment by removing the systematic image drifts through registration of the different images. In this way no absolute velocity measurements would be possible, but different relative rotational velocities could be measured. This is the technique used in Paper III to obtain the first measurements of radial differential rotation.

Finally, it should also be possible to measure latitudinal differential rotation by displacing the observing location away from the center of the disk to the north and to the south, once a system is accurately guiding on the solar limbs. By then computing the power spectra at different latitudes, different rotational velocities could be measured.

Alternatively, it is still possible to measure relative differential rotation with the present observing equipment by removing the systematic guiding difficulties discussed in the text. Spectral estimates ranging down to 8000 km effective depth were included in this analysis.

at Sacramento Peak. Hence the measured drift velocity did not correspond to the true solar rotation velocity. For accurate measurements of the Sun's absolute rotation rate on a daily basis, on the other hand, the observing telescope should accurately track the solar limbs so that the field of view would be constantly pointing at the center of the visible solar disk. In this way the measured drift velocity would in fact be equal to the total rotational velocity across the disk center.
the diode array hardware to accommodate the requirements of this experiment. We gratefully acknowledge several helpful discussions with Dr. Robert Howard, who also read the manuscript critically.

REFERENCES


ROGER K. ULRICH: Department of Astronomy, MSA 8979, University of California, Los Angeles, 405 Hilgard Avenue, Los Angeles, CA 90024

EDWARD J. RHODES, JR.: Downs Laboratory of Physics, California Institute of Technology, Pasadena, CA 91125

FRANZ-LUDWIG DEUBNER: Fraunhofer-Institut, Schöneckstrasse 6, D-79 Freiburg Im Breisgau, Federal Republic of Germany

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