ON THE THEORETICAL SIGNIFICANCE OF DENSITY MEASUREMENTS IN XUV FLARE KERNELS

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Received 1978 May 30; accepted 1978 August 15

ABSTRACT

Current methods of diagnosis of soft X-ray kernels in flares often indicate electron densities in excess of $10^{12} \text{ cm}^{-3}$. In particular, use of the forbidden-to-intercombination-line-intensity ratio in He-like ions leads to $n_e \geq 10^{14} \text{ cm}^{-3}$. The physical implications of these high densities are discussed in terms of the energetics and dynamics of flare mechanisms, and it is concluded that $n_e \approx 10^{14} \text{ cm}^{-3}$ is impossible. Possible sources of error in the forbidden-line method are then discussed and it is concluded that neither radiation nor plasma turbulence can explain the high-density results, though each can play a role in some transition rates. This leaves transient ionization equilibrium as the only possible explanation and implies that kernels represent direct evidence of a dynamic field dissipation process in flares.

Subject headings: plasmas — Sun: X-rays — turbulence

I. INTRODUCTION

One of the most interesting results of recent observational studies of solar flares is that the XUV emission seems to be concentrated in small dense kernels, often within an overall loop geometry. Typical XUV emission emanates from a source of temperature $T \approx 10^7 \text{ K}$ in which the density is estimated by the methods described below (cf. Svestka 1976, pp. 138–140).

a) Conductive Cooling Time $\tau_{\text{cond}}$

Culhane, Vesecky, and Phillips (1971) and subsequent authors have considered cooling processes in X-ray flares and concluded that thermal conduction usually dominates. For a source size $L$ and density $n$, the conductive cooling time is approximated by, with conductivity $\kappa(T)$, $\tau_{\text{cond}} \approx \frac{3nkT}{\kappa(T)L^2}$, where $k$ is the Boltzmann constant. The density $n$ is then determined by observing $T$ (from the line ratios or the continuum slope), while estimating $L$ (or an upper limit to it) and identifying $\tau_{\text{cond}}$ with the observed source decay time $\tau_x$. With the assumption of a Coulomb conductivity $\kappa_c(T)$, typical results have been $n \geq 10^{10}$–$10^{11} \text{ cm}^{-3}$ (e.g., Craig 1973; Phillips, Neupert, and Thomas 1974). Insofar as the source may involve a characteristic size much smaller than the observed estimate $L$, this may be a severe underestimate. On the other hand, if the source plasma is turbulent so that $\kappa \ll \kappa_c$ or if energy is supplied to the source continuously (i.e., $\tau_{\text{cond}} \ll \tau_x$), then an overestimate will result.

b) Observation of Source Emission Measure $n^2V$ and Volume $V$

For optically thin collisionally excited emissions, the intensity is proportional to $n^2Vf(T)$ where $f(T)$ is found by a steady-state collisional equilibrium computation. Thus if $n^2V$ is inferred from a line (or continuum) intensity and $T$ is known, then an (upper limit) estimate of $V \approx L^3$ immediately gives a value for $n$ (lower limit). Again, results around $n \approx 10^{10}$–$10^{11} \text{ cm}^{-3}$ are frequently quoted (e.g., Widing and Cheng 1974; Neupert, Thomas, and Chapman 1974), but values as high as $n \approx 3 \times 10^{12} \text{ cm}^{-3}$ have been obtained in a small flash phase kernel (Brueckner 1976).

c) Transient Ionization Equilibrium

If the source plasma is heated in a time shorter than the time scale needed to establish collisional ionization equilibrium, then both of the above methods may be invalidated (on the latter time scale), by deviation of transition rates from their assumed equilibrium values. On the other hand, this effect can in principle itself lead to a density estimate by observation of the time taken to approach equilibrium transition rates. As yet only lower limits $n \geq 10^{10} \text{ cm}^{-3}$ have been obtained by this method (e.g., Phillips et al.).

d) Collisional Suppression of Forbidden Lines

At sufficiently high densities, the collision time of an atom with electrons may become less than the lifetime of a forbidden radiative transition, which is consequently suppressed. This was first pointed out by Gabriel and Jordan (1969) in connection with the intensity ratio of the forbidden $3S-1S$ to intercombination $3P-3S$ lines in He-like ions, the density sensitivity here being due to collisional interchange between $3S$ and $3P$. The basic method has since been elaborated and generalized to other ion types (e.g., Gabriel and Jordan 1971; Jordan 1974; Loulergue and Nussbaumer 1974; Mason 1978). An important feature of these methods is that they do not require any assumption concerning source size. Indeed, combination of the inferred density with the emission measure deduced from resonance-line in-
tensities (for example) allows inference of \( L \approx V^{1/3} \). Thus, in conditions of collisional equilibrium, this approach should be the most reliable. However, such collisional interpretation of the suppression of the forbidden line in the He-like ion of S \(_{15}\), formed around \( 10^7 \) K, leads to densities \( n \gtrsim 10^{14} \) cm\(^{-3}\) in some flares (e.g., Neupert 1971; Kestenbaum et al. 1977), which seems implausibly high. Likewise, the size scale implied by the emission-measure value of Kestenbaum et al. (1977), viz., \( L \approx 20 \) km, seems absurdly small. In § II we show on quantitative theoretical grounds that \( n \gtrsim 10^{14} \) cm\(^{-3}\) is indeed physically impossible, and in § III we consider possible sources of error in these measurements. Finally, in § IV we suggest a dynamic situation which we consider as theoretically likely and potentially capable of explaining the spectral data through transient effects.

II. PHYSICAL DIFFICULTIES WITH HIGH DENSITIES

An X-ray kernel at \( T \approx 10^7 \) K with a density \( n \gtrsim 10^{14} \) cm\(^{-3}\) has a thermal energy density \( u = 3 \) \( nkT = 4 \times 10^8 \) ergs cm\(^{-3}\). Production of such an energy density even by total annihilation of a magnetic field in situ would require a field (\( B^2/8 \pi \geq n \)), \( B \gtrsim 3000 \) gauss, which is comparable to the largest photospheric fields ever measured. Extension of such fields into the low corona, where X-ray sources appear to lie (Svestka 1976, p. 112), seems hardly feasible even in the small energy release volume required in models such as Spicer’s (1977). In fact, the volume from which the above field must be drawn to sustain an X-ray kernel is much larger than the volume of the emitting kernel itself, as may be seen on dynamical grounds. Since the gas pressure corresponding to equation (2), viz., \( P \approx 3 \times 10^8 \) dyn cm\(^{-2}\), is comparable to photospheric values, a coronal kernel cannot be in pressure equilibrium with its surroundings along the field lines. Thus, even if conductive cooling is inhibited by plasma turbulence (see §§ III, IV), the kernel will disperse and cool by free expansion on a dynamical time scale \( \tau_{\text{dyn}} \approx L/\tau_s \approx 0.1 \) s, where \( \tau_s \) is the sound speed at \( T = 10^7 \) K. Since X-ray kernels have lifetimes \( \tau_s \geq 10^4 \) s, the volume of field annihilation must be \( \tau_s/\tau_{\text{dyn}} \approx 10^8 \) times larger than the instantaneous volume of the kernel itself to supply enough energy and mass to sustain the kernel in addition. The degree of compression of coronal plasma required to achieve a density of \( 10^{14} \) cm\(^{-3}\) does not seem feasible.

Since fields of the order 3000 gauss may exist at photospheric levels, one might ask whether dense X-ray kernels could originate there. That is, we might envisage heating of photospheric material, initially at \( n_0 \approx 10^{17} \) cm\(^{-3}\), \( T_0 \approx 6000 \) K, for example, up to \( T = 10^7 \) K by which time it has expanded to \( n \approx 10^{14} \) cm\(^{-3}\). This can be excluded, however, because of the high radiative losses of this dense plasma. First, at the onset of heating the emission of H\(^+\) ions would have to be overcome, which is extremely difficult and would result in optical emissions much in excess of those observed (Machado, Emslie, and Brown 1978). Second, at higher temperatures the plasma must be carried over the maximum in the radiative loss curve (McWhirter, Thonemann, and Wilson 1975) at \( T_1 \approx 5 \times 10^4 \) K (for constant pressure heating). At this temperature the kernel would have \( n_1 \approx 10^{16} \) cm\(^{-3}\), dimensions \( L_1 \approx V_1^{1/3} \approx 5 \) km, and be radiating at a rate \( E_{\text{rad}} \approx n_1^2 V_1 F(T_1) \approx 5 \times 10^{17} \) ergs s\(^{-1}\). This luminosity is far in excess of that in the final X-ray kernel itself and would be in the UV, which is again not observed to be the case (e.g., Emslie et al.; Donnelly and Kane 1978). Furthermore, the maximum rate at which magnetic energy could be supplied to such a small volume is \( E_{\text{max}} \approx (B_1^2/8\pi) V_1^2 L_1^3 \approx 2 \times 10^{14} B_1^3 \), where \( V_1 \) is the Alfvén velocity. Consequently, to offset the radiative losses at \( T_1 \), a field \( B_1 \approx 60,000 \) gauss would in fact be needed, which is absurd. Though this value could be reduced by inclusion of optical thickness effects and by change of geometry of the kernel from our near-cubical assumption, it would be increased by the fact that \( n_1 \) would be higher than the value at constant pressure we have used. In any case the required field \( B_1 \) varies only as the \( 1/3 \) power of the estimated losses. Thus it appears that a photospheric origin for X-ray kernels with \( n \approx 10^{14} \) cm\(^{-3}\) can be entirely excluded. Since a coronal location also does not seem viable, we conclude that a kernel density \( \sim 10^{14} \) cm\(^{-3}\) is not physically realistic. We therefore now consider possible sources of error in § IId which lead to this density value.

III. SOURCES OF ERROR IN THE LINE INTENSITY RATIO

METHOD OF DENSITY MEASUREMENT

Considering the particular line pair observed by Kestenbaum et al. (1977), we see that the forbidden line could be suppressed by photoexcitation (e.g., to \( ^3P \)) or photoionization out of the \( ^4S \) level, mimicking a high density. Either of these processes requires a strong incident photon flux in the near-UV such as we might expect in the flare situation. However, without calculating the details of the case in hand, we estimate from the results of Shull and McCray (1977) on optical forbidden-line suppression in the strong radiation fields of quasars, that the necessary flare UV flux would be totally excessive unless concentrated near the wavelength of the \( ^4S-^4P \) transition. While such concentration would occur coincidentally for one UV line, it seems improbable as an explanation for the systematic density values obtained for distinct He-like ions (e.g., Neupert 1971).

For several reasons we can be rather certain that a substantial level of plasma turbulence is present in at least some regions of the flaring plasma. (1) The ability of most magnetic energy release mechanisms to attain the rates observed in flares depends on a turbulent reduction in the electrical conductivity (e.g., Sweet 1950). (2) The fluxes of fast electrons required to produce hard X-ray bursts, whether nonthermally or thermally, are likely to generate ion-sound turbulence (Brown and Melrose 1977; Brown, Melrose, and Spicer 1978) and possibly also Langmuir waves (see Spicer and Davis 1975; Smith 1975). (3) Langmuir waves associated with type III burst electron streams have been observed directly in space (Lin 1974). (4) As discussed further below, modification of optical line profiles gives a

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direct indication of the presence of turbulent electric fields (Davis 1974, 1977).

Since the frequencies of plasma waves (excluding electromagnetic waves) are much smaller than $kT/h$ ($h$ = Planck’s constant), such waves cannot directly produce the transitions which govern the ionization equilibrium and which produce the radiations ($\nu \approx kT/h$) by which we characterize the plasma. However, the enhancement of electric microfield fluctuations (above their normal level in a thermal plasma) can broaden or split energy levels and so modify transition rates by mixing levels (e.g., Griem 1964, 1974; Davis 1974, 1977; Davis and Jacobs 1975). The maximum broadening $\Delta \nu$ of a level is of the order of the frequency of the waves involved and so, under typical astrophysical conditions, may be taken to be the electron plasma frequency $\nu_p \approx 10^8 n^{1/2}$. For densities in the range $10^{16}-10^{14}$ cm$^{-3}$, the wave broadening at optical wavelengths is $\Delta \nu / \nu_0 = \nu_p / \nu_0 \approx 10^{-4}-10^{-5}$ and so is potentially observable. The broadening of optical lines in this way and its use as a plasma diagnostic tool is treated at length by Griem (1974) and has been applied to the flare problem by Davis (1974, 1977), Tsytovich (1973), Davis and Jacobs (1975), and Spicer and Davis (1975). Specifically, these authors point out that the Stark broadening of the hydrogen Balmer lines in flares very probably results from turbulent electric fields rather than from collisional Stark broadening, as has usually been assumed in optical flare density estimates (e.g., Fritze-Svestkova and Svestka 1967; Svestka 1976). Thus the normally adopted optical electron densities $10^{12}-10^{13}$ cm$^{-3}$ may be much too high.

At X-ray energies, on the other hand, the line broadening will clearly be much smaller, viz., $\Delta \nu / \nu_0 \approx 10^{-6}$, and so not directly observable. However, for levels which are intrinsically very narrow, i.e., metastable levels, the plasma turbulent line broadening may be sufficient to modify transition rates from them. Since the great majority of transitions involved in ionization equilibrium calculations and in measurement of temperatures are resonance transitions, the density determination methods discussed in §§ 1a, Ib, and Ic may not be much modified by plasma turbulence at X-ray temperatures. (This may not necessarily be so, since Jacobs, Davis, and Kappel (1967) have shown that dielectronic recombination in Fe XXIII may be substantially modified by turbulence.) The method in § Id, however, depends essentially on the use of a forbidden transition rate and is a likely candidate for modification by plasma turbulence. Here we merely make a crude assessment of the effect involved without detailed analysis of the atomic processes involved. The importance of turbulent electric microfields depends on their rms field ($\langle E^2 \rangle^{1/2}$), which is related to the wave energy density $W$ by $\langle E^2 \rangle^{1/2} = (8\pi W)^{1/2}$, where for a typical maximum value of $W \approx 10^{-4} n^2 kT$ for Langmuir waves (limited by soliton collapse—e.g., Smith 1977) we have $\langle E^2 \rangle^{1/2} \approx 2(n/10^{18})^{1/2}$ esu around $T = 10^6$ K. This may be compared with the normal thermal rms field of electron fluctuations, viz. (see Davis 1977, for example), $\langle E^2 \rangle^{1/2} = 0.4(n/10^{12})^{3/2}$, so that

$$\frac{\langle E^2 \rangle^{1/2}}{\langle E^2 \rangle^{1/2}} \approx 5 \left(\frac{n}{10^{12}}\right)^{-1/4}.$$ (1)

We therefore conclude that, over the entire density range $10^9 < n < 10^{10}$ of interest here, turbulent electric microfields will dominate collisions in the Stark broadening of metastable levels. Consequently it is possible that plasma turbulence will have some effect on the overall ionization balance insofar as this depends on forbidden transitions. However, its effect on individual forbidden transitions (e.g., $3s^{-1}p$) will be to intensify them by increasing the mixing of the states involved. Since the collisional interchange ($e.g.$, $3s^{-3}p$) process will be unaffected, the problem of forbidden-line suppression seems to be worsened rather than removed by invoking turbulence.

Therefore it seems that the problem of high observed densities can only be explained by systematic errors in the evaluation of the collisional processes themselves. The overpopulation of $3p$ level relative to $1p$ can result under transient conditions with a large number of high energy electrons in non-Maxwellian electron energy distribution, before collisional equilibrium is attained. However, a calculation of just how transient conditions will affect line ratios relevant to density determination is beyond the scope of this Letter, though the observed effect seems possible in principle. Finally, it must be recognized that even the indirect methods in §§ 1a and Ib of density determination often demand densities as high as $10^{11}-10^{12}$ cm$^{-3}$ at which the collisional relaxation time does not exceed a few seconds. There then arises the paradoxical situation that a transient effect with a time scale of a few seconds is required to explain an observation of many minutes' duration (Kestenbaum et al. 1977). The resolution of this apparent paradox is best illustrated by reconsidering the flare dissipation process itself.

IV. Dynamic Dissipation in Kernels of $n \approx 10^{11}-10^{12}$ cm$^{-3}$

In view of the preceding discussion, it seems that $n \approx 10^{11}-10^{12}$ cm$^{-3}$ is the most likely estimate of X-ray kernel densities. These are quite feasible on energetic grounds, since (see § II) the corresponding thermal energy densities $u = 3nkT \approx 4 \times 10^4 - 4 \times 10^5$ ergs cm$^{-3}$, correspond to annihilation of reasonable coronal fields, viz., $B \approx 100-300$ gauss. In addition these densities may be achieved by feasible degrees of compression of preflare coronal material. The radiative cooling time will be $\tau_{\text{rad}} \approx 3nkT/n^2 F(T) \approx 40-400$ s, where we have taken $F(T)$ from McWhirter et al.

For a small emission measure $n^2 V \approx 10^6$ cm$^{-2}$ the typical size of the source is $L \approx V^{1/3} \approx (n^2 V)^{1/3}/n^{2/3} \approx 500-2000$ km, which allows us to estimate the conductive cooling time,

$$\tau_{\text{cond}} \approx \frac{3k}{\kappa} \left(\frac{n^2 L^3}{n^{1/3}}\right)^{1/3} \approx 2-3$$ s .

We are grateful to A. H. Gabriel for drawing our attention to this fact.
Thus, unless turbulence reduces the thermal conductivity by about two orders of magnitude, energy must be supplied continuously to offset conductive losses through the burst duration ($10^7-10^8$ s). Furthermore, the gas pressures are still well in excess of coronal values, and so the kernel will expand along the field lines and so cool convectively in a time

$$\tau_{\text{dyn}} = \frac{L}{v_s} = \frac{(n^2V)^{1/2}}{n^{3/2}(kT/m_p)^{1/2}} \approx 2-8 \text{ s.}$$

(3)

It follows from relations (2) and (3) that densities $n \approx 10^{17}-10^{18}$ cm$^{-3}$, and indeed densities as low as $10^{18}$ cm$^{-3}$ still require that both new energy and new plasma be supplied continuously to sustain the burst through its duration. This indicates that the dissipation process is a dynamic one and offers an explanation of the presence of transient ionization equilibrium throughout the burst. For although the burst emission lasts $\tau_X \approx 10^6$ s, any particular mass of plasma remains hot and compressed only for a time $\tau_{\text{dyn}}$ or $\tau_{\text{cond}}$, so that it effectively ceases to emit before it can completely reach collisional ionization equilibrium. We envisage two types of situation in which its condition may arise.

**a) Steady-State Reconnection**

In the original neutral sheet picture of field annihilation (e.g., Sweet 1969), magnetic energy flows steadily into the sheet and is converted into thermal energy of the plasma in the sheet, with some simultaneous compression. Here the replenishment of energy comes from the annihilating field and the replenishment of mass from the plasma carried into the sheet from the sides by the frozen-in field, while the hot gas escapes from the sheet edges (cf. Brown and Melrose 1977). In this case, therefore, the observed X-ray kernel would occupy a static spatial location but would comprise a mass of plasma flowing steadily through the annihilation region, emitting briefly as it does so. In practice it is the problem of ejecting the hot plasma sufficiently fast from the sheet edges which makes the sheet model less viable than dynamic models (Priest 1976).

**b) Dynamic Reconnection**

A much more efficient release of energy than the sheet can be achieved by having dynamic dissipation of magnetic field successively in a series of small regions within a sheared geometry (e.g., Syrovatskii 1966; Spicer 1977). In such a case the X-ray “kernel” would comprise a set of small volumes which are successively heated by dynamic annihilation of field, the heated plasma expanding away from each and possibly contributing to triggering of the next site. In this case the X-ray plasma is not “replenished” at a single site but replaced by emission at the next site. For an X-ray burst to last 500 s, equation (3) shows that about 100 such sites would be involved, each having an instantaneous volume, say, $10^9$ cm$^3$, when at soft X-ray temperatures (the kernel may originate at even higher temperatures in smaller volumes producing an impulsive hard X-ray spike as it cools and expands by propagation of a collisionless conduction front). The total volume involved would be $10^{19}$ cm$^3$ with typical dimension of such a volume, $L_{\text{tot}} \approx 5000$ km ($\approx 7^\circ$), is still consistent with observations of an apparently single small kernel near the limits of current spatial resolution.

One of the authors (J.C.B.) wishes to acknowledge the support and hospitality of the High Altitude Observatory, National Center for Atmospheric Research, during his visit and also partial support from the Skylab Solar Workshop Series on Solar Flares, which are sponsored by NASA and NSF and managed by the High Altitude Observatory. This Letter has benefited from the authors’ discussions with D. S. Spicer, D. F. Smith, A. H. Gabriel, and H. E. Mason.

**REFERENCES**


