A MODEL FOR X-RAY EMISSION FROM LOOP PROMINENCES

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Abstract. A study is made of X-ray line emission observed during the developing stages of a set of post-flare loop prominences. The time behaviour of the line emission can be described by a model consisting of two flux tubes containing plasma heated impulsively at the flash phase; the plasma cools by radiation and by conduction to the chromosphere. These ideas are extended to the possible formation of Hα prominences from low-lying hot loops.

1. Introduction

Post-flare loop prominences are occasionally observed following a very large flare. They consist of a system of loops which continually form one above the other over the flare site, lower loops successively fading from view. The entire development of the loop system may take several hours, although the lifetime of an individual loop is less than about 30 min. The appearance in Hα of loop prominences has been extensively reported on; Bruzek (1964) in particular has given a full account. A striking feature is the wide line profile of Hα, which Jefferies and Orrall (1965a) have attributed to jet-like streaming. More recently, the spatial coincidence of Hα loops with coronal features in the green [Fe xiv] line has been recognized (McCabe, 1973; Fisher, 1974). Soft X-ray bursts occur simultaneously with loops (Teske, 1971), indicating the presence of even higher excitation conditions. This fact is strikingly confirmed by OSO-7 monochromatic images in ultraviolet and X-ray wavelengths of loops on the limb (Thomas, 1975); loop structures with heights of $7 \times 10^9$ cm are visible in ions up to Si xiii, formed at a temperature of $10^7$ K.

Models for loop prominences have been proposed by Jefferies and Orrall (1965b) and Olson and Lykoudis (1967). Jefferies and Orrall argue that the mass and energy of loop prominences are too great for them to have condensed out of a permanent coronal enhancement, and suggest an alternative mechanism in which the mass and energy are supplied by flare-accelerated protons. In Olson and Lykoudis' model, magnetic fields first compress contained plasma, which then loses energy by radiation and by conduction along field lines. This model is intended to explain loop prominences up to a height of 30 000 km, though in fact most loop systems attain much greater heights.

In this paper, we examine X-ray and Hα observations of post-flare loops following a flare on or very near the west limb on May 13, 1971. We shall attempt to explain the X-ray observations, which consist of time-varying intensities of lines due to a wide range of ions by one or more loops containing hot plasma. We assume that the heating of the plasma takes place impulsively at the flash phase,
but we leave unspecified the heating process. After the flash phase, the plasma cools by radiation and by conduction along magnetic field lines. We ask whether such a model accounts for the time variations of the X-ray emission, and if so, whether the model then has plausible values for linear dimensions, mass and energy. We use the same ideas in trying to explain the origin of H$\alpha$ loops.

2. The Observations

The post-flare loops we discuss were associated with a class 1N flare on May 13, 1971 near or on the west limb at latitude $12^\circ$ N. Observations were made at Haleakala Observatory, Maui, from the start of the flare at about 1750 U.T. till about 2150 U.T. when the loop prominences were still in progress but had passed their maximum development. These observations consist of graded height spectra (which we do not discuss here) and filter-grams made in H$\alpha$ and the coronal green line ($\lambda$5303 Å). A spray occurred shortly after the first indications of activity on the limb, reaching greatest brightness at about 1803 U.T. The loop prominences can be seen as a small mound near the base of the spray perhaps as early as 1808 U.T.; they slowly brightened until about 1930 U.T. The loops climbed in altitude at a rate which decreased with time in agreement with Bruzek's (1964) description; but the rates are slightly smaller than he quotes: 3 km s$^{-1}$ at 1900 U.T. and 1 km s$^{-1}$ at 2150 U.T. The loops attained an altitude of $4 \times 10^9$ cm at 2150 U.T.

Soft X-ray observations of this event from the OSO-6 spacecraft have been supplied to us by Dr G. A. Doschek at the U.S. Naval Research Laboratory. They were made with uncollimated crystal spectrometers operating in the wavelength ranges 0.6–4 Å (LiF crystal), 2–9 Å (EDDT) and 6–14 Å (KAP). Each of these wavelength ranges was scanned once every seven minutes. Data are available from the time the spacecraft emerged from eclipse at 1801 U.T. until 1856 U.T., and again from 1935 U.T. We have selected various prominent lines in these spectra and, by summing photon counts over the profile of each line, have derived the time variations of the relative intensity. Dead-time corrections were applied to all data. An absolute intensity calibration is available only for the KAP spectrometer; Dr Doschek has given us the necessary conversion factors as functions of wavelength, but he cautions us that an accuracy of not better than 50% is expected.

Flare spectra in these wavelength regions are characterized by resonance lines of hydrogenic and helium-like stages of abundant elements. Table I lists the lines chosen for analysis. In the case of the helium-like ions Mg xi, S xv, Ca xix and Fe xxv, three transitions, $1s^{2}1S\rightarrow 1s2p^{4}P$, $1s2p^{3}P$ and $1s2s^{3}S$, fall very close together; the photon count rates for the entire complex were summed. A recombination edge in the detector efficiency falls within this complex for Si xiii, so that the intensity of the $1^{1}S\rightarrow 2^{1}P$ and $1^{1}S\rightarrow 2^{3}P$ transitions is greatly diminished compared with $1^{1}S\rightarrow 2^{3}S$; in this case we measured counts in only the $1^{1}S\rightarrow 2^{3}S$ transition.
TABLE I
X-ray lines chosen for analysis

<table>
<thead>
<tr>
<th>Spectrometer</th>
<th>Ion</th>
<th>Transition(s)</th>
<th>Wavelength (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LiF</td>
<td>Fe xix</td>
<td>$1^1S-2^1P, 2^3P, 2^3S$</td>
<td>1.85–1.88</td>
</tr>
<tr>
<td></td>
<td>Ca xix</td>
<td>$1^1S-2^1P, 2^3P, 2^3S$</td>
<td>3.17–3.21</td>
</tr>
<tr>
<td>EDDT</td>
<td>S xv</td>
<td>$1^1S-2^1P, 2^3P, 2^3S$</td>
<td>5.04–5.10</td>
</tr>
<tr>
<td></td>
<td>Si xix</td>
<td>$1^1S-2^3S$</td>
<td>6.74</td>
</tr>
<tr>
<td>KAP</td>
<td>Si xiv</td>
<td>1s–2p</td>
<td>6.18</td>
</tr>
<tr>
<td></td>
<td>Mg xi</td>
<td>1s–2p</td>
<td>8.42</td>
</tr>
<tr>
<td></td>
<td>Mg xi</td>
<td>$1^1S-2^3P, 2^3P, 2^3S$</td>
<td>9.13–9.32</td>
</tr>
</tbody>
</table>

Fluxes have been derived for all the lines in Table I and are plotted in Figures 1 and 2. Absolute fluxes (units: photons cm$^{-2}$ s$^{-1}$) are given for lines observed with the KAP spectrometer, while for the EDDT and LiF spectrometers (for which no absolute calibration is available) values of photon counts s$^{-1}$ are plotted.

Dr J. P. Castelli of the Sagamore Hill Observatory has sent us radio records of the event. These are in the form of single-frequency observations in the range 245 to 35 000 MHz from 1730 U.T. to 2000 U.T. A very weak burst at roughly the same time as the spray and flash phase occurred on frequencies between 245 and 4995 MHz; this burst is commented on by Machado et al. (1972). There was more significant activity at about 1840 U.T. at low frequencies, and a much larger burst at frequencies 245–8800 MHz occurred at 1900 U.T., lasting for an hour. There was no other Hα activity at these later times, so these bursts seem to be associated

Fig. 1. Fluxes (in absolute units of photons cm$^{-2}$ s$^{-1}$) of Mg xi (9.2 Å), Mg xii (8.4 Å) and Si xiv (6.2 Å) lines, observed with KAP spectrometer on OSO-6 for the May 13, 1971 loop-prominence event. The smooth curve drawn through the Si xiv points represents the time variations of this line observed with the uncalibrated EDDT spectrometer. The flash phase ($t=0$) is assumed to be 1801 U.T.
Fig. 2. Fluxes of Si XII (6.74 Å), S XV (5.1 Å), Ca XIX (3.3 Å) and Fe XXV (1.9 Å) lines observed with the uncalibrated EDDT and LiF spectrometers. Each curve has been arbitrarily placed with respect to the flux axis.

with the flare whose flash phase was at 1801 U.T. Gergely and Kundu (1974) report a moving type IV burst at decametric wavelengths starting at 1812 U.T., while a coronal depletion event was observed by Hansen et al. (1974).

3. Theory

3.1. The model and basic equations

The images obtained with the OSO-7 XUV spectroheliograph of loop prominences (Thomas, 1975) suggest that the X-ray emission arises from hot plasma contained in one or more magnetic flux tubes above the flare site. As a starting point for our model, we suppose that the X-ray emission comes from plasma contained in one such loop which has been heated during the flash phase to a uniform temperature, high enough to emit the observed lines. After the flash phase, we assume that no further energy is supplied apart from the small amount of mechanical energy always being delivered to the corona from the chromosphere. The plasma then cools by radiation and conduction.

Unlike Olson and Lykoudis’ (1967) treatment, then, we ignore the details of how the plasma was heated. We have done this on the basis of Olson and
Lykoudis' own findings that the type of compression the plasma undergoes at the start of the flare in their model does not greatly affect the way in which the plasma subsequently cools. We do assume that the heating is impulsive. Evidence that this is physically not unrealistic is provided by the frequent occurrence of very short-lived bursts of hard X-ray and microwave radio emission during the first few minutes of the development of a flare (see, e.g., Kane, 1969; Frost, 1969); such bursts, indicating particle energies of up to $\sim 100\text{ KeV}$, show that an impulsive accelerating mechanism is present. Our other assumption – that the temperature distribution along the loop is initially uniform – may be justified by the fact that, owing to the high thermal conductivity of coronal material along field lines, any temperature distribution quickly relaxes to one in which the loop is almost isothermal over much of its length except for the sections in contact with the chromosphere where the temperature rapidly falls.

A simple model along the lines we propose was presented by Culhane et al. (1970). They considered conduction and radiation separately as cooling mechanisms, deducing that conduction plays the predominant role. We improve on their treatment by considering both cooling mechanisms. Further, we calculate numerically the temperature distribution along the loop's length, rather than assume, as they did, a uniform temperature gradient.

Figure 3 shows schematically the model X-ray source to be used in this work. The semi-length of the loop is $L$, and $s$ is a length variable measured from one of the footpoints. Time $t$ is taken to start at the instant the heating phase is over. The temperature $T$ is a function both of $s$ and of $t$. We take the area of the loop cross section $A$ to be a function of $s$; we discuss later what form for $A(s)$ we assumed.

We consider loop sizes $L$ of up to $2 \times 10^{10}$ cm. If the top of the loop is about $0.75L$ above the photosphere, we find that the height of a loop with $L = 2 \times 10^{10}$ cm is equal to a pressure scale height for a temperature of $2.4 \times 10^6$ K.

![Diagram](image-url)

Fig. 3. Schematic representation of X-ray-emitting model loop. $T =$ temperature; $A =$ cross-sectional area; $s, t$ are distance and time variables; and $L$ is the loop's semi-length.
The temperatures we consider are generally much higher than this, so a constant particle density \((n = n_e + n_H = 2n_e)\) is justified. However, temperatures of only a few \(10^5\) K or less will exist at the loop footpoints, and so there will be an increase of density if the gas pressure is equalized in the loop. We discuss this further below.

On the above assumptions, the rate of change of energy per unit volume of the loop is given by

\[
3nk \frac{\partial T}{\partial t} = \nabla \cdot [K(T) \nabla T] - \mathcal{P}(T) + E_{\text{mech}},
\]

where \(k\) is Boltzmann's constant, \(K(T)\) the thermal conductivity of the plasma, \(\mathcal{P}(T)\) the radiated power over all wavelengths per unit volume, and \(E_{\text{mech}}\) the mechanical energy delivered to unit volume of the loop from the chromosphere. For an unmagnetized plasma, \(K(T)\) is given by Delcroix and Lemaire, (1969)

\[
K(T) = 1.8 \times 10^{-5} \frac{T^{5/2}}{\ln \Lambda} \text{ erg cm}^{-1} \text{ s}^{-1} \text{ K}^{-1},
\]

where \(\ln \Lambda\) is the Coulomb logarithm, a slowly varying function of \(T\) and \(n\). With a magnetic field present, as in our case, this expression for \(K(T)\) is the thermal conductivity along the field lines, while perpendicular to the field conduction is practically suppressed. We used Tucker and Koren's (1971) calculations of \(\mathcal{P}(T)/n^2\) for \(T > 10^6\) K, and those of Cox and Tucker (1969) for \(T < 10^6\) K. The resulting curve only slightly varies with temperature at \(T > 3 \times 10^6\) K, but rises sharply at lower temperatures. These works agree reasonably well with recent calculations by Raymond et al. (1976) and Kato (1976); the largest differences occur between \(10^6\) K and \(10^7\) K, where the curve of Raymond et al. is higher, and that of Kato lower, than that of Tucker and Koren. To evaluate \(E_{\text{mech}}\), we supposed that mechanical energy from the chromosphere, with an energy flux equal to the quiet-sun value of \(2 \times 10^5\) erg cm\(^{-2}\) s\(^{-1}\) (Athay, 1971), is delivered via the loop footpoints and is dissipated uniformly throughout the loop's volume; this gives \(E_{\text{mech}} = 2 \times 10^5/L\) erg cm\(^{-3}\) s\(^{-1}\). Substituting for \(K(T)\) and \(E_{\text{mech}}\) and making use of the geometry variable \(s\), Equation (1) becomes

\[
3nk \frac{\partial T}{\partial t} = \frac{1}{A(s)} \frac{\partial}{\partial s} \left( K_0 A(s) T^{5/2} \frac{\partial T}{\partial s} \right) - \mathcal{P}(T) + 2 \times 10^5/L \text{ erg cm}^{-3} \text{ s}^{-1},
\]

where \(K_0 = 1.8 \times 10^{-5}/\ln \Lambda\).

To resolve this second-order partial differential equation for \(T(s, t)\) we need two boundary conditions and one initial condition. We chose the boundary conditions as follows:

(a) \(T(0, t) = 10^4\) K, i.e. the footpoints of the loop are maintained at a chromospheric temperature;

(b) \((\partial T/\partial s)_{s=L} = 0\), i.e. the temperature gradient is zero at the top of the loop.

As mentioned earlier, we have taken the loop to be isothermal at \(t = 0\), i.e. the
end of the heating phase. Thus the initial condition is simply \( T(s, 0) = T_0 \); we choose \( T_0 \) to be a free parameter whose value matches as nearly as possible the X-ray observations.

It was mentioned earlier that if the gas pressure is equalized in the loop, we may expect an increased density at the loop footpoints where the temperature approaches the chromospheric value. A density increase results in an increased radiation loss, and so the cooling will be more rapid than that evaluated by the above analysis. However, even with this increase of radiative energy loss, conductive energy flow from the top of the loop should always predominate in the energy transport equation unless the loop temperature is generally low. The neglect of this effect should not therefore be serious.

The assumption that energy transport takes place by conduction along the field lines may be justified if, in the terminology of Alfvén and Fälthammar (1963, p. 161), the plasma is medium-density, i.e. if for the conducting electrons \( \rho \ll \lambda \ll L \), where \( \rho \) is the Larmor radius of the electrons and \( \lambda \) their mean free path. We may take \( \lambda \) to be \( \nu t_D \), where \( \nu \) is the electron velocity and \( t_D \) the deflection time as defined by Spitzer (1962, p. 132). Using Spitzer's expression we obtain

\[
\lambda = \frac{6.22 \times 10^{-19} v^4}{n \ln \Lambda} \text{ cm.}
\]  

(4)

With \( n = 10^{10} \text{ cm}^{-3} \) and a field strength of 10 G (values probably typical of an X-ray flare plasma), the condition \( \rho \ll \lambda \) is always satisfied. But with loop sizes comparable to those of the H\( \alpha \) loops, i.e. \( L = 5 \times 10^8 \text{ cm} \), \( \lambda \) is a large fraction of the physical dimension for velocities greater than \( 5 \times 10^9 \text{ cm s}^{-1} \). In such a case, transport of energy is imprecisely described by Equation (3). However, we will assume the validity of this equation since, for the temperatures we deal with, a velocity of \( 5 \times 10^9 \text{ cm s}^{-1} \) is at least twice the velocity at which the Maxwellian distribution is at a maximum.

3.2. Computational procedure

The temperature distributions \( T(s, t) \) have been obtained by solving Equation (3) numerically using various values of \( n, L \) and \( T_0 \). In our models, we have considered densities \( n \) between \( 10^9 \text{ cm}^{-3} \) and \( 10^{11} \text{ cm}^{-3} \), a range which includes values obtained from optical emission in loop prominences (Jefferies and Orrall, 1961; Zirin and Acton, 1967) and values from X-ray flares generally (see references cited by Phillips et al., 1974). We took values of \( L \) from \( 3 \times 10^9 \) cm, the semi-length of the May 13 H\( \alpha \) loops, to \( 2 \times 10^{10} \) cm, approximately the maximum semi-length of H\( \alpha \) loops in Bruzek's (1964) survey. Once \( n \) has been chosen, there is some constraint on the value of \( L \); this is because the X-ray emission, fixed at the observed value (where known), will depend upon \( n^2 \) and the volume, and the value for the loop's diameter which is of the order of \( L/5 \) to preserve the loop shapes as indicated, e.g., by the OSO-7 X-ray images.
A linear form for the loop cross-sectional area \( A(s) \) was chosen: \( A(s) = (1 + \alpha s/L) A(0), \) with \( \alpha \) taking values between zero (i.e. constant cross-section) and ten. The function \( \mathcal{P}(T)/n^2 \) was approximated by straight-line segments at \( T > 3 \times 10^6 \) K, where the temperature dependence is slight; for lower temperatures, where the function varies rapidly, a table of values was used.

Equation (3) was solved by the method of finite differences using an implicit method described by Carnahan et al. (1969). We obtained solutions for temperature \( T_i \) at the \( i \)th distance point \( s_i \) and \( j \)th time step \( t_j \), having first chosen values of \( n, L, T_0 \) and \( \alpha \). We set the distance increments \( \delta s = s_i - s_{i-1} \) equal to one another, notwithstanding the much larger temperature gradient near \( s = 0 \). We found that, for a variety of models, increasing the number of distance steps between \( s = 0 \) and \( s = L \) from 60 to 300 made no sensible difference to the temperature solutions. Although altering the time increments \( \delta t = t_j - t_{j-1} \) from 5 to 60 s made little difference also, \( \delta t \) was allowed to vary between these limits according to the temperature gradient halfway up to the loop, \( \delta t \) being smallest for relatively high values of \( \delta T/\delta s \).

The X-ray line emission, which in the case of coronal plasma is due to electron collisional excitation, may be evaluated from the temperature solutions \( T(s, t) \) as follows. The flux at earth \( (\delta F) \) of line radiation from a volume element \( (\delta V) \) of the Sun's atmosphere at temperatures \( T \) is given by

\[
\delta F = 4.3 \times 10^{-31} \frac{f}{W} \frac{n_E}{n_H} G(T) n^2 \delta V \text{ photons cm}^{-2} \text{ s}^{-1},
\]  

(5)

where

\[
G(T) = \frac{n_{\text{ion}}}{n_E} \frac{e^{-W/kT}}{\sqrt{T}} P(W/kT)
\]  

(6)

(see, e.g., Phillips, 1975). \( n_{\text{ion}} \) is the number density of the emitting ion, \( n_E \) that of all ionization stages of the particular element; \( f \) and \( W \) are the oscillator strength and excitation energy (eV) of the transitions; and \( P \) is a slowly varying function, related to the Gaunt factor, defined by van Regemorter (1962). The flux \( F(t) \) of line radiation coming from the whole loop at time \( t \), with the geometry of Figure 3, is therefore

\[
F(t) = 2 \times 4.3 \times 10^{-31} \frac{f}{W} \frac{n_E}{n_H} n^2 \times \frac{L}{0} \int A(s) G[T(s, t)] \, ds \text{ photons cm}^{-2} \text{ s}^{-1}.
\]  

(7)

For the transitions in Table I we used the tabulations of Mewe (1972) for the \( G(T) \) functions. These values have almost the same dependence on \( T \) as those of Tucker and Koren (1971), though the absolute values differ by up to a factor of two. Improved data on the excitation cross sections have been made available since both these works appeared, but a more detailed calculation of the atomic
factors did not seem worth while in view of the uncertainties in the observed fluxes.

The time dependence of the theoretical fluxes of the X-ray lines listed in Table I was calculated for a range of models having different values of $n$, $L$, $T_0$ and $\alpha$. $T_0$ was chosen so that the fluxes of the Si $\text{xiv}$, Mg $\text{xii}$ and Mg $\text{xii}$ lines (those observed with the KAP spectrometer, the only instrument for which an absolute calibration exists) near the flash phase ($t = 0$) were approximately matched. First-trial values of $n$ and $L$ were chosen. We used as a guide for arriving at more refined values the following dimensional-analysis argument. At high enough temperatures (say $T > 5 \times 10^6$ K), conduction exceeds radiation as a cooling mechanism owing to the fact that the temperature dependence of $K(T)$ is strong (Equation (2)) while that of $\mathcal{P}(T)/n^2$ between $3 \times 10^6$ K and $10^8$ K is weak (e.g., Tucker and Koren, 1971). From (3), then, we may form a ‘condonation time’, given by

$$\tau_{\text{cond}} \sim \frac{N^2 L^2}{K_0 T^{5/2}},$$

where $T$, $N$ and $L$ are characteristic values of temperature, density and length. Decreasing the density or the length thus decreases $\tau_{\text{cond}}$, i.e. hastens the decline of temperature and therefore of the flux of lines due to high-temperature ions. Adjustments were made to the values of $n$ and $L$ so that the calculated flux curves agreed as nearly as possible with the observed.

4. Results

Using the procedure described in Section 3, we calculated the time dependence of the X-ray lines in Table I for models having different values of $n$, $L$, $T_0$ and $\alpha$. Models consisting both of a single loop and of two separate loops were considered. In what follows, we shall regard 1801 U.T. as the time of the flash phase, i.e. $t = 0$, when the loop or loops are assumed to be heated.

4.1. Single-loop Models

The intensity ratios for the Mg $\text{xii}$, Mg $\text{xii}$ and Si $\text{xiv}$ lines at 1801 U.T. (approximately 2.6 : 2.8 : 1.0) and Mewe's theoretical curves suggest $T_0 = 11 \times 10^6$ K. This temperature is too low, however, to produce line radiation at 1.9 Å, due to Fe $\text{xxv}$ and Fe $\text{xxiv}$ satellite transitions, which is observed to last for 35 min. Such emission only becomes observable for a typical flare density and volume at $T > 14 \times 10^6$ K. If we are to retain a single-loop model, we must assume that the highest temperature in the loop is at least $14 \times 10^6$ K for 35 min, and that there are inaccuracies in either the KAP spectrometer efficiency curve or the atomic factors used in Mewe's calculations.

We therefore examined various models in which $T_0$ took values between $15 \times 10^6$ K and $22 \times 10^6$ K. With $L = 3 \times 10^9$ cm, equal to that of the H$\alpha$ loops, a too-rapid cooling results, even when the density is raised to $3 \times 10^{11}$ cm$^{-3}$ and
when the area parameter $\alpha$ takes non-zero values. In Figure 4, we depict the observed and calculated time dependence of the Mg xii, Si xiv and Fe xxv line fluxes.

Larger values of $L$ were chosen in an attempt to slow down the rate of cooling. It was found possible to match, e.g., the Si xiv curve. Figure 5 shows that the dependence of line emission calculated from one such model, having $L = 1.3 \times 10^{10}$ cm, $n = 2 \times 10^{10}$ cm$^{-3}$, $T_0 = 16 \times 10^6$ K and $\alpha = 10$. As can be seen the Mg xii line rises for a long period of time before reaching a maximum. The rise of the Mg xii line flux and that of other cool ions is due to the increasing volume of low-temperature material as the loop cools.

Thus, no matter how we adjust the parameters, the calculated time dependence of the lines is always at variance with the observations. It is possible that the question can be resolved by relaxing our earlier assumption that no energy is supplied after the flash phase. We note in this respect Rust and Bar's (1973) evidence from magnetic-field calculations that energy input continued throughout
the lifetime of a loop-prominence event they studied. To account for continuous heating of the flare plasma, we might have followed Strauss and Papagiannis (1971), who have described hard X-ray flare emission by a loop model like the one considered here, but include a source term, $Q(s, t)$, in their energy transport equation; this term is adjusted so that both the observed rise of X-ray emission and the X-ray spectrum are reproduced. However, we have chosen to retain our earlier assumption that energy is supplied impulsively only at the flash phase and, guided by the appearance of the OSO-7 X-ray images, to consider a model consisting of two loops instead of one.

4.2. Two-loop models

As with the single-loop model, we require of the two-loop model that it reproduce the X-ray observations, at least to within the errors of the observations and of the atomic calculations. The observations consist of the relative fluxes of seven lines as a function of time, and absolute fluxes for three of these lines. The fluxes of all lines decline approximately exponentially, as can be seen from
Figures 1 and 2. Thus, in matching the observations with model calculations, we may speak in terms of reproducing ten observational parameters, viz. the e-folding times of fluxes for all seven lines and the absolute fluxes of three lines. If the two loops are each to have arbitrary shapes and sizes, a total of ten parameters, viz. \( n, L, T_0, \alpha \) and the maximum cross-sectional area for each loop, are to be determined. The problem thus has zero degrees of freedom. We may increase the number of degrees of freedom by adopting values for \( \alpha \) and the ratio of loop length to maximum diameter from OSO-7 X-ray images; the loops visible in those images indicate that \( \alpha = 10 \) and that the maximum loop diameter is about \( L/5 \). This then gives us six free parameters to determine, viz. \( n, L, \) and \( T_0 \) for each loop.

We found the parameters for the hotter loop by assuming that it is solely responsible for line emission from the ions Ca xix and Fe xxv. As with the second of the two models discussed in Section 4.1 (see Figure 5), the flux curves for cool ions (e.g. Mg xii) will, as a consequence of the high initial temperature, start from low values and slowly rise with time. The second loop must therefore have a sufficiently low temperature that the time behaviour soon after \( t = 0 \) of line emission from the cool ions approximately matches the observations; that is, the flux curve should start at a high value, pass through a maximum at \( t = 400 \) s, then decline slowly. Furthermore we demand that the line emission from ions of intermediate temperature (e.g. Si xiv) from this loop, when added to that of the first loop, produces a smooth curve that decays at the observed rate.

To produce Fe xxv line emission that would persist for 35 min as observed, the first loop must be very dense or very long. A model with \( n = 7 \times 10^9 \) cm\(^{-3}\), \( L = 2 \times 10^{10} \) cm and \( T_0 = 25 \times 10^6 \) K gives a reasonable fit to both the Fe xxv and Ca xix line emission (Figure 6). The line emission from the cooler ions was calculated for various models which had much lower starting temperatures. The Mg xii curve can be matched with a model loop having the same length as the first loop, but with \( n = 1.6 \times 10^{10} \) cm\(^{-3}\) and \( T_0 = 9 \times 10^6 \) K. The emission from this loop, when added to that of the first loop, gives reasonable fits to the time behaviour of the Mg xii, Si xiv, Si xiii and S xv lines and agrees to about a factor of two with the absolute fluxes of the Si xiv and Mg xii lines. Figure 6 illustrates the observed fluxes and those calculated from this two-loop model for five of the lines. In this figure, the fluxes of the Mg xii and Si xiv lines are absolute, i.e. the observed curves have not been moved up or down to agree with the model curves, whereas they have for the Si xiii, Ca xix and Fe xxv lines, no calibration being available for the corresponding detectors.

The two loops in this model are considerably longer than the observed H\( \alpha \) loops. At the time of the X-ray observations, the H\( \alpha \) loops have an altitude of about \( 3 \times 10^9 \) cm, i.e. \( L = 4 \times 10^9 \) cm, whereas the model loops have had to be made five times longer to prolong the model X-ray emission to agree with the observed curves.

The energy and mass of the two loops appear to be reasonable. With the
temperatures at their initial values ($25 \times 10^6$ K and $9 \times 10^6$ K), the total energy is $3.4 \times 10^{31}$ ergs, the energy content of each loop being roughly equal. The mass of the loops is $9.7 \times 10^{15}$ g, which is very close to estimates by Jefferies and Orrall (1965b) for the total mass flow of Hα material in a loop prominence.

There is a significant difference between the observed and calculated curves for the Mg xi line. The observed curve reaches a maximum at $t = 500$ s, while the calculated curve for the two-loop model rises slowly to its maximum at $t = 4000$ s. However, the absolute fluxes agree to within a factor 2.5 at all times. (The non-flaring active-region contribution to the flux of this line, estimated from a spectral scan before the flare, is negligible.) Since the model curve lies below the observed up to $t = 2500$ s, we could remove the discrepancy by adding a third loop which emits the Mg xi line for about this length of time. This loop should be sufficiently cool that it does not produce lines due to higher-temperature ions, the

Fig. 6. Observed fluxes of Mg XII, Si XIV and Fe XXV lines compared with those from a two-loop model. The two loops have the following parameters: (1) $L = 2 \times 10^{10}$ cm, $T_0 = 25 \times 10^6$ K, $n = 7 \times 10^9$ cm$^{-3}$; (2) $L = 2 \times 10^{10}$ cm, $T_0 = 9 \times 10^6$ K, $n = 1.6 \times 10^{10}$ cm$^{-3}$. Each has $\alpha = 10$, maximum loop diameter = $L/5$. 
observed fluxes already agreeing with those calculated from the two-loop model: this suggests that the initial temperature of the third loop is \( \sim 5 \times 10^6 \) K.

The addition of a third loop may appear to be an arbitrary means of arriving at agreement with the X-ray observations. However, supporting evidence that a multiplicity of hot loops is present in a loop-prominence event is provided by the OSO-7 X-ray images, which show at least three distinct loop structures.

Although we have matched the X-ray observations with a model consisting of three loops, we do not assert that this is a unique solution. For example, the temperature decay of the loop that has \( T_0 = 25 \times 10^6 \) K may be reproduced with a shorter loop having a greater density. However, the amount of cool-ion emission (which depends on \( n^2 \times \text{volume} \)) at late stages in the event will be affected by changes in \( n \) and \( L \). If the loop diameter is a constant fraction of \( L \), as assumed earlier, the emission measure is proportional to \( n^2 L^3 \). Thus there is some constraint on the values of \( n \) and \( L \) we choose. However, the restriction is only loose since, owing to the observational errors, we need only attain about a factor-of-two agreement with the absolute fluxes. Furthermore, we have no evidence that the loop diameter is strictly proportional to the length. Thus, the model discussed here is only a possible means of reproducing the X-ray observations.

We conclude that it is possible to describe the X-ray observations with magnetic loops containing material heated impulsively at the flash phase; the observed behaviour of the X-ray emission can be accounted for by conductive and radiative cooling of the loops.

4.3. Radio Emission

The radio activity after about 1820 U.T. is much more significant than that occurring at the time of the spray and initial stages of the loop prominence. However, the bursts at these later times may not be associated with the flare event proper but rather with the coronal depletion event discussed by Hansen et al. (1974). Certainly the moving type IV burst observed by Gergeley and Kundu (1974) at decametric wavelengths seems unconnected with the loop prominences since the emission was at very high altitudes, about two solar radii above the photosphere, placing them far above even the two large loops considered in Section 4.2 to explain the X-ray emission. The much weaker burst at 1750–1820 U.T., on the other hand, has a similar time profile to the X-rays and reaches a maximum within a few minutes of the spray event. We investigated whether thermal radio emission from the two-loop model of Section 4.2 could account for this burst.

Using formulae for the free-free absorption coefficient from Kundu (1965), and assuming a path equal to the maximum diameter of either loop (\( 2 \times 10^{10}/5 \) cm), we calculate that the source is optically thin for \( T \gtrsim 2 \times 10^6 \) K. We estimated the free-free emission from an optically thin source from formulae given by Allen (1973), obtaining 18 s.f.u. for the sum of the two loops, independent of frequency. This is rather more than half the observed values – 27 ± 1 s.f.u. – at 1415, 2695
and 4995 MHz. In view of many uncertainties in the calculation, we regard this as satisfactory agreement. With the density of each loop equal to $\approx 10^{10} \text{ cm}^{-3}$, the plasma frequency is 900 MHz; we should thus expect diminished emission at frequencies lower than this. This is in fact observed, so we conclude that the two-loop model, assumed optically thin to radio wavelengths, can satisfactorily account for the radio burst.

4.4. Relation to $\text{H}_\alpha$ Loops

The two-loop model discussed in Section 4.2 is sufficient to explain the X-ray observations, but the data we have available do not preclude the existence of other loops having initial temperatures of less than $\approx 5 \times 10^6 \text{ K}$. In particular, an entire nest of loops might be present, and their contributions to the line emission we considered earlier would still be negligible. We recall that each loop in the two-loop model has $L = 2 \times 10^{10} \text{ cm}$, a value which greatly exceeds that of the $\text{H}_\alpha$ loops for this event, viz. about $3 \times 10^9 \text{ cm}$. If indeed there were loops down to the size of the $\text{H}_\alpha$ loops, it is possible that the $\text{H}_\alpha$ loops 'condense' from the hot loops. In this process the hot loops cool by conduction and radiation to a temperature of $\approx 10^6 \text{ K}$; at this point the radiative power loss sharply increases, maintaining a high value until the temperature has fallen to 15 000 K, when the radiative losses again diminish. Neutral hydrogen then radiates and the $\text{H}_\alpha$ prominences appear.

The time for an $\text{H}_\alpha$ loop so to form from a loop at, say $5 \times 10^6 \text{ K}$ initially, should show a dependence on length, and therefore height, of the loop, since at least part of the cooling is by conduction. If radiative losses were very small compared with conductive losses for most of the cooling time, the plasma would cool in a time of the order of $\tau_{\text{cond}}$, given by Equation (8). If, in this equation, $L$ is proportional to loop height, and if the density is the same for all loops in the nest, the time for an $\text{H}_\alpha$ loop to form should be proportional to the height squared. This is observed to be so for the May 13 loops for which measurements were made from $\text{H}_\alpha$ filtergrams taken between 1820 and 2151 U.T. The plot, shown in Figure 7, is closely parabolic. This also seems to be the case for most of the loop prominences discussed by Bruzek (1964).

We examined this idea more closely by constructing model loops with various densities and lengths but each having an initial temperature of $5 \times 10^6 \text{ K}$. In these calculations, the cooling was assumed to be by conduction and radiation, at constant density, in exactly the same way as for the calculations of Sections 4.1 and 4.2. The assumption of constant density is more questionable in this case since, with the temperature varying by a factor of 300, we may expect a density increase. We discuss this below.

The calculated results are compared with the observed height-time relation in Figure 7. We have plotted, for particular densities, the height of the model loop, approximated by $0.75 L$, against time for the peak temperature $T(L, t)$ to reach $\approx 15 000 \text{ K}$. In each case, the curves are drawn through three or four points, each
point corresponding to the cooling of a model loop. For \( n < 8 \times 10^8 \) \( \text{cm}^{-3} \), the radiative losses were so small that they were balanced by the mechanical energy input term; the temperature asymptotically approaches \( \sim 10^6 \) K. This is to be expected since quiet-coronal loop structures having this temperature and density are observed for long periods of time without their condensing to form prominences. For \( n \geq 8 \times 10^8 \) \( \text{cm}^{-3} \), the calculations indicate that condensation will occur, and that the time to do so increases with height; but the relations \( \tau_{\text{cond}} \propto L^2 \) is not confirmed. We found that radiation losses at these low temperatures are never negligible compared with conductive losses, particularly at high densities. This accounts for the fact that the condensation times are inversely related to \( n \) and are only weakly dependent on \( L \) for large values of \( n \). These conclusions are not greatly affected by the values chosen for the starting temperature in the range \( 3 \times 10^6 \) to \( 10^7 \) K. If, as mentioned earlier, the density increases as the loop cools to very low temperatures, the radiation losses will be larger, and so, for large \( n \), dependence on \( L \) will be even weaker, i.e. the curves will be more nearly vertical.

Olson and Lykoudis (1967) have made similar calculations to these, and in their plots of \( \text{H} \alpha \) loop height against time of formation they have reproduced the observed shape more successfully than we have. This is despite the fact that the temperatures, densities and lengths of their model loops have values like those considered here. The main difference for the differing behaviour of the calculated results seems to be in the treatment of the radiative power loss function. This function, \( P(T)/n^2 \), has been approximated by \( 1.75 \times 10^{-23} \) ergs \( \text{cm}^3 \text{s}^{-1} \) in their work, while we took a temperature-dependent form that is considerably larger for \( T < 2 \times 10^6 \) K.

The form of the radiative power loss function is much more critical for these calculations, in which the temperatures are relatively low, than in those of Sections 4.1 and 4.2. The high values of the function between \( 10^5 \) and \( 10^6 \) K are
mainly due to line radiation from oxygen and neon ions, and so the abundance of these two elements is of some importance. Thus, differences between the curves of Cox and Tucker (1969), the work we have used, and those of Raymond et al. (1976) at $T = 6 \times 10^5$ K are attributable to a factor-of-six difference in the neon abundance. Again, cooling by ions, neglected by Cox and Tucker, is important between 1 and $2 \times 10^6$ K. Our calculations suggest that, if the radiative loss function took lower values than were assumed here for $T > 10^6$ K, and higher values at lower temperatures, a height-time relation like that observed may be obtained. For, unless the density were very high, the plasma would cool from 5 to $1 \times 10^6$ K principally by conduction, but would rapidly cool from $1 \times 10^6$ K by radiation; the total cooling time would then be of order $\tau_{\text{cond}}$, and so a height-squared relation would result.

5. Summary and Conclusions

We have described models for the source of X-rays during a loop-prominence system on May 13, 1971. We investigated models consisting of one or more flux tubes containing plasma heated impulsively at the flare's flash phase; the plasma is assumed to be conductively connected to the chromosphere, so that cooling proceeds by radiation and by conduction along field lines. Single-loop models were found to be inadequate if the assumption that the plasma receives energy only at the flash phase is to be retained: the calculated line fluxes always show a time behaviour that is quite different from that observed. A model consisting of two loops, each of semi-length $2 \times 10^{10}$ cm, can match to within the measurement uncertainties (about a factor of two) both the absolute fluxes of the Mg xii and Si xiv lines and the observed time behaviour of all lines except Mg xi. The Mg xi line can be reproduced by adding a third, short loop. The two-loop model is not unique; but some constraint on the loop's density and length is provided by the known absolute fluxes of the Mg xi, Mg xii and Si xiv lines.

The calculated thermal radio emission agrees to within a factor of two with that observed for a weak burst at the time of the flash phase.

The loops in the two-loop model are much larger than the observed Hα loops, but other loops with dimensions down to those of the Hα prominences could be present; the loops could be as hot as $\sim 5 \times 10^6$ K without their adding to the emission of the X-ray lines considered here. We investigated whether the Hα prominences could condense from these nested loops as they cool from $5 \times 10^6$ K to Hα-emitting temperatures, viz. $\sim 15 000$ K. The radiative power loss function we used is such that radiative cooling at these temperatures is never small compared with conductive cooling. We calculate that the Hα loops would form at successively higher altitudes, thus giving the climbing behaviour of loop prominences, but that, contrary to what is observed, the rate of climbing increases with time. Thus, either the loop model considered here for the X-ray source is not applicable to the Hα loops, or significant errors remain in the published values of the radiative power loss function.
Our main conclusion from this study is that, for this particular event, it is possible to explain the X-ray emission as coming from two large loops above the Hα prominences that are heated impulsively at the flash phase; that is, a continuing input of energy is not required.

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